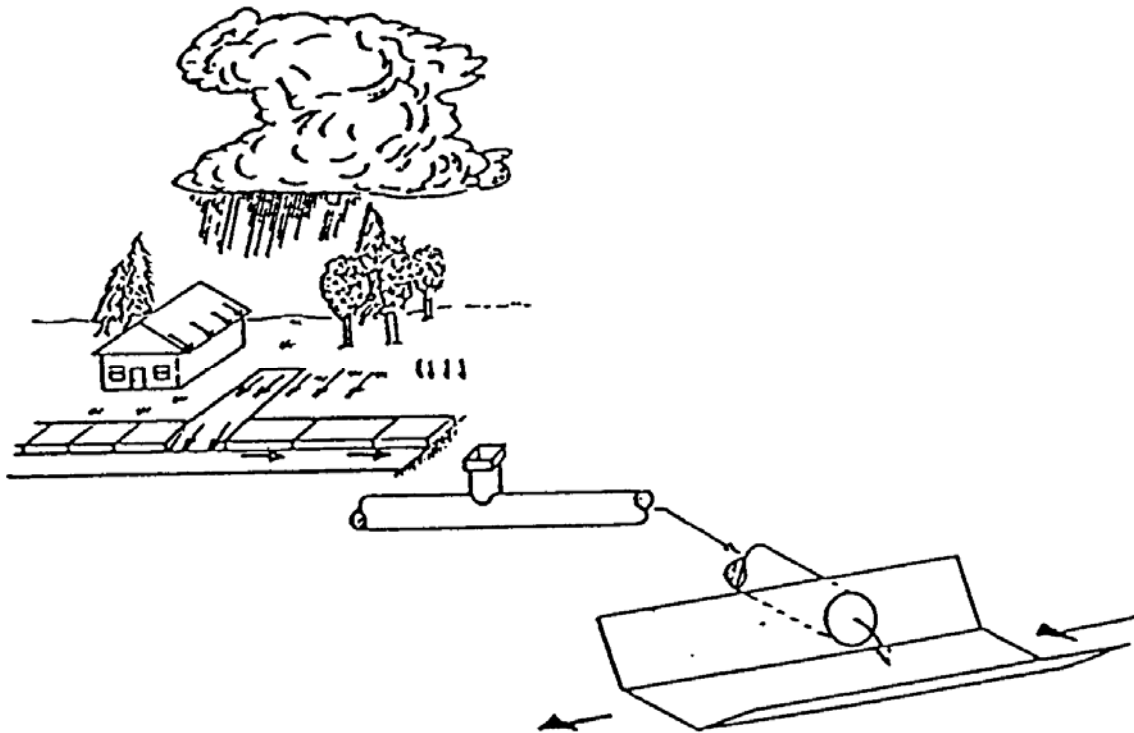




**US Army Corps
of Engineers**
Hydrologic Engineering Center

Introduction and Application of Kinematic Wave Routing Techniques Using HEC-1



July 1993

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

The public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to the Department of Defense, Executive Services and Communications Directorate (0704-0188). Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number.

PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ORGANIZATION.

1. REPORT DATE (DD-MM-YYYY) July 1993		2. REPORT TYPE Training Document		3. DATES COVERED (From - To)	
4. TITLE AND SUBTITLE Introduction and Application of Kinematic Wave Routing Techniques Using HEC-1				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Robert MacArthur, Johannes J. DeVries				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) US Army Corps of Engineers Institute for Water Resources Hydrologic Engineering Center (HEC) 609 Second Street Davis, CA 95616-4687				8. PERFORMING ORGANIZATION REPORT NUMBER TD-10	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/ MONITOR'S ACRONYM(S)	
				11. SPONSOR/ MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES .					
14. ABSTRACT This document discusses the application of the kinematic wave routing method in the flood hydrograph package (HEC-1) for analyzing urban runoff processes. The material is presented in two chapters and a comprehensive example. Chapter 1 presents introductory material necessary for an understanding of the theory, assumptions, equations and numerical methods incorporated into HEC-1 for kinematic wave flood routing. The physical processes of the urban runoff and Streamflow routing are discussed briefly and related to the kinematic wave capabilities in HEC-1. Chapter 2 explains methods of applying kinematic wave routing techniques using HEC-1. Data requirements along with specific methods of applying kinematic wave routing techniques to runoff problems in urban hydrology are discussed. The chapters have been prepared so the user can either read one or both. A user interested only in the theory, or only in application procedures, can read the appropriate chapter.					
15. SUBJECT TERMS Precipitation-runoff model, kinematic wave routing					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES 68	19a. NAME OF RESPONSIBLE PERSON
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER

Introduction and Application of Kinematic Wave Routing Techniques Using HEC-1

July 1973

US Army Corps of Engineers
Institute for Water Resources
Hydrologic Engineering Center
609 Second Street
Davis, CA 95616

(530) 756-1104
(530) 756-8250 FAX
www.hec.usace.army.mil

TD-10

Introduction and Application of Kinematic Wave Routing Techniques Using HEC-1

Table of Contents

Chapter	Page
Foreword	v
Notation	vii
1 An Introduction to Kinematic Flow Approximations	
1.1 Introduction	1
1.2 Modeling Unsteady Flow Using the Kinematic Wave Approach	1
1.3 Review of the Basic Types of Flow	3
1.3.1 Steady versus Unsteady Flow	4
1.3.2 Uniform Versus Varied Flow	4
1.4 Development of Governing Equations	4
1.4.1 St. Venant Equations	4
1.4.2 General Assumptions	5
1.5 Discussion of the General Concepts and Structure of the Kinematic Wave Flood Routing Technique	6
1.6 The Kinematic Wave Form of the Momentum Equation is a Simple Stage Discharge Relationship	8
1.7 Development of α and m for Various Cross Section Shapes	10
1.7.1 Overland Flow Relationship	11
1.7.2 Collector and Main Channel Routing Relationships	12
1.7.3 Determination of α_c and m_c for Collectors and Streams	13
1.7.4 Triangular Sections	14
1.7.5 Rectangular Sections	15
1.7.6 Trapezoidal Sections	16
1.7.7 Circular Sections	16
1.8 Numerical Solution of the Kinematic Wave Equations	16
1.9 Finite Difference Solutions of the Kinematic Wave Equations	17
1.10 Standard Form of the Kinematic Wave Equations	19
1.11 Conservation Form of the Equation	21
1.12 Accuracy of the Finite Difference Solution	22
1.13 References	23
2 Application of Kinematic Wave Routing Techniques Using HEC-1	
2.1 Introduction	25
2.2 Basin Modeling	26
2.3 Elements Used in Kinematic Wave Calculations	26
2.4 Overland Flow Elements	27
2.5 Loss Rate Selection	27
2.6 Correction for Unconnected Impervious	30
2.7 Selection of Overland Flow Parameters	31
2.8 Collector Channel	33
2.9 Selection of Collector Channel Parameters	34

Table of Contents (continued)

Chapter	Page
2.10 Main Channel	34
2.11 Selection of Main Channel Parameters	36
2.12 Selection of Channel Cross Section	36
2.13 Summary and Conclusions.....	37
2.14 References	37
Appendix	
A An Example of Kinematic Wave Methods.....	39
B Sample Data Tabulation Form.....	53

List of Figures

Figure Number	Page
1 Visualization of Dynamic and Kinematic Waves	2
2 Possible Types of Open Channel Flow [Thomas, 1975]	3
3 Elements Used in Kinematic Wave Calculations	7
4 Relationships Between Flow Elements	7
5 Typical Urban Drainage Pattern	9
6 The Rising Hydrograph - Variation with k , for $F=1$ [Liggett/Woolhiser, 1967]	10
7 Two Basic Channel Shapes and Their Variations Used by the HEC-1 Program for Kinematic Wave Stream Routing	14
8 Characteristic Curves on a Fixed $\Delta x - \Delta t$ Grid.....	18
9 Space-Time Grid Used for Finite Difference Method	20
10 Graph for Determining Composite Curve Number	30
11 Discharge Versus Flow Area for Various Cross Sections	35
12 Fitting Channel Cross Sectional Shape	37
A-1 Basin for Example Problem.....	39
A-2 HEC-1 Input File Using Kinematic Wave Method.....	40
A-3 Computed Hydrograph at Outlet of Subbasin 1 and at Outlet of Entire Basin	41
A-4 HEC-1 Output File for Example Problem	42
A-5 Computed Hydrographs at Outlet with, and without Urbanization.....	49
A-6 Graphical Representation of Sensitivity to Changes in Surface Roughness	50
A-7 Graphical Representation of Sensitivity to Changes in Overland Flow Length.....	51

List of Tables

Table Number		Page
1	Effective Roughness Parameters for Overland Flow	12
2	Parameters for Kinematic Wave Example Problem.....	28
3	Runoff Curve Numbers for Selected Agricultural, Suburban and Urban Land Use	29
4	Effective Resistance Parameters for Overland Flow	32
A-1	Parameters for Kinematic Wave Example Problem.....	40
A-2	Sensitivity to Changes in Surface Roughness.....	50
A-3	Sensitivity to Changes in Overland Flow Length	51
A-4	Sensitivity to Changes in NDXMIN for Subbasin 1	52

Foreword

This document discusses the application of the kinematic wave routing method in the Flood Hydrograph Package (HEC-1) for analyzing urban runoff processes. The material is presented in two chapters and a comprehensive example. Chapter 1 presents introductory material necessary for an understanding of the theory, assumptions, equations and numerical methods incorporated into HEC-1 for kinematic wave flood routing. The physical processes of the urban runoff and streamflow routing are discussed briefly and related to the kinematic wave capabilities in HEC-1.

Chapter 2 explains methods of applying kinematic wave routing techniques using HEC-1. Data requirements along with specific methods of applying kinematic wave routing techniques to runoff problems in urban hydrology are discussed. The chapters have been prepared so the user can either read one or both. A user interested only in the theory, or only in application procedures, can read the appropriate chapter.

The basic approach for the subdivision of the basin into the various elements listed here was developed and coded through discussions with John Peters, Art Pabst, and Paul Ely of the Hydrologic Engineering Center (HEC). The original development and programming of the kinematic wave routing routines in computer program HEC-1 was done under contract by Resource Analysis, Incorporated, under the direction of Dr. Brendan Harley. Subsequently, David Goldman modified the computational scheme so that the solution of the kinematic wave equations is more stable and provides the user with information on the accuracy of the solution.

Robert MacArthur originally wrote Chapter 1, and Johannes J. DeVries originally wrote Chapter 2. Paul Detjens modified Chapters 1 and 2 to reflect the changes made to the program. He also created a new example, Appendix A, which demonstrates the use of the kinematic wave option. Troy Nicolini was instrumental in organizing this version of TD-10.

Notation

Term	Definition	Dimensions
A	cross sectional area of flow (Equation 7)	ft ²
A _c	cross sectional area of flow for collector channel elements (Equations 14 and Equation 24)	ft ²
A _m	cross sectional area of flow for main channel and stream elements	ft ²
A _{BASIN}	total surface area of the basin; $A_{BASIN} = \sum A_{subbasin}$	mi ²
A _{subbasin}	surface area of a subbasin	mi ²
A _p	percentage of the subbasin area that would be pervious	%
A _i	percentage of the subbasin area that would be impervious	%
c	shallow wave celerity, $c = \sqrt{gD}$	ft/sec
D	hydraulic depth of channel	ft
D _c	diameter of circular collector channel (Figure 7)	ft
f	infiltration rate (Equations 1, 11 and 12)	cfs/ft ²
F	Froude number = $\frac{V}{\sqrt{gD}}$ (Equation 6)	N-D*
g	acceleration of gravity (Equations 2 and 4)	ft/sec ²
i	rainfall intensity (Equations 1, 11 and 12); when used as a (subscript) in Figure 9, i indicates spatial location on a finite difference grid	N-D*
j	when used as a subscript in Figure 9, j indicates temporal location on a finite difference grid	N-D*
k	dimensionless kinematic flow number (Equation 6)	N-D*
L	channel length, length of overland flow plane (Equation 6); when used as a subscript, L indicates lateral inflow (Equations 1, 2 and 4)	ft N-D*

*N-D - Non-dimensional

Term	Definition	Dimensions
L_o	length of overland flow element (Figure 3)	ft
L_c	length of collector element (Figures 3 and 4)	ft
L_m	length of main channel element (Figures 3 and 4)	ft
L_{o_1}	length of Type 1 overland flow element (Figure 4)	ft
L_{o_2}	length of Type 2 overland flow element (Figure 4)	ft
m	when used as a subscript, m indicates properties pertaining to main channel elements (Figures 3 and 4)	N-D*
m	kinematic routing coefficient (Equation 3)	N-D*
m_o	kinematic routing coefficient for overland flow elements (Equations 10 and 12)	N-D*
m_c	kinematic routing coefficient for collector channel elements (Equations 14, 19, 20, 21, 22, 23 and 24)	N-D*
n	Manning's resistance coefficient (Equation 7)	sec/ft ^{1/3}
N	effective roughness parameter for overland flow (Equation 8 and Table 1)	sec/ft ^{1/3}
o	when used as a subscript, o indicates properties pertaining to overland flow elements	N-D*
P	wetted perimeter	ft
q	discharge per unit width of channel (Equations 1 and 9)	cfs/ft
q_c	discharge per unit width of collector channel (Figures 3 and 4)	cfs/ft
q_o	discharge per unit width of overland flow element (Figures 3 and 4)	cfs/ft
q_L	total lateral inflow per unit length of channel	cfs/ft ²
q_{o_1}	discharge per unit width of overland flow strip from Type 1 flow elements (Figure 4)	cfs/ft
q_{o_2}	discharge per unit width of overland flow strip from Type 2 flow elements (Figure 4)	cfs/ft

*N-D - Non-dimensional

Term	Definition	Dimensions
Q	discharge (Equations 3, 4, 7 and 8)	cfs
Q _{OUT}	total discharge from a subbasin (Figures 3 and 4)	cfs
Q*	dimensionless discharge (Figure 6)	N-D*
R	hydraulic radius - A/P (Equation 7)	ft
S _f	friction slope defined by Manning's equation (Equation 1)	N-D*
S _o	average bottom slope (Equations 2, 4, 6 and 7, and Figure 3)	N-D*
S _c	average bottom slope for main collector channel elements (Equations 15 and 19, and Figure 3)	N-D*
S _m	average bottom slope for main channel elements (Figure 3)	N-D*
t	time (Figures 8 and 9, and Equations 1, 2, 11 and 13)	sec
t*	dimensionless time (Figure 6)	N-D*
Δt	time step used in finite difference equations (Equations 29, 31 and 32, and Figures 8 and 9)	sec
u	x-component of mean velocity (Equations 2 and 4)	ft/sec
v	x-component of velocity from lateral inflow into a channel (assumed negligible) (Equation 2)	ft/sec
V	mean cross section velocity = Q/A	ft/sec
w	bottom width of typical collector or main channel (Figure 7)	ft
x	longitudinal distance (Equations 1 and 2), spatial direction (Figures 8 and 9)	ft
Δx	spatial stepping distance used in finite difference equations (Equations 25 and 27, and Figures 8 and 9)	ft
y	mean depth in St. Venant equation (Equations 1, 2 and 4)	ft
y _c	mean depth of flow in collector channel elements (Equations 16 and 17, and Figure 7)	ft
z	side slope for generalized trapezoidal cross section used for collector or main channel routing (Equations 16, 17, 18, 19 and 22, and Figure 7)	N-D*

*N-D - Non-dimensional

α kinematic wave routing parameter for a particular cross sectional shape, slope and roughness (Equations 3, 10, 14, 19, 20, 21, 22 and 23) $(\text{ft}^2/\text{sec})/\text{ft}^m$

Chapter 1

An Introduction To Kinematic Flow Approximations

1.1 Introduction

Rapid urbanization of river basins in and around metropolitan areas has forced land and water resources planners and hydrologists to develop a variety of methods for analyzing problems in urban hydrology. Problems involving both design and management decisions are often so complex as to require application of mathematical models. The mathematical models which have been most commonly used rely on basic unit-graph techniques to model the application and distribution of precipitation as rainfall or snowmelt, compute rainfall and snowmelt losses and excesses, and determine subbasin outflow hydrographs. Although these models which are based on the development of a representative unit hydrograph are frequently applied and have been used successfully, it is difficult to associate physical properties of the basin to be modeled to the parameters necessary to develop a unit hydrograph. It is even more difficult to define some of the parameters such as the Clark T_c and R or Snyder C_t and C_p for basins that have no recorded data. Because it is very important to develop the best representation of the actual urban runoff situation when analyzing urban storm water runoff problems, it is desirable to relate runoff processes directly to measurable geographic features of the basin. It is also desirable for the modeling technique to be able to reproduce nonlinear runoff characteristics rather than being limited to linear responses such as those developed by unit graph techniques.

The kinematic wave method of routing overland and river flows was chosen as an additional routing option for use in the HEC-1 program for several reasons. First, although simple in form, kinematic wave theory offers the benefits of nonlinear response without needing an unduly complicated or costly solution procedure. Second, for the purposes of modeling unsteady overland flow, any model will require considerable parameter adjustment to account for the complexities of the basin and the specific flows that occur within the basin. The kinematic wave method relates basin and flow characteristics directly to the two routing parameters, α and m . The parameters α and m are directly related to the shape of the channel, the boundary roughness and the slope of the channel or overland flow surface. There also have been several previous studies that have developed sets of appropriate values for these parameters for a large range of flow and boundary conditions. Third, numerical techniques used to simulate overland and river flows can only approximate the actual response of real systems because of the complex nature of natural drainage basins and because simplifications must be made to the mathematics to make the model efficient and economical to execute. The kinematic wave approximation has been proven to be an accurate and efficient method of simulating storm water runoff from small basins for both overland flow and stream channel routing [Overton and Meadows, 1976].

1.2 Modeling Unsteady Flow Using the Kinematic Wave Approach

Kinematics is defined as the study of motion exclusive of the influences of mass and force, in contrast with dynamics, in which these influences are included. Flood waves can be

identified as either of two separate kinds of wave phenomena: the dynamic wave and the kinematic wave. Although both of these kinds of waves are initially present, certain characteristics of a watershed can make kinematic waves the dominant characteristic of a flood event.

When inertial and pressure forces are important, "dynamic waves" govern the movement of long waves in shallow water, like a large flood wave in a wide river [Stoker, 1957]. When the inertial and pressure forces are not important to the movement of the wave, "kinematic waves" govern the flow. For this latter flow condition, the weight component (the force in the direction of the channel axis due to the weight of the fluid flowing downhill in response to the action of gravity) is approximately balanced by the resistive forces due to channel bed friction (in most cases this is represented by Manning's equation). Flows of this nature (kinematic waves) will not be accelerating appreciably and the flow will remain approximately uniform along the channel. No visible surface wave will be noticeable and the passage of the flood wave, as depicted in Figure 1, will be seen by an observer on the bank as an apparently uniform rise and fall in the water surface elevation over a relatively long period of time with respect to the size of the specific subbasin being analyzed. Therefore, kinematic flows are often classified as uniform unsteady flows.

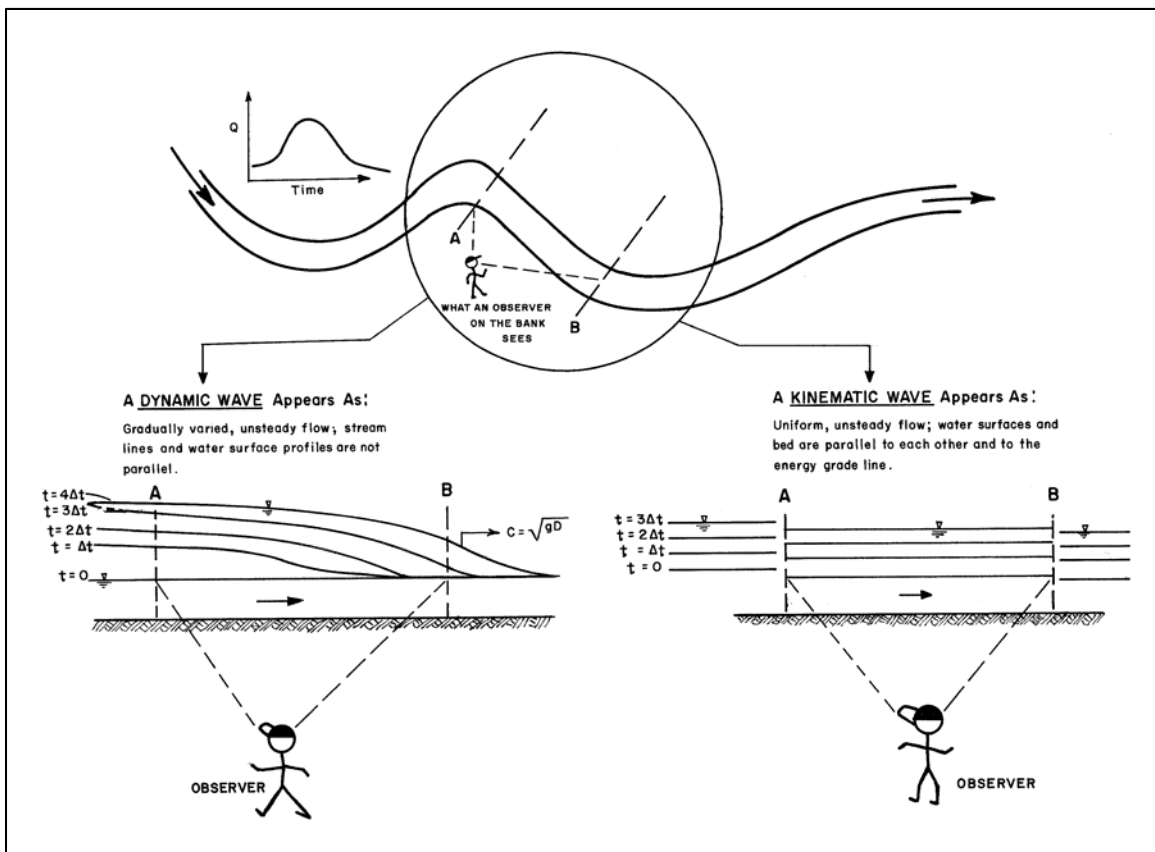


Figure 1 Visualization of Dynamic and Kinematic Waves

Dynamic waves [Lighthill/Whitham, 1995] normally have much higher velocities and attenuate more quickly than kinematic waves; flood flows are generally dominated by kinematic waves. Even though any surface disturbance will send a "signal" downstream at the speed of small gravity waves, this "signal" will be too weak to be detected at any considerable distance

downstream. Therefore, a main flow change or "signal" is carried as a kinematic wave at much slower velocities and the speed of flood waves may be approximated by the speed of kinematic waves. In this context, a kinematic wave represents the characteristic changes in discharge, velocity and water surface elevation with time as observed at any one location on an overland flow plane or along a stream channel.

The speed of small gravity waves occurring in shallow open channels is often referred to as wave celerity, c (see page vi), where the variable D is the channel hydraulic depth [Chow, 1959]. The ratio of the flow velocity, $V=Q/A$, to the celerity is called the Froude number, F . Therefore, the Froude number also represents the ratio of inertial forces to gravity forces.

Flows with Froude numbers greater than one are classified as supercritical flows [Chow, 1959], and surface waves are unable to move in the upstream direction because the flow velocity is greater than the wave celerity, c . Flows with Froude numbers greater than two tend to be unstable, which may affect the accuracy and applicability of steady flow assumptions to these flows. However, the characteristics of kinematic waves [Lighthill/Whitham, 1955] dominate over those of dynamic waves for flows with Froude numbers that are less than or equal to two (e.g., $F \leq 2$). In fact, they found that for $F < 1$, dynamic waves decay exponentially with respect to a time constant they chose to define as $V/[gS(1 - F^2/2)]$, where, S , is the channel or surface runoff slope. Therefore, one may conclude that kinematic waves will ultimately dominate the flow characteristics occurring for overland flows and small watershed channel flows when the flow Froude number is less than two. This brief introduction to kinematic wave approximations leads to the following discussion and definition of uniform unsteady flows.

1.3 Review of the Basic Types of Flow

As a brief review, several basic types of open channel flow that are most commonly experienced will be discussed to remind the reader of some of the important differences and characteristics of flows found in practice [Thomas, 1975]. Figure 2 presents several possible types of open channel flow.

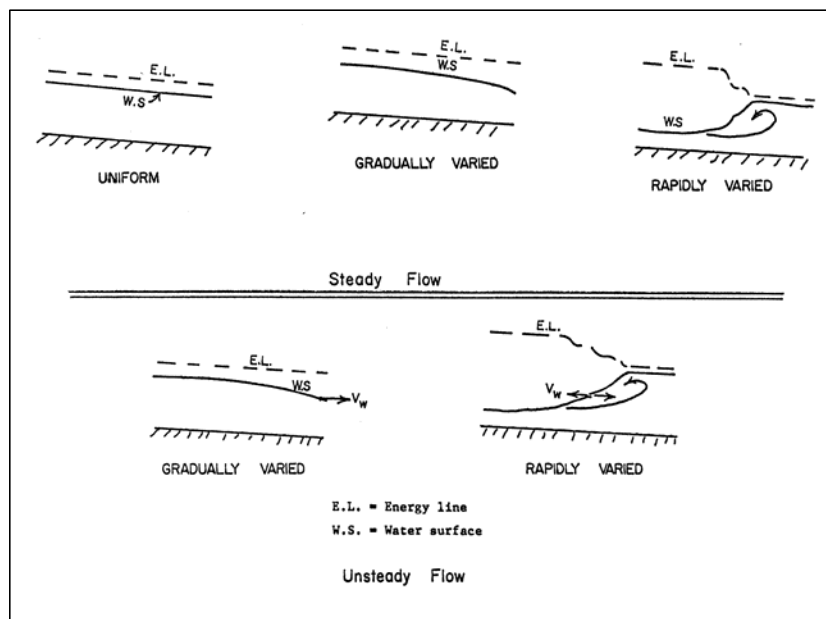


Figure 2 Possible Types of Open Channel Flow [Thomas, 1975]

1.3.1 Steady versus Unsteady Flow

If the change in velocity with respect to time at a given location is zero, the flow is called **steady flow**. Otherwise, the flow is classified as **unsteady flow**. Therefore, unsteady flows require the consideration of time as an additional variable.

1.3.2 Uniform versus Varied Flow

If the change in channel flow velocity, V , with respect to distance along the channel, x , is zero (i.e., $dV/dx = 0$) for a given period of time, the flow is called **uniform**. Otherwise, the flow is **nonuniform** and the relationship between kinetic energy and potential energy will be changing along the channel. When the flow is uniform, the water surface will be parallel to the channel bottom. If the flow is not uniform, the slope of the water surface will be slightly different than the slope of the channel bottom.

If the rate of change of the water surface slope is not visible to the eye, the flow may be considered as **gradually varied**. **Rapidly varied** flows demonstrate apparent and rather large water surface slope changes (such as at a hydraulic jump). Rapidly varied flow requires special treatment and will not be considered herein.

1.4 Development of Governing Equations

Although the basic differential equations capable of describing one-dimensional gradually varied unsteady flow were originally developed a century ago, they have only been recently applied (within the last thirty to forty years) to general hydrologic engineering problems because it was not possible to solve these equations efficiently without a high-speed computer.

The mechanics of unsteady open channel flow may be expressed mathematically in terms of the equations developed in 1870 by St. Venant. Equations 1 and 2 are partial differential equations that may be derived from the basic principles of conservation of mass and momentum. Various derivations of these equations have been presented [Chow 1959], [Henderson 1966], [Strelkoff 1969], [Fread 1976].

1.4.1 St. Venant Equations

$$\text{Continuity} \quad \frac{\partial y}{\partial t} + \frac{\partial q}{\partial x} = q_L + (i - f) \quad (\text{channels of unit width}) \quad (1)$$

$$\text{Momentum} \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial y}{\partial x} = g (S_o - S_f) - q_L \frac{(u - v)}{y} \quad (2)$$

where: g = acceleration of gravity (ft/sec²)
 y = mean depth (ft)
 q = discharge per unit width of channel (cfs/ft)
 x = distance measured in downstream flow direction (ft)
 t = time (seconds)

- u = x-component of mean velocity (ft/sec)
- i = rainfall intensity (cfs/ft²)
- f = infiltration rate (cfs/ft²)
- S_o = average bottom slope (ft/ft)
- S_f = friction slope defined by the Manning equation
- q_L = total lateral inflow per unit length of channel (cfs/ft²)
- v = the x-component of velocity for lateral inflow. (This is assumed to be negligible to the total momentum balance for channel routing and is therefore zero).

The four terms in Equation 1 and the five terms in Equation 2 are known successively as:

Terms in the Continuity Equation (Equation 1)

- rate of rise term
- storage term
- lateral inflow per unit length
- intensity of excess rainfall

Terms in the Momentum Equation (Equation 2)

- acceleration term
- velocity head term
- depth taper term
- bed slope minus friction term
- lateral inflow term

Prior to presenting the details of the kinematic flow method that have been incorporated into the HEC-1 program, it is important to discuss the basic assumptions and requirements associated with the equations used for gradually varied unsteady flows. If these basic assumptions are **not** valid for the intended flow conditions, then alternate methods of simulating the flow should be sought. (For further definitions of terms, refer to the **Notation** section, pages vii - ix.)

1.4.2 General Assumptions

In the development of the general unsteady flow equations it is assumed that the flow is one dimensional in the sense that flow characteristics such as depth and velocity are considered to vary only in the longitudinal x-direction of the channel. Additional basic assumptions necessary for the validity of the equations include:

- (1) the velocity is constant and the water surface is horizontal across any section perpendicular to the longitudinal flow axis;
- (2) all flows are gradually varied with hydrostatic pressure prevailing at all points in the flow such that all vertical accelerations within the water column can be neglected;
- (3) the longitudinal axis of the flow channel can be approximated by a straight line, therefore, no lateral secondary circulations occur;

- (4) the slope of the channel bottom is small (less than 1:10);
- (5) the channel boundaries may be treated as fixed noneroding and nonaggrading;
- (6) resistance to flow may be described by empirical resistance equations such as the Manning or Chezy equations;
- (7) the flow is incompressible and homogeneous in density.

The assumptions found in items 1 through 7 have been shown [Strelkoff, 1969] to be valid and applicable for most open channel flows occurring in natural rivers and streams. The validity of the restrictions presented in assumption 5, are not easily evaluated. Research is currently under way to estimate the overall affects of this assumption. It has also been found that overland flows such as those associated with storm water runoff can be described by the kinematic wave form of Equations 1 and 2 without violation of the seven previous assumptions [Lighthill/Whitham, 1955] [Liggett/Woolhiser, 1967]. If lateral flows are much more significant than the main channel flow then these local effects may require special attention and changes to methods currently used in HEC-1.

1.5 Discussion of the General Concepts and Structure of the Kinematic Wave Flood Routing Technique

A general conceptual description of the methods of modeling storm water runoff using kinematic wave simplifications will be presented here prior to discussing the specific equations and algorithms that are used. This will help the reader understand how the model "visualizes" the actual physical characteristics of the basin and how it responds to rainfall. Specific mathematical relationships and numerical solution techniques will be presented in a following section.

Analyses of surface runoff problems are complicated not only by the nature of specific storm events that occur but perhaps to a greater extent, by the nature and complexity of the watershed or urban area being analyzed. Description of the local physical characteristics, geometry, and response of the system could become a monumental task if one were to include every minute detail. The concept incorporated into the HEC-1 program involves simulating the natural complexities of a basin with a number of simple elements, such as overland flow planes, stream segments, and lengths of representative storm drain or sewer pipe as shown in Figures 3 and 4. If the previously listed gradually varied unsteady flow assumptions are not violated, then combinations of these basic elements have proven to be quite representative of the actual behavior of most systems. Measured responses [Wooding, 1966] from three natural drainage areas were compared with calculated results obtained from a model that used a simple kinematic flow routing procedure similar to that found in HEC-1. Conclusions were that although the geometry of natural catchments is far more complicated than that of the simple model, the agreement between computed and actual discharge hydrographs is quite good.

To simulate the response of a complex watershed to precipitation from storm events a mathematical model made up of combinations of simple geometric components is constructed. Successful application of this approach begins with the description of unsteady, uniform flow over an idealized planar overland flow element of unit breadth (called an overland flow strip) for

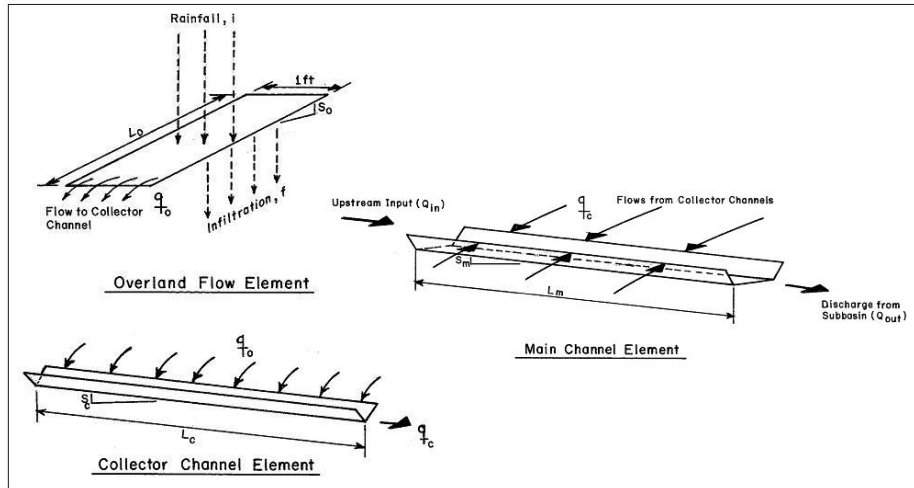


Figure 3 Elements Used in Kinematic Wave Calculations

a number of given boundary conditions. A boundary condition represents known or assumed flow conditions that are specified by the user. Then, similar relationships for routing channel flows resulting from runoff from overland flow elements are developed. Once these relationships have been developed, combinations of simple elements can be made to describe basin and subbasin responses to storm events. Figure 4 summarizes the relationship between the three different types of elements.

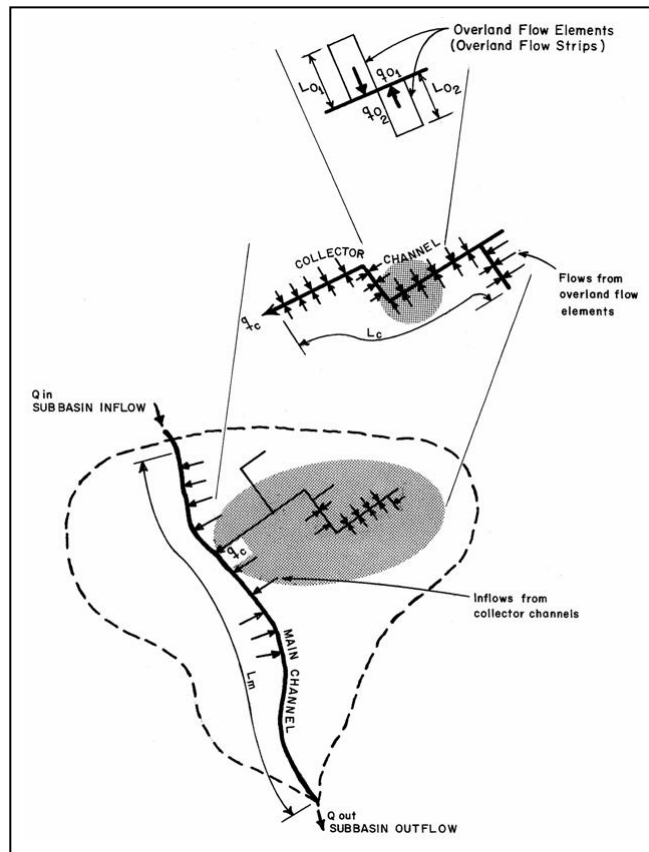


Figure 4 Relationships Between Flow Elements

Notice that overland flows are handled separately from the channel flow, because overland runoff demonstrates specific shallow flow properties which lead to a form of the equations of motion that differ from the form required for collector channel and main channel flows. These flows are calculated individually and then combined properly to preserve continuity and accuracy. This computational method has been shown to be quite efficient [Harley, et. al., 1972].

Figures 3 and 4 present a schematic representation of this approach. The governing equations used for this combined overland flow and river channel routing procedure are derived from the general St. Venant equations for unsteady gradually varied open channel flow.

For the overland flow portion of the model, shallow surface water runoff assumptions are applied to Equations 1 and 2, resulting in a simple kinematic wave form of the equations. Similar simplifications are found to be valid and useful for the channel routing portions as well. Attempting the rigorous mathematical representation of these complex phenomena (runoff and channel routing through the natural topography) would require exceedingly small spatial and temporal detail and result in a very large system of simultaneous equations. The kinematic wave form of the St. Venant equations provides a simplified description of the physical system in terms of surfaces and channels with homogeneous properties. The important concept of the overland portion of the model is that water is distributed over a wide area and at very shallow average depths until it reaches a well-defined collector channel. In urban areas, two general types of overland flow surface are usually present: pervious and impervious. The mechanics of flow over both kinds of surfaces are similar; however, the slopes, flow lengths, roughnesses, and loss rates will differ. Rooftops, parking lots, and paved surfaces such as streets, are described as impervious areas. Lawns, fields, parks, etc., are pervious areas. The user stipulates the percentages of the total subbasin area that are impervious and pervious. The model develops the runoff flows from the rainfall intensity and loss rates specified for the pervious and impervious areas in the basin. After the overland runoff is routed down the length of the overland flow strip it is then distributed uniformly along the collector system, which represents rivulets, channels, gutters, sewers, and storm drains such as shown in Figures 3 and 5. Once the runoff flows enter the collector system (Figure 5) they move through it picking up additional (uniformly distributed) lateral inflow from adjacent runoff strips. These collector flows eventually reach a main channel where they are then routed as open channel stream flows through the system.

The next sections will review the basic concepts associated with the kinematic wave simplification of the momentum equation and present some criteria [Lighthill/Whitham, 1955] that relates to its validity for various flow conditions. Following sections will then present the details of the three different segments of flow routing (overland, collector and stream routing) and how they can be applied to problems in urban hydrology.

1.6 The Kinematic Wave Form of the Momentum Equation is a Simple Stage-Discharge Relationship

Recall that kinematic waves occur when the dynamic terms in the momentum equations are negligible. This allows one to assume that the bed slope is approximately equal to the friction slope ($S_o = S_f$). Under these conditions and if there is no appreciable backwater effect, the discharge can be described as a function of depth of flow only, for all x and t .

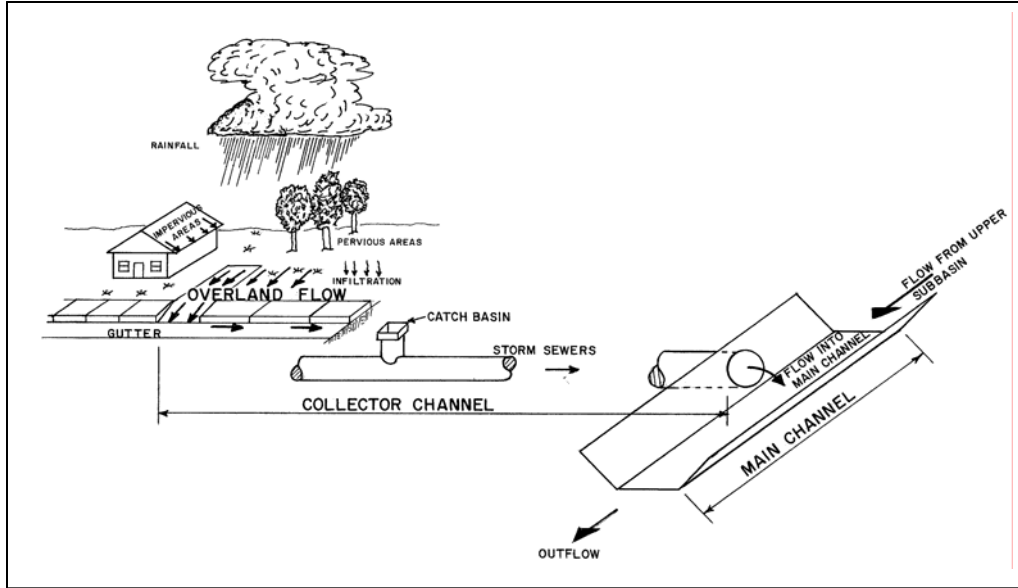


Figure 5 Typical Urban Drainage Pattern

$$Q = \alpha y^m \quad (3)$$

Q is discharge in cfs and α and m are kinematic wave routing parameters which are directly related to the basin and flow characteristics. This can be best appreciated by using an approach of normalizing the momentum equation with a steady uniform discharge called Q_n [Henderson, 1966]. Rearrangement of the normalized form of Equation 2 yields:

$$Q = Q_n \left[1 - \frac{1}{S_o} \left(\frac{\partial y}{\partial x} + \frac{u}{g} \frac{\partial u}{\partial x} + \frac{1}{g} \frac{\partial u}{\partial t} + \frac{q_L u}{gy} \right) \right]^{-\frac{1}{2}} \quad (4)$$

If the sum of the terms to the right of the minus sign is much less than one (i.e., pressure, inertia and local inflow are relatively small compared to S_o), then unsteady flows are nearly uniform and may be approximated by a series of normal flows, e.g.,

$$Q \approx Q_n \quad (5)$$

Normal flows of this nature can be described by a depth-discharge relationship, such as Equation 3 [Overton/Meadows, 1976]. This describes kinematic flow and provides a simple method for calculating flows from storm water runoff.

For Froude numbers less than two [Lighthill/Whitman, 1955], the dynamic component decays exponentially and the kinematic wave ultimately dominates. As was mentioned in the introduction, this means that no visible surface wave is observed; only the rise and fall of the water surface can be seen Figure 1 (page 2). A study [Liggett/Woolhiser, 1967] on the characteristics of a rising hydrograph for a large variety of flow conditions has shown that the dynamic component in Equation 4 will be dampened enough to be neglected, provided that:

$$k = \frac{S_o L}{y F^2} > 10 \quad (6)$$

where: $F = \frac{V}{\sqrt{gY}}$

y = mean depth

S_o = bed slope

k = dimensionless kinematic flow number

L = the length of the plane

Practical evaluation of k is difficult because it may be hard to estimate L, y or F precisely for natural flow conditions without collecting field measurements. Results from the study [Liggett/Woolhiser, 1967] allow Equation 2 (page 4) to be greatly simplified. These results are summarized in Figure 6 (where Q* and t* are dimensionless discharge; Q* = (q_L L/Vg) and time t* = (tV/L), respectively). For a value of k of 10, an approximate ten percent error in the calculated discharge hydrograph would result by deleting the dynamic terms from the momentum equation. Notice that as k increases above ten, however, that the error in discharge decreases rapidly. A "true kinematic" solution results as k approaches infinity, but for engineering purposes flows characterized by k > 10 can be approximated reasonably well with the kinematic wave form of the momentum equation, e.g., Equation 3. Continuing research will attempt to develop practical methods of evaluating k and, therefore, the applicability of kinematic wave simplifications from typical basin data and flow conditions.

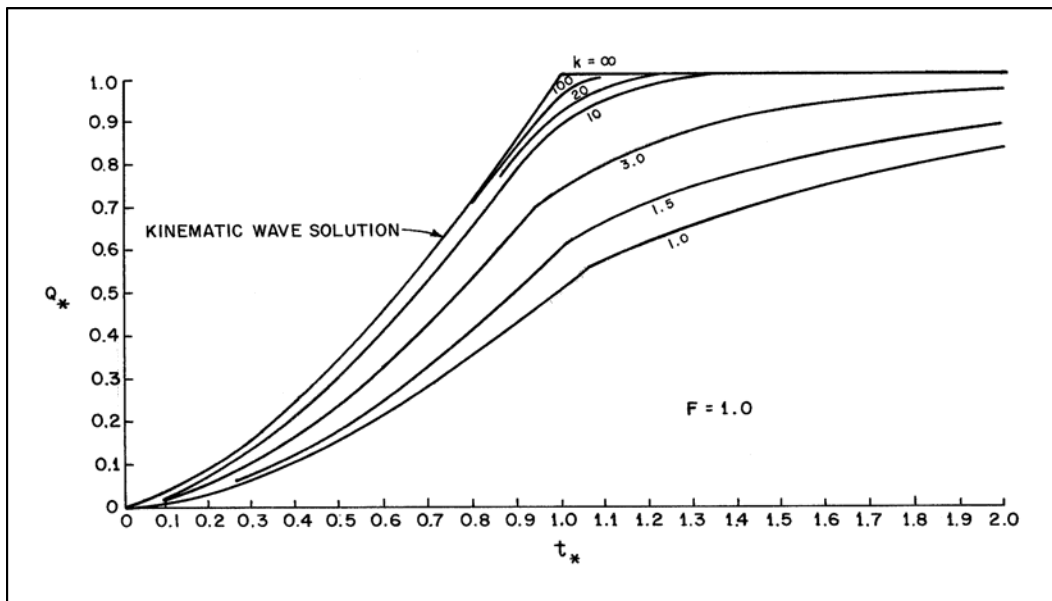


Figure 6 The Rising Hydrograph - Variation with k, for F=1 [Liggett/Woolhiser, 1967]

Derivation of the relationships used for overland flow, collectors and main channels will be presented next.

1.7 Development of α and m for Various Cross Section Shapes

The following sections will present example derivations of α 's and m's for the different cross-sectional shapes considered by HEC-1.

1.7.1 Overland Flow Relationship

The kinematic wave equation for an overland flow segment on a wide plane with very shallow flows can be derived from Manning's equation and Equation 3. Consider the flow down an overland flow strip of unit breadth as shown in Figure 3 (page 6). The steady discharge from a unit strip can be described with Manning's relationship.

$$Q = \frac{1.486}{n} R^{2/3} S_o^{1/2} A \quad (7)$$

where for very shallow flows that are at a depth of y_o , R (the hydraulic radius) and A are simply $(y_o \cdot 1)/1$ and $y_o \cdot 1$ respectively. Substitution of these values for R and A into Equation 7, yields:

$$Q = \frac{1.486}{N} S_o^{1/2} y_o^{5/3} \quad (8)$$

Notice also that Manning's n has been replaced with an appropriate N that describes the properties of the runoff surface being modeled. Values of N are usually greater than Manning's ' n '. Table 1 presents suggested ranges of values for N for various surface conditions. These values were obtained from previous field and laboratory investigations for overland flow.

Because the discharge represented by Equation 8 is per unit breadth one can substitute the previously defined q for Q (see Equation 2 for units, page 4) to obtain discharge in terms of flow per unit breadth.

$$q = \left(\frac{1.486}{N} S_o^{1/2} \right) y_o^{5/3} \quad (9)$$

Rewriting Equation 3 in terms of discharges per unit width where the subscripts "o" indicate variables associated with overland flows.

$$q_o = \alpha_o y_o^{m_o} \quad (10)$$

where: $\alpha_o = \left(\frac{1.486}{N} S_o^{1/2} \right)$
 $m_o = 5/3$
 $S_o =$ average slope of overland flow element
 $y_o =$ mean depth for overland flow
 $\alpha_o =$ conveyance for particular runoff surface, slope, and roughness

Because there are two unknowns in Equation 10 another relationship is required for mathematical closure and a complete solution. A form of the continuity equation (Equation 1, page 4) provides the necessary second equation to complete the solution.

$$\frac{\partial y_o}{\partial t} + \frac{\partial q_o}{\partial x} = (i - f) \quad (11)$$

where: $(i - f) =$ rate of excess rainfall (rainfall-infiltration) in ft/sec

- q_o = discharge per unit width in cfs/ft
 t = time in seconds
 y = mean depth of overland flow in feet

Together Equations 10 and 11 form the complete kinematic wave equations for overland flow. If Equation 10 is substituted into Equation 11 one obtains:

$$\frac{\partial y_o}{\partial t} + \alpha_o m_o y_o^{(m_o-1)} \frac{\partial y_o}{\partial x} = i - f \quad (12)$$

which has only one dependent variable so that it can be solved to give a relationship for y_o in terms of x , t , and the excess rainfall intensity ($i - f$). Once y_o is found, it can be substituted back into Equation 10 to obtain a value for q_o . This procedure provides the necessary information to be able to determine the time dependent discharge from the overland flow elements.

Table 1
Effective Resistance Parameters for Overland Flow

Surface	N value	Source
Asphalt/Concrete*	0.05 - 0.15	a
Bare Packed Soil Free of Stone	0.10	c
Fallow – No Residue	0.008 - 0.012	b
Conventional Tillage – No Residue	0.06 - 0.12	b
Conventional Tillage – With Residue	0.16 - 0.22	b
Chisel Plow – No Residue	0.06 - 0.12	b
Chisel Plow – With Residue	0.10 - 0.16	b
Fall Disking – With Residue	0.30 - 0.50	b
No Till – No Residue	0.04 - 0.10	b
No Till (20-40 percent residue cover)	0.07 - 0.17	b
No Till (60-100 percent residue cover)	0.17 - 0.47	b
Sparse Rangeland with Debris:		
0 Percent Cover	0.09 - 0.34	b
20 Percent Cover	0.05 - 0.25	b
Sparse Vegetation	0.053 - 0.13	f
Short Grass Prarie	0.10 - 0.20	f
Poor Grass Cover on Moderately Rough	0.30	c
Bare Surface		
Light Turf	0.20	a
Average Grass Cover	0.4	c
Dense Turf	0.17 - 0.80	a,c,e,f
Dense Grass	0.17 - 0.30	d
Bermuda Grass	0.30 - 0.48	d
Dense Shrubbery and Forest Litter	0.4	a

Legend: a) Harley (1975), b) Engman (1986), c) Hathaway (1945), d) Palmer (1946), e) Ragan and Duru (1972), f) Woolhiser (1975). [Hjemfelt, 1986]

1.7.2 Collector and Main Channel Routing Relationships

For the collector system (which represents rivulets, storm drains, and sewer pipes) and stream channel segments, simple cross section shapes have been used to simulate prototype channels. It has been found that appropriate usage of simple triangular, trapezoidal, and circular channel shapes can provide an accurate representation of the response of the prototype.

Flows entering the collectors and the stream channels can consist of both flows from upstream sections and lateral inflows from adjacent catchment surfaces. Their slope, length, cross sectional dimensions, shape, and Manning's 'n' value, describes these representative channels. The standard Manning's 'n' is used here because collector and streamflows behave more as normal open channel flows. The basic form of the equations for kinematic wave routing of collector and streamflows are similar to those developed for shallow overland flow (Equations 10 and 11). The kinematic wave equations for collector and streamflow routing are:

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q_c}{\partial x} = q_o \quad (13)$$

$$Q_c = \alpha_c A_c^{m_c} \quad (14)$$

where: A_c = cross sectional area of flow in ft²
 Q_c = discharge in cfs
 q_o = lateral inflow per unit length in cfs/ft from overland flow strips
 t = time in seconds
 x = distance along the stream in feet
 α_c, m_c = kinematic wave parameters for a particular cross sectional shape, slope and roughness

The reader should note that the subscripted variables used above are for a typical collector channel and are indicated as such with the subscript c. Identical relationships would be used for routing in the main channel but one may wish to identify them separately with a subscript such as m to indicate main channel (refer to the Notation section, pages vii - ix).

1.7.3 Determination of α_c and m_c for Collectors and Streams

Values of α_c and m_c will be different for each differently-shaped cross section and will vary with effective Manning's 'n' and channel slope as well. The **basic** channel shapes considered by the HEC-1 program are the trapezoid and circle. Variation of the side slope and bottom width for the trapezoidal section allows one to develop rectangular and triangular channel shapes as well. These shapes are presented in Figure 7.

As was done for the development of the overland flow parameters α_o and m_o , one needs to define the proper values for α_c and m_c for the stream and collector system. Rather than derive all the α_c and m_c values for each differently shaped section, an easy-to-follow development for a simple triangular section will be presented as an example. The results for

the remaining shapes will merely be presented because their derivations follow the same procedures.

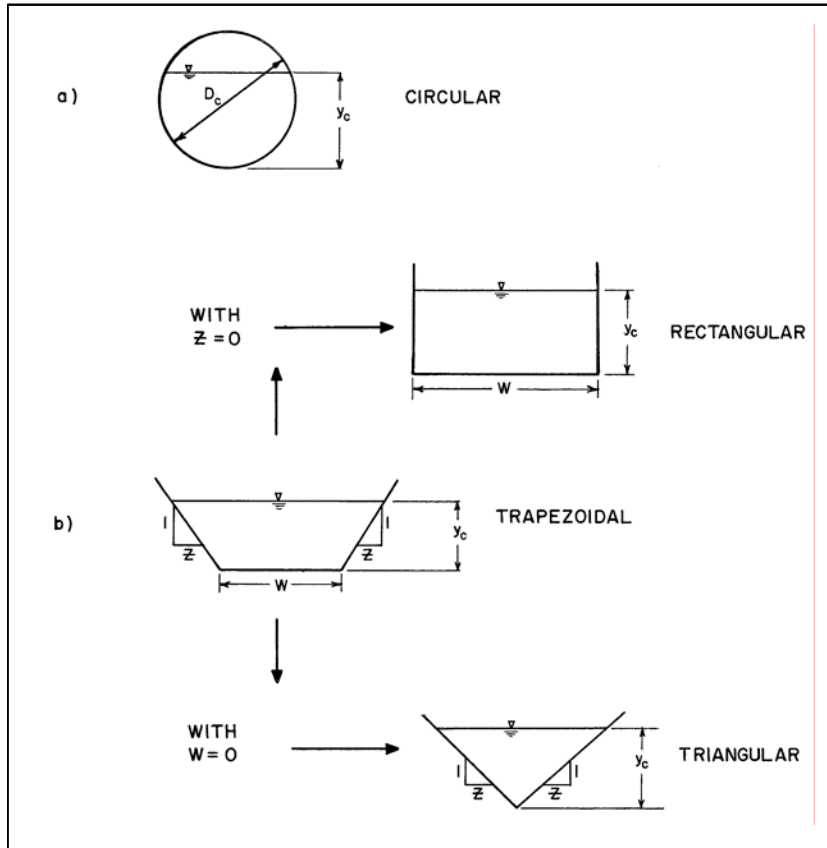


Figure 7 Two Basic Channel Shapes and Their Variations Used by the HEC-1 Program for Kinematic Wave Stream Routing

1.7.4 Triangular Sections

Derivation of α_c and m_c for a triangular channel shape begins with the description of R_c and A_c in Equation 7 (page 11) for a triangular shaped cross section. The reader should refer to Figure 7 and to the definition of terms after Equations 13 and 14.

$$R_c = A_c/P_c = (\text{Area of triangular cross section}/\text{Wetted Perimeter}) = \text{Hydraulic Radius}$$

$$\begin{aligned} \text{where: } P_c &= 2(\sqrt{1+z^2}) \cdot y_c &&= \text{wetted perimeter} \\ A_c &= \frac{1}{2}(2y_c \cdot z) \cdot y_c = zy_c^2 &&= \text{cross-sectional area} \\ z &= \text{side slope ratio} \end{aligned}$$

Now the values of A_c and R_c may be substituted into Equation 7.

$$Q_c = \frac{1.486}{n} S_c^{1/2} \frac{A_c^{2/3}}{P_c^{2/3}} A_c = \frac{1.486 S_c^{1/2}}{n} \frac{A_c^{5/3}}{P_c^{2/3}} \quad (15)$$

Defining Φ as $\frac{1.486S_c^{1/2}}{n}$, and substituting the appropriate value for A_c and P_c into Equation 15 and simplifying, gives

$$Q_c = \Phi \frac{z^{5/3} y_c^{10/3}}{(2^{2/3}) (1+z^2)^{1/3} y_c^{2/3}} = \Phi \frac{z^{1/3} (z^{4/3} y_c^{8/3})}{(2)^{2/3} (1+z^2)^{1/3}} \quad (16)$$

$$= \frac{1.486 S_c^{1/2}}{1.587 n} \left(\frac{z}{1+z^2} \right)^{1/3} (z \cdot y_c^2)^{4/3} \quad (17)$$

$$\text{or } Q_c = \frac{0.94 S_c^{1/2}}{n} \left(\frac{z}{1+z^2} \right)^{1/3} A_c^{4/3} \quad (18)$$

Equation 8 (page 11) may now be written in the form of Equation 14:

$$Q_c = \alpha_c A_c^{m_c} \quad (19)$$

$$\text{where: } \alpha_c = \frac{0.94 S_c^{1/2}}{n} \left(\frac{z}{1+z^2} \right)^{1/3}$$

$$m_c = \frac{4}{3}$$

1.7.5 Rectangular Sections

A rectangular shape is obtained by stipulating that z in Figure 7b (page 14) is zero. This produces a channel w feet wide with vertical walls. This shape may represent man-made channels and rectangular concrete drain sections. Following similar procedures, which produced Equations 18 and 19, two separate relationships for rectangular shapes are easily derived. One relationship is for a very wide rectangular channel where w is much greater than the depth y_c , and the other relationship is for a rectangular channel that may have comparable depths and breadths, e.g., $w \approx y_c$.

For the wide shallow channel case:

$$\alpha_c = \frac{1.486 S_c^{1/2}}{n} \cdot w^{-2/3} \quad (20)$$

$$\text{and } m_c = \frac{5}{3}$$

For the rectangular channel where $w \approx y_c$:

$$\alpha_c = \frac{0.72S_c^{1/2}}{n} \quad (21)$$

and $m_c = \frac{4}{3}$

1.7.6 Trapezoidal Sections

The trapezoidal section is one of the two basic sections considered by the general HEC-1 code. Modifications of the trapezoidal shape produced the rectangular and triangular alternatives defined above. When describing a trapezoidal section it is important to define the side slopes z and the channel bottom widths w accurately (Figure 7, page 14). It is not possible to derive a simple relationship for α_c and m_c from the geometric properties alone, so it becomes necessary to fit α_c and m_c to the Manning equation at two or more depths y_c and use numerical techniques of fitting the kinematic equation to these values to obtain values for α_c and m_c for various flow conditions.

Kinematic wave equation: $Q_c = \alpha_c A_c^{m_c}$

Manning equation for a trapezoid: $Q_c = \frac{1.486S_c^{1/2}}{n} (A_c')^{5/3} \left[\frac{1}{w + zy_c(1 + z^2)^{1/2}} \right]^{2/3} \quad (22)$

where: A_c' = the area of the effective cross section at depth y_c

values of m_c vary from 4/3 for a triangular section to 5/3 for a wide rectangular section.

1.7.7 Circular Sections

Circular sections can be used to model storm or sewer pipes in urban areas. The following relationships for α_c and m_c were derived [Resource Analysis, Inc., 1975] for typical circular sections such as that shown in Figure 7a (page 13), which apply to pipe sections flowing less than 90 percent full.

$$\alpha_c = \frac{0.804S_c^{1/2}}{n} D_c^{1/6} \quad (23)$$

and $m_c = 1.25$
 $D_c =$ the diameter of the circular section in feet

1.8 Numerical Solution of the Kinematic Wave Equations

The HEC-1 program solves the kinematic wave equations using finite difference numerical techniques. The details of these techniques, which have been perfected after years of development and testing, are available from several sources [Harley, 1975], [Resource Analysis, Inc., 1975], [Bras, 1973]. As with standard numerical procedures, time is discretized

in constant steps of Δt and distance in steps of Δx . The rainfall excess ($i - f$) is assumed constant within each time step Δt , but does change from time step to time step, to simulate the variability occurring within a storm event.

Recall that Equations 13 and 14 were the kinematic wave equations for collector and streamflow routing. If Equation 14 is substituted into Equation 13, the following relationship, which has A_c as the only dependent variable, will be obtained

$$\frac{\partial A_c}{\partial t} + \alpha_c m_c A_c^{(m_c-1)} \frac{\partial A_c}{\partial x} = q_o \quad (24)$$

Numerical solution of Equation 24 produces a relationship for A_c in terms of x , t , and q_o . These values of A_c may then be substituted into Equation 14 to solve for Q_c . This simple procedure (described previously for overland flows) allows the calculation of Q_c as a function of the segment length L_c and time t . Therefore, one can describe the discharge hydrograph from each of the segments that are of length L_c . If these discharges, Q_c , represented discharges from local collector channels, then HEC-1 will distribute them uniformly as lateral inflow into the main channel or stream as shown in Figures 3 and 4. Calculation of the resulting discharge hydrograph from the main channel or stream is then performed in an identical manner as was just presented. The equations used would be identical to Equations 23 and 24 except the subscript c would be replaced with an m everywhere throughout Equations 23 and 24. This provides a simple straightforward procedure for calculating first the overland flows, then the flows through the collector channel system and finally the discharges in the main stream.

Detailed development of the specific finite difference equations, the coding procedures and boundary requirements can be found in the following references [Lighthill and Whitham, 1955]; [Bras, 1973]; [Resource Analysis, Inc., 1975]; [Hydrologic Engineering Center, 1979]. The coding modifications that were made to include the kinematic wave routing procedure in HEC-1 was done by Resource Analysis, Inc., under the direction of Dr. B. Harley. The current version of HEC-1 uses an improved algorithm for solving the KW equations [HEC, 1990]. The algorithms used are based on those developed for the MITCAT Catchment Simulation Model [Resource Analysis, Inc., 1975]. The following section presents a brief review of the numerical methods and procedures utilized by the HEC-1 program to perform kinematic wave routing.

1.9 Finite Difference Solutions of the Kinematic Wave Equations

The movement of a flood wave down a stream or along an overland flow surface can be followed by monitoring the times and locations where specific water surface elevations and discharges occur. For example, if lateral inflow is assumed to be zero for a moment, a flood wave can be followed in time by noting the times when a flow of a given magnitude, or a stage of a given magnitude occurs at successive downstream stations along the channel. Flood routing methods, such as the kinematic wave method used in HEC-1, depend upon certain numerical techniques to solve the governing equations, which describe the movement, stage, and discharge characteristics of a flood wave as it propagates downstream. Discussions of several of the different methods currently in use for hydrologic engineering studies are available [Overton/Medows, 1976] [Mahmood/Yevjevich, 1975]. The numerical method currently employed by the HEC-1 program to solve the governing equations is a finite difference method.

A finite difference method (FDM) presents a "point wise approximation" to the governing partial difference equations. The FDM uses simple difference equations, which replace the partial differential equations for an array of stationary grid points, located in the space-time ($x-t$) plane (Figure 8). The intersections of the lines in Figure 8 define the time and space points at which the discharge and water surface elevation are computed. Lines parallel to the x -axis are called time lines, and lines, which are parallel to the t -axis, are space lines. The regular pattern formed by the intersections of time and space lines is called the "computational network." Those points that are marked by solid dots represent computation points (called nodes). Solutions to the governing equations via the FDM will be computed at each of these nodes. Computations advance along the downstream direction for each time step Δt until all the flows and stages are calculated along the entire distance L . Then the computation is advanced ahead in time by one Δt and the computations for discharge and water surface elevation are performed once again.

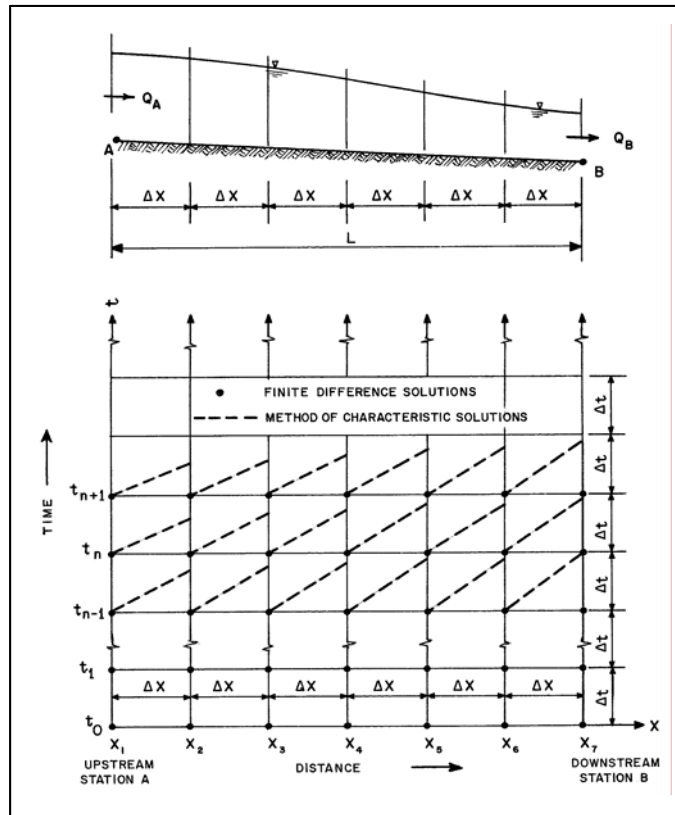


Figure 8 Characteristic Curves on a Fixed $\Delta x - \Delta t$ Grid

Also shown on Figure 8 are the solution curves (the dashed lines) that represent solutions obtained from another method called the "method of characteristics." The method of characteristics is **not** an available alternate solution technique in HEC-1. As can be seen in Figure 8, the solution curves (called characteristic curves) do not always intersect at a node where evenly spaced time and space lines intersect. Because of this, additional interpolation would be required to obtain solutions at evenly spaced nodes. To avoid this interpolation, the method of characteristics is not used in HEC-1. It is mentioned here though, because the characteristic curves represent locations in the $x-t$ plane where specific flow properties, such as wave celerity c , remain constant for each time step Δt . The importance of this is shown below when the specific numerical solution methods (the standard form and conservation form of the governing equations) are explained.

There are detailed methods available for the approximation of derivatives by finite differences [Carnahan, et. al., 1969]. By simply combining the appropriate finite difference approximations for first- and second-order derivatives, complete partial differential equations, such as Equations 14 and 24 can be recast in terms of finite differences instead of partial derivatives. These new equations are approximations of the original equations but are now in a form that can be easily handled numerically, especially with the aid of high-speed computers. As an example of the procedure; the first-order partial derivative $\partial Q/\partial x$ is approximated using a backward finite difference method:

$$\frac{\partial Q}{\partial X} \approx \frac{\Delta Q}{\Delta x} \approx \frac{Q_{i,j} - Q_{i-1,j}}{\Delta x} \quad (25)$$

The following sections will discuss how similar finite-difference approximations are applied to solve the governing kinematic wave equations in space and time.

The governing equations developed in the previous sections consisted of a pair of equations for each of the different kinds of flow elements; e.g., Equations 10 and 12 for overland flow elements, Equations 13 and 14 for collector elements, and two additional equations identical to Equations 13 and 14 (only with different subscripts) for the main channel routing elements. Rather than treat each of these three pairs of equations separately, the solution details will be developed for one set only because they are basically all the same. HEC-1 handles each of these three different kinds of flow elements by using the following computational sequence: computations start with the determination of overland flows which are then input as uniformly distributed lateral inflows into the collector elements which, in turn, modify the flows and distribute these collector flows uniformly and laterally along the main channel. The main channel routes the final flood wave through the subbasin. A combination of several subbasins thus allows for the complete description of an entire basin during a storm event.

1.10 Standard Form of the Kinematic Wave Equations

Consider Equation 13, which is the continuity equation and Equation 14 which relates flow Q_c within a collector element to the collector channel cross-sectional area A_c . It is assumed that the kinematic wave coefficients α_c and m_c are constant for any given system of channel elements. Differentiation of the flow Equation 14 with respect to x and substitution of this into Equation 13 gives:

$$\frac{\partial A_c}{\partial t} + \alpha_c m_c A_c^{m_c-1} \frac{\partial A_c}{\partial x} = q_o \quad (26)$$

Notice that Equation 26 is the same as the previously derived Equation 24. Referring to Figure 9, the area A_c is known at points on the space-time grid for times prior to the current time (designated by the index j) and at points in space prior to the current location (indicated by the index i). Therefore, the index (i,j) corresponds to the current time and space coordinates. Future times and space locations that are advanced by one Δt and Δx are indicated as $j+1$ and $i+1$, respectively. Similarly, one previous time and space location would correspond to a point on the space-time grid indexed by $(i-1, j-1)$. In this way, each point (or node) in the space-time grid can be indicated by a double subscript of i 's and j 's. This allows one to rewrite partial

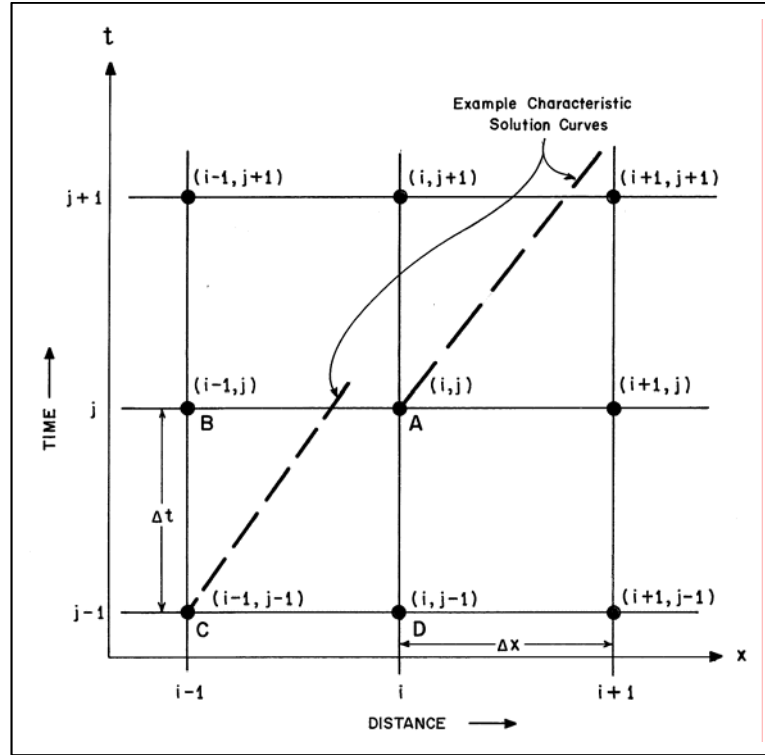


Figure 9 Space-Time Grid Used for Finite Difference Method

differential equations in terms of finite difference approximations of known quantities (located at previous times and space points) and unknown quantities (current time and space points). Therefore, the area A_c at point B in Figure 9 would be designated as $A_{c(i-1,j)}$, at point C, $A_{c(i-1,j-1)}$, at point D, $A_{c(i,j-1)}$ and so forth. With this background, one can now express the governing equations in terms of finite differences using the previously defined indexing scheme and solve for the values of A_c and Q_c at the "current point A" in Figure 9.

$$\alpha m A^{m-1} \frac{\partial A}{\partial x} \approx \alpha m A^{m-1} \frac{\Delta A}{\Delta x} = \alpha m \left(\frac{A_{(i,j-1)} + A_{(i-1,j-1)}}{2} \right) \left(\frac{A_{(i,j-1)} - A_{(i-1,j-1)}}{\Delta x} \right) \quad (27)$$

where the differential of the area A in the x-direction is taken as the difference between the values known at points C and D in Figure 9. Also, the area term which is raised to the (m-1) power in Equation 27 is considered to be an average area between points C and D in Figure 9.

Consider the time derivative term $\frac{\partial A}{\partial t}$ in Equation 26. It is evaluated between points A and D (Figure 9), thus:

$$\frac{\partial A}{\partial t} \approx \frac{\Delta A}{\Delta t} = \left(\frac{A_{(i,j)} - A_{(i,j-1)}}{\Delta t} \right) \quad (28)$$

The lateral inflow term q is handled as an average lateral inflow, which occurs within a time step Δt and is defined as q here to simplify the final form of the equation.

$$q \approx \frac{q_{(i,j)} + q_{(i,j-1)}}{2} = \bar{q} \quad (29)$$

Combining Equations 27, 28 and 29 produces the complete finite difference form of the original partial differential Equation 26:

$$\frac{A_{(i,j)} - A_{(i,j-1)}}{\Delta t} + \alpha m \left(\frac{A_{(i,j-1)} + A_{(i-1,j-1)}}{2} \right)^{(m-1)} \left(\frac{A_{(i,j-1)} - A_{(i-1,j-1)}}{\Delta x} \right) = \bar{q} \quad (30)$$

$A_{i,j}$ is the only unknown quantity in Equation 30 which can, therefore, be isolated and directly solved for.

$$A_{(i,j)} = \bar{q} \Delta t + A_{(i,j-1)} - \alpha m \frac{\Delta t}{\Delta x} \left(\frac{A_{(i,j-1)} + A_{(i-1,j-1)}}{2} \right)^{(m-1)} (A_{(i,j-1)} - A_{(i-1,j-1)}) \quad (31)$$

Once $A_{i,j}$ is known, the corresponding flow $Q_{i,j}$ can be computed from Equation 14 (page 13). This provides a straightforward method of computing the time varying discharges along the channel.

1.11 Conservation Form of the Equation

The previous "standard form" of the equations applies in most situations where the average wave celerity, c , is less than the ratio of the computational space to the time step, e.g., $c < (\Delta x/\Delta t)$. This corresponds to a solution below a diagonal connecting points C and A in Figure 9 (page 20). When c is less than $\Delta x/\Delta t$, this procedure provides an accurate approximation for the kinematic wave characteristics of a flood wave, and points C, D and A are used to determine $A_{i,j}$.

However, if c is greater than $\Delta x/\Delta t$, is it possible for flood wave characteristics to propagate more rapidly through space and time than the numerical approximation method can account for them. This would correspond to a solution found above a diagonal connecting A and C in Figure 9. (This numerical stability criterion can be associated with the familiar Courant condition for stability of explicit finite difference schemes.) Because of the potential wave instability, an alternate form of the approximate finite difference equations is needed.

The "conservation form" of the governing equations is therefore applied when the average celerity c of the flood wave is greater than the ratio $\Delta x/\Delta t$. In this form, the temporal derivatives are evaluated between points B and C in Figure 9 rather than A and D, while the spatial derivatives are evaluated between points B and A rather than C and D. In this way, rapidly advancing flood wave characteristics can be handled more accurately. HEC-1 checks the stability of the wave for each time step. If it is stable, the standard form of the equations is used; if not, the conservation form of the equations is used.

The conservation form of the spatial derivative of discharge will be evaluated between points B and A in Figure 9 (page 20):

$$\frac{\partial Q}{\partial x} \approx \frac{\Delta Q}{\Delta x} = \frac{Q_{(i,j)} - Q_{(i-1,j)}}{\Delta x} \quad (32)$$

and the temporal derivative of area will be evaluated between points B and C.

$$\frac{\partial A}{\partial t} \approx \frac{\Delta A}{\Delta t} = \frac{A_{(i-1,j)} - A_{(i-1,j-1)}}{\Delta t} \quad (33)$$

The substitution of these new "conservation form" derivatives into the continuity equation produces:

$$\frac{Q_{(i,j)} - Q_{(i-1,j)}}{\Delta x} + \frac{A_{(i-1,j)} - A_{(i-1,j-1)}}{\Delta t} = \bar{q} \quad (34)$$

Solving for the only unknown, $Q_{i,j}$ gives

$$Q_{(i,j)} = Q_{(i-1,j)} + \bar{q}\Delta x - \frac{\Delta x}{\Delta t} (A_{(i-1,j)} - A_{(i-1,j-1)}) \quad (35)$$

If desired, $A_{i,j}$ can be determined from the flow equation.

$$A_{(i,j)} = \left(\frac{Q_{(i,j)}}{\alpha} \right)^{1/m} \quad (36)$$

1.12 Accuracy of the Finite Difference Solution

The accuracy and stability of the finite difference scheme depends on approximately maintaining the relationship $c(\Delta t) = \Delta x$, where c is the average kinematic wave speed in an element. The kinematic wave speed is a function of flow depth, and, consequently, varies during the routing of the hydrograph through an element. Since Δx is a fixed value, the finite difference scheme utilizes a Δt variable internally to maintain the desired relationship between Δx , Δt and c . However, HEC-1 performs all other computations at a fixed time interval specified by the user. Necessarily, the variable Δt hydrograph computed for a subbasin by the finite difference scheme is interpolated to the user specified computation interval prior to other HEC-1 computations. The resulting interpolation error is displayed in both intermediary and summary output. These interpolation error summaries are shown in the example problem in Appendix A.

The accuracy of the finite difference scheme depends on the selection of the distance increment, Δx . The distance increment is initially chosen by the formula $\Delta x = c\Delta t_m$ where c , in this instance, is an estimated maximum wave speed depending on the lateral and upstream inflows. Δt_m is the time step equal to the minimum of, (1) one third the travel time through the reach, the travel time being the element length divided by the estimated wave speed, (2) one fourth the upstream hydrograph rise time, and (3) the user specified computation interval. Finally, Δx is chosen as the minimum of the computed Δx and $L/NDXMIN$, where L is the length of the channel reach and $NDXMIN$ is a user specified number of Δx values to be used by the

finite difference scheme ($5 \leq \text{NDXMIN} \leq 50$ for overland flow planes, and $2 \leq \text{NDXMIN} \leq 50$ for channels). Once calculated, Δx remains constant over the routing through the channel reach.

Consequently, the accuracy of the finite difference solution depends on both the selection of Δx and the interpolation of the kinematic wave hydrograph to the user specified computation interval. The default selection of the Δx value by the program will probably result in an error of less than five percent. Based on the certainty of the parameter assumptions, this is probably accurate enough for engineering purposes. The user may wish to check the accuracy by altering NDXMIN and comparing the results (see Appendix A). More importantly, the user should always check the error in interpolating to the user specified computation interval as summarized at the end of the HEC-1 output. Reducing the computation interval, NMIN, may reduce this interpolation error.

This concludes the introductory development of the general kinematic flow equations and a brief discussion of their numerical solution techniques. Chapter 2 will present additional background information and the details necessary for the effective application of these methods to solve problems in urban hydrology.

1.13 References

Abbot, B. B., "Methods of characteristics," in **Unsteady Flow in Open Channels**, K. Mahmood and V. Yevjevich, eds., Water Resources Publications, Fort Collins, Colorado, 1975.

Alley, W.M. and Smith, P.E., **Distributed Routing Rainfall-Runoff Model**, Open-File Report 82-344, U.S. Geological Survey, Reston, Virginia, 1987.

Bras, R. L., **Simulation of the Effects of Urbanization on Catchment Response**. These presented to M.I.T, Cambridge, Massachusetts, 1973.

Carnahan, B. J., Luther, H. and Wilkes, J., **Applied Numerical Methods**, John Wiley & Sons, Inc., New York, 1969.

Chow, Ven Te, **Open-Channel Hydraulics**, McGraw-Hill Book Company, New York, 1959.

Crawford, N.H., and R.K. Linsley, **Digital Simulation in Hydrology – Stanford Watershed Model IV**, Stanford University, Department of Civil Engineering, Technical Report No. 39, Stanford, California, 1966.

Fread, D.L., **Theoretical Development of An Implicit Dynamic Model**, report to the Office of Hydrology, National Weather Service, Maryland, December 1967.

Harley, B. M., **Use of the MITCAT Model of Urban Hydrologic Studies**, presented at the Urban Hydrology Training Course at the Hydrologic Engineering Center, Davis, California, September 1975.

Harley, B.M., F. E. Perkins, and P.S. Eagleson, **A Modular Distributed Model of Catchment Dynamics**, Ralph M. Parson Laboratory for Water Resources and Hydrodynamics, Technical Report No. 133, M.I.T., Cambridge Massachusetts, 1972.

Henderson, F.M., **Open Channel Flow**, MacMillan Publishing Company, Inc., New York., 1966.

Hromadka, T.V. and DeVries, J.J., "Kinematic wave and computational error," **Journal of Hydraulic Engineering**, ASCE Volume 114, No 2, 1988, pages 207-217.

Hydrologic Engineering Center (HEC), **HEC-1 Flood Hydrograph Package, User's Manual**, U.S. Army Corps of Engineers, Davis California, 1990.

Leclerc, Guy and Schaake, J.C., Jr., "Methodology for assessing the potential impact of urban runoff and the relative efficiency of runoff control alternative"; Ralph M. Parsons Laboratory, Report No. 167, Massachusetts Institute of Technology, page 257, 1973.

Liggett, J.A. and Woolhiser, D.A., "The use of the shallow water equations in runoff computations," **Proceedings Third Annual American Water Resources Conference**, San Francisco, California, 1967.

Lighthill, M.J. and Whitham, G.B., "Kinematic waves," **Proceedings Royal Society London**, Volume 229, 1955, pages 281-316.

Linsley, Kay K., Jr., Kohler, Max A., and Paulhus, Joseph L.H., **Hydrology for Engineers**, 2nd ed. McGraw-Hill Book Company, New York, 1975.

Mahmood, K. and Yevjevich, V., **Unsteady Flow in Open Channels**, Water Resources Publications, Fort Collins, Colorado, 1975.

Miller, W.A. and Cunge, J.A., "Simplified equations of unsteady flow," **Unsteady Flow in Open Channels, Volume I**, K. Mahmood and V. Yevjevich, eds., Water Resources Publications, Fort Collins, Colorado, 1975.

Overton, D.E., "Kinematic flow on long impermeable planes," **Water Resources Bulletin**, Volume 8, No 6, 1972, pages 1198-1204.

Overton, D.E. and Meadows, M.E., **Stormwater Modeling**, Academic Press, New York, 1976.

Resource Analysis, Incorporated, **MITCAT Catchment Simulation Model, Description and User's Manual**, Version 6, Cambridge, Massachusetts, September 1975.

Stoker, J.J. (1957). **Water Waves**, Interscience, New York.

Strelkoff, T., "One-dimensional equations of open-channel flow," **Proceedings Journal of the Hydraulics Division**, ASCE, May 1969.

Thomas, W.A., **Water Surface Profiles**, IHD Volume 6, Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, California, July 1975 (out of print).

Wooding, R.A., "A hydraulic model for the catchment stream problem," **Journal of Hydrology**, Volume 3, 1965.

Woolhiser, D.A., "Simulation of unsteady overland flow," in **Unsteady Flow in Open Channels**, K. Mahmood and V. Yevjevich, eds., Water Resources Publications, Fort Collins Colorado, 1975.

Chapter 2

Application of Kinematic Wave Routing Techniques using HEC-1

2.1 Introduction

HEC-1 contains options for using kinematic wave theory to compute subbasin outflow hydrographs and to route hydrographs through a stream reach [HEC, 1990]. These options provide an alternative to the unit hydrograph method for determining direct runoff. They also provide a streamflow routing technique, which can be used in place of the Muskingum, modified Puls, and other methods available in HEC-1. As with any type of hydrologic model, however, it is imperative that the modeler checks the performance of his modeling effort against observed data. Use of the model without a procedure for verifying its ability to correctly simulate the behavior of a given basin is strongly discouraged.

The purpose of this chapter is to describe how the kinematic wave method in HEC-1 may be applied. A discussion of kinematic wave theory as it relates to this program is given in Chapter 1 of this document. Several papers are recommended for general background on the kinematic wave method [Lighthill/Whitman, 1955] [Harley, et. al., 1972] [Wollhiser, 1975].

One of the attractive features of the kinematic wave approach to rainfall-runoff modeling is that the various physical processes of the movement of water over the basin surface, with the attendant infiltration, flow into stream channels, and flow through the channel network are considered. Parameters such as roughness, slope, catchment length and areas, and stream channel dimensions are used to define the processes.

The various features of the irregular surface geometry of the basin are generally approximated by either of two types of basic elements: (1) an overland flow element, and (2) a stream or channel flow element. In the modeling process described here, one or two overland flow elements (designated as overland flow strips) are combined with one or two channel flow elements to represent a subbasin. An entire basin is modeled by linking the various subbasins together.

Because the descriptions of the various elements comprising the model are directly related to physical parameters, the model can be easily modified to reflect changing land uses. This makes kinematic-wave-type models very useful for urban studies because the effects of increasing urbanization can be accounted for by changing the parameters describing the basin.

The various topics covered in this chapter include basin modeling procedures, a description of the elements used in kinematic wave calculations, and procedures for selecting the parameters. An example problem, shown in Appendix A, is presented to illustrate HEC-1 input and output data, and the effects of changes to numeric values of the parameters are discussed.

2.2 Basin Modeling

The modeling process starts with a description of the topologic structure of the basin: drainage basin boundaries, stream and drainage channels, and the logical relationships between the drainage areas and the channels. The definition of the drainage boundary will depend on the objective of the study being conducted, as well as the topological character of the basin. Studies dealing with urban hydrology usually require delineation of subbasins that are smaller than two square miles in extent (about five square kilometers). Studies dealing with the effects of channel modifications may permit use of large areas; however, as the area is increased the assumptions required to apply the kinematic wave method become more tenuous. A typical urban drainage system is shown in Figure 5 (page 8). Rain falls on two general types of surfaces: (1) those that are essentially impervious, such as roofs, driveways, parking lots and other paved areas; and (2) pervious areas, most of which are covered with vegetation and have numerous small depressions that produce local storage of rainfall. It is assumed in the model that water initially travels over these surfaces as sheet flow; however, in a relatively short distance the water begins to collect in small streams or rivulets and the process of stream or channel flow begins. For impervious areas, the distance to the first channel (e.g., a gutter) is typically thirty to one hundred feet. For pervious surfaces, the longest distance a drop of water must travel to reach a channel is on the order of one hundred to several hundred feet.

Water collected by the street gutters, travels no more than a few hundred feet until it enters catch basins, which are connected, to sewers. These sewers are typically 1.5 to 2 feet in diameter for the local drains. The local drains are connected in turn to larger and larger drains, which feed the main storm, drain. In many areas the main storm drains are open channels or streams. In major urban areas, the main storm drains are often large closed-conduit sections, usually designed to flow only partially full. The kinematic wave routing approach, which assumes open channel flow, is therefore appropriate.

There are certain weaknesses inherent in the kinematic wave routing approach, which should be kept in mind by the modeler. These include the following: (1) in kinematic wave routing, the theory does not provide for attenuation of the flood wave. As a consequence, peak flows may be over-estimated. (2) Surcharging of storm drains frequently occurs during major storm events, but no explicit provision for surcharging is provided by the method. (3) Also, ponding and local storage of water during major events is not accounted for. This might include overbank storage, ponding in the streets, etc., which occur during large storms of primary interest and may not occur to as great a degree in the events available for calibration and verification of the model. The modeler, therefore, should analyze the program results to see if this is happening.

2.3 Elements Used in Kinematic Wave Calculations

The runoff process described above is idealized in HEC-1 through the use of the following flow elements: (1) one or two typical overland flow elements, (2) a typical collector channel element, and (3) a main channel element. These generally provide the necessary detail for modeling the runoff process in urban basins. Schematic drawings of these elements are shown in Figure 3 (page 6). Figure 4 (page 7) illustrates the relationships between the flow elements. The elements are specified to represent typical features of the basin, and thus the parameters chosen for the individual elements should be representative of the entire subbasin.

The runoff simulation process is automatically expanded from the typical elements to the whole subbasin by the program. Because land use and development practices are usually very similar within a selected hydrologic unit, assigning a single value to a given parameter usually gives good results.

2.4 Overland Flow Elements

The basic overland flow element is simply a sloping rectangular plane surface upon which the rain falls. It is modeled as a strip of unit width (one foot or one meter wide). Some of the rainfall is lost by infiltration; the remainder runs off the lower edge of the plane into a channel. Infiltration losses may vary with time or be constant, and a different loss rate can be specified for each flow strip. The selection of appropriate loss rates is discussed in the next section. The fraction of the element that is impervious can also be specified.

The basic kinematic-wave-analysis concept used in HEC-1 allows the use of either one or two overland flow surfaces, each discharging into a collector channel. For example, one element could represent all areas that are essentially impervious, with short lengths of flow (L_o) to the point where the flow becomes channel flow. This element would represent driveways, roofs, street surfaces, etc.

The other overland flow element could represent areas that are pervious and have higher resistance to flow, such as lawns, fields, and wooded areas. In general, the catchment flow lengths and roughness coefficients will be much greater for these areas. Again, the value of L_o to be used is the representative maximum distance for water to travel as overland flow for this type of land surface.

The user of this method should think of the overland flow strips as representing typical flow surfaces rather than actual planar surfaces, except when very small areas (such as one city lot) are being considered. It is only at these very small scales that the mean surface slope and actual area and length come close to fitting the basic theoretical concept.

The following data are needed as input to HEC-1 to describe each overland flow strip:

- L_o - typical overland flow length
- S_o - representative slope
- N - roughness coefficient (Table 1, page 11)
- A_{o1}, A_{o2} - The percentages of the subbasin area which the overland flow surface represents (two possible types for each subbasin)

Methods to determine these parameters are discussed in the following sections. It is suggested that the data first be tabulated on a form for the entire basin and then entered into an input data file for HEC-1. A typical data tabulation sheet is given in Table 2.

2.5 Loss Rate Selection

Loss rates are a very important component of the kinematic wave model, and must be specified for the overland flow strips. Unfortunately, these loss rates are often difficult to exactly determine. When the analysis focuses on an ungaged basin, as is often the case in

Table 2
Parameters for Kinematic Wave Example Problem

	Percent Total Subbasin Area	L (ft)	S	Roughness	Channel Shape	Channel Size (ft)	Z	Collector System Area (mi ²)	Loss Rate (in/hr)
Subbasin 1 (1.5 mi²)									
1. Overland Flow Strip	100	500	0.04	0.40	--	--	--	--	0.50
2. Collector Channel	--	1,500	0.025	0.10	Triangle	--	1	0.40	--
3. Main Channel	--	3,500	0.010	0.05	Triangle	--	4	--	--
Subbasin 2 (1.2 mi²)									
1. Overland Flow Strip 1	20	50	0.06	0.30	--	--	--	--	0.02
2. Overland Flow Strip 2	80	180	0.01	0.40	--	--	--	--	0.20
3. Collector Channel	--	2,100	0.008	0.020	Triangle	--	1	0.35	--
4. Main Channel	--	4,000	0.003	0.025	Trapezoid	2	2	--	--

urban hydrology, the SCS curve number method is the most popular. This curve number technique [SCS, 1975] involves estimating a curve number from soil hydrologic groups identified on a soil survey map. The SCS has recommended using the curve numbers shown in Table 3 for use in urban areas.

Many of these curve numbers for urban land use are based on percent impervious data. The curve numbers presented here are actually weighted averages using a curve number of 98 for impervious regions, and a value between thirty-nine and eighty for the pervious areas. For the kinematic wave model, the pervious loss rate should be selected based on pervious area, such as open space, not on the land use under study. If the pervious curve number is selected based on land use, the effect of the percent impervious will be counted twice and result in an overestimation of runoff.

An example will help clarify this point:

For both unit hydrograph and kinematic wave models, determine the correct loss rate for a subbasin with ¼-acre residential lots on a soil group of A. The land use in the subbasin is considered to be uniform.

- Unit Hydrograph Model: the curve number corresponding to ¼ acre lots on soil group A is sixty-one (Table 3). This would be the correct loss rate to use for the subbasin.
- Kinematic Wave Model: This can be modeled using two overland flow strips, one representing impervious areas (CN=98), the other representing the pervious areas, such as open space and lawns. A curve number of thirty-nine would be appropriate for these pervious areas on soil group A. The average ¼-acre lot is 38% impervious (Table 3), which means 38% of the basin area would have a curve number of ninety-eight, and the remaining 62% would have a curve number of thirty-nine. Thus, ninety-eight and thirty-nine would be the appropriate loss rates to use on the two overland flow strips.

Note that with this data, a composite curve number can be calculated:
 $(0.38) * 98 + (0.62) * 39 = 61$

Table 3
Runoff Curve Numbers for Selected Agricultural,
Suburban, and Urban Land Use

Antecedent Moisture Condition II, and $I_a = 0.25$ [SCS,1975]

Land Use Description	Hydrologic Soil Group			
	A	B	C	D
Cultivated Land ¹ : without conservation treatment	72	81	88	91
with conservation treatment	62	71	78	81
Pasture or Range Land: poor condition	68	79	86	89
good condition	39	61	74	80
Meadow: good condition	30	58	71	78
Wood or Forest Land: thin stand, poor cover, no mulch	45	66	77	83
good cover ²	25	55	70	77
Open Spaces, Lawns, Parks, Golf Courses, Cemeteries, etc.				
good condition: grass cover on 75 percent or more of the area	39	61	74	80
fair condition: grass cover on 50 to 75 percent of the area	49	69	79	84
Commercial and Business Areas (85 percent impervious)	89	92	94	95
Industrial Districts (72 percent impervious)	81	88	91	93
Residential: ³				
Average Lot Size				
Average Percentage Impervious⁴				
1/8 acre or less	65			
1/4 acre	38			
1/3 acre	30			
1/2 acre	25			
1 acre	20			
Paved Parking Lots, Roofs, Driveways, etc. ⁵	98	98	98	98
Streets and Roads:				
Paved with curbs and storm sewers ⁵	98	98	98	98
Gravel	76	85	89	91
Dirt	72	82	87	89

¹ For a more detailed description of agricultural land use curve numbers refer to National Engineering Handbook, Section 4, Hydrology, Chapter 9, August 1972.

² Good cover is protected from grazing and litter and brush cover soil.

³ Curve numbers are computed assuming the runoff from the house and driveway is directed towards the street with a minimum of roof water directed to lawns where additional infiltration could occur.

⁴ The remaining pervious areas (lawn) are considered to be in good pasture condition for these curve numbers.

⁵ In some warmer climates of the country a curve number of 95 may be used.

Since this composite curve number (used for the unit hydrograph model) includes the percent impervious, it should not be used to model the pervious areas in the kinematic wave model. Using this curve number would result in an overestimation of the impervious runoff.

Another concern in the selection of loss rates is the proper consideration of unconnected impervious areas. In developed areas of the watershed, most impervious areas are directly connected to storm drain systems. However, some impervious areas may drain onto pervious surfaces, which reduce the volume of the resulting runoff. For example, the part of a roof, which drains into a backyard or landscaping, would be considered as unconnected impervious area because the total volume of the resulting runoff would probably not enter the storm drain system. The effects of unconnected imperviousness should be considered when determining loss rates. Site inspection or sensitivity analysis could be used to determine the extent of unconnected imperviousness. Section 2.6 discusses how to correct for unconnected imperviousness.

2.6 Correction for Unconnected Impervious

The effects of unconnected impervious can be accounted for by using a composite curve number. This curve number (CN) is a function of the pervious area CN, the percent impervious area, and the percent of impervious area that is unconnected. Figure 10 shows a graph that can be used to determine the composite curve number [McCuen, 1989].

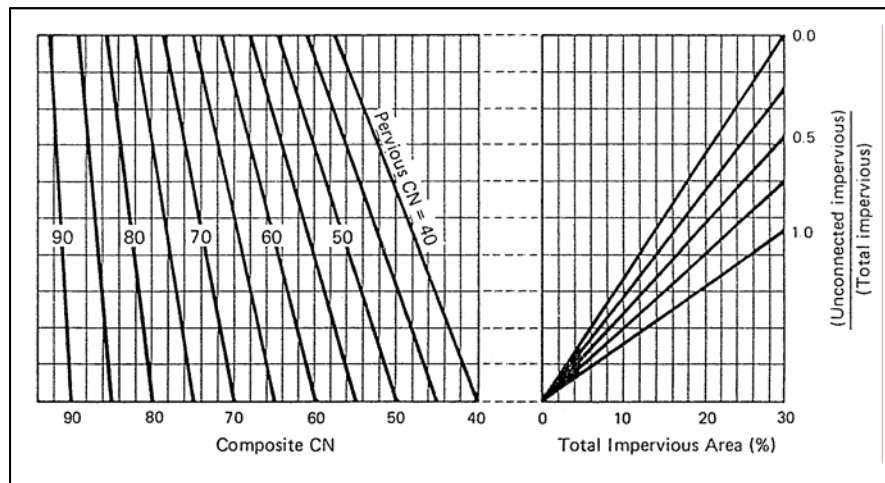


Figure 10 Graph for Determining Composite Curve Numbers

On the right hand side of the graph, find the intersection of the total percent impervious and the percent unconnected. Move horizontally to the left from this intersection until the intersection with the line representing the pervious (not composite) CN. The number directly below this second intersection is the desired composite CN.

This adjustment is not without limitations. For the composite CN to be valid, the total percent impervious must be less than thirty percent. If the total percent impervious is too large, the additional runoff from the unconnected areas cannot be assumed to be uniformly distributed on the pervious area. Also, as the percent impervious increases, the remaining pervious regions become quickly saturated and thus less effective in infiltrating water.

To use this composite curve number in a kinematic wave model, it must be broken down into its pervious and impervious components. The impervious CN remains 98 and the total percent impervious is assumed to be the same, so the following equation can be used to calculate the adjusted pervious curve number.

$$X = \frac{CN_c - 98 * f}{1 - f} \quad (37)$$

where: X = adjusted pervious curve number
 CN_c = composite curve number (Figure 10)
 f = total percent impervious 0 ≤ f ≤ 1

This adjusted pervious curve number can then be used as the loss rate for the pervious overland flow strip. A curve number of 98 can be used for the impervious flow strip.

2.7 Selection of Overland Flow Parameters

The area is the simplest quantity to specify; the area of each element is given as a percentage of the total area of the subbasin. If a single element is used, one hundred percent is specified. If two elements are used, the sum of the percentages should be one hundred percent.

The slope is a value, which is representative of the slope of the path that the water takes on its way to the collector channel. It may differ from the mean topographic slope for the catchment, and it is usually strongly related to the type of land use or development. For an urban setting, a single slope-value could be used for all areas of similar building practice, even though the mean ground slopes vary significantly.

As discussed in Chapter 1, the kinematic wave equations for the wide flow planes of the overland flow elements are based on Manning's equation for flow in a wide channel:

$$q_o = \alpha_o y_o^{m_o} \quad (38)$$

where: q_o = flow per unit width
 y = flow depth
 α_o = kinematic wave parameter
 m_o = kinematic wave parameter

For this situation:

$$m_o = 5/3 \text{ and } \alpha_o = (1.49/N)S^{1/2}$$

where: S_o = slope
 N = surface roughness coefficient similar to Manning's 'n' for channel flow

Because the nature of sheet flow with very small depths over rough surfaces differs markedly from streamflow, these resistance factors have much different values than the Manning's 'n' values used in streamflow computations. Values of N found to be appropriate are given in

Table 4. Use of Manning's 'n' instead of these roughness coefficients will result in exaggerated values for q_0 on the overland flow strips.

Table 4
Effective Resistance Parameters for Overland Flow

Surface	N value	Source
Asphalt/Concrete*	0.05 - 0.15	a
Bare Packed Soil Free of Stone	0.10	c
Fallow - No Residue	0.008 - 0.012	b
Conventional Tillage - No Residue	0.06 - 0.12	b
Conventional Tillage - With Residue	0.16 - 0.22	b
Chisel Plow - No Residue	0.06 - 0.12	b
Chisel Plow - With Residue	0.10 - 0.16	b
Fall Disking - With Residue	0.30 - 0.50	b
No Till - No Residue	0.04 - 0.10	b
No Till (20 - 40 percent residue cover)	0.07 - 0.17	b
No Till (60 - 100 percent residue cover)	0.17 - 0.47	b
Sparse Rangeland with Debris:		
0 Percent Cover	0.09 - 0.34	b
20 Percent Cover	0.05 - 0.25	b
Sparse Vegetation	0.053 - 0.13	f
Short Grass Prairie	0.10 - 0.20	f
Poor Grass Cover on Moderately Rough	0.30	c
Bare Surface		
Light Turf	0.20	a
Average Grass Cover	0.4	c
Dense Turf	0.17 - 0.80	a,c,e,f
Dense Grass	0.17 - 0.30	d
Bermuda Grass	0.30 - 0.48	d
Dense Shrubbery and Forest Litter	0.4	a

*Asphalt/Concrete n value for open channel flow 0.01-0.016

Legend: a) Harley (1975), b) Engman (1986), c) Hathaway (1945), d) Palmer (1946), e) Ragan and Duru (1972), f) Woolhiser (1975). [Hjemfelt, 1986]

A critical parameter in the overland flow element description is the flow length L_o . A proper specification of L_o is vital, since it is the most important parameter in determining the response characteristics of the overland flow elements. The overland flow length can be looked on as the maximum length of the path taken by a representative water drop to reach a channel where it first moves as streamflow. It is thus the distance for overland flow to reach a tributary or local channel, such as the street gutter.

Fortunately, in many natural basins and urban catchments, close examination of the full drainage system reveals that the small-scale drainage patterns are quite similar throughout the entire basin. The value of L_o appropriate for such a situation will not vary greatly over the basin. However, the actual values of L_o , which give the correct runoff response for the basin, must be verified through comparison of model output with measured data.

2.8 Collector Channel

The collector channel element is used to model the flow in its path from the point where it first becomes channel flow to the point where it enters the main channel. The inflow to the collector channel is taken as a uniformly distributed flow along the entire length of the channel. This correctly represents the situation where overland flow runs directly into the gutter, and also provides a reasonable approximation of the flow inputs into the storm drain system from individual catch basins and tributary collector pipes distributed along the collector channel.

The kinematic wave equations (developed in Chapter 1) are the continuity equations for unsteady channel flow with lateral inflow, and Manning's equation. The continuity equation is:

$$\frac{\partial A_c}{\partial t} + \frac{\partial Q_c}{\partial x} = q_o \quad (39)$$

where: Q_c = the channel flow in cfs
 A_c = flow cross-sectional area in square feet
 q_o = lateral inflow in cfs/ft to the channel
 x = distance along the channel in feet and t is the time in seconds

Manning's equation is written in the form:

$$Q_c = \alpha_c A_c^{m_c} \quad (40)$$

where the kinematic wave routing coefficients α and m are a function of the channel geometry. The general expression for α is:

$$\alpha_c = \frac{K \sqrt{S_c}}{n} \quad (41)$$

where: K = a constant that depends on the channel geometry
 S_c = channel slope
 n = Manning's roughness coefficient

A change in either n or S_c will change the value of α_c used in the calculations. It should be noted that the model is more sensitive to changes in n than to S_c because α_c depends on the first power of n while it is proportional to the square root of S_c .

The value of the exponent m_c for trapezoidal channels ranges from 4/3 when the trapezoid has a base width of zero (triangular shape) to 5/3 for a very wide rectangular shape. For a channel with a circular cross section, m_c is taken as 1.25. See Chapter 1 for the derivation of these parameters.

The following data are needed as input to describe the collector channel system (refer to Figure 7, page 14):

- The surface area drained by a single representative collector channel (e.g., gutter plus storm drain), A_c .

- The collector channel length (total length of gutter plus length of storm drain), L_c .
- The channel shape (either a circular section or some variant of a trapezoid).
- The pipe diameter or the trapezoid bottom width and side slope, of appropriate.
- The channel slope, S_c and
- Manning's 'n'.

2.9 Selection of Collector Channel Parameters

Looking at a drainage map for the basin and selecting a typical collector system for each subbasin can determine the characteristics of the collector channel components. This single typical collector channel is used to represent all the collector channels in the subbasin.

The area associated with the collector system can be determined from the map. This is an area in square miles (or square km) rather than the percentage of the subbasin area. It does not have to be an integer multiple of the subbasin area.

The collector channel length is taken as the longest flow path from the upstream end of the collector system to its outlet at the main channel. This length should include the distance the water will travel as gutter flow.

The channel shape and size will usually change along the length of the channel; however, a single shape must be chosen to represent the channel along its entire length. This is not as great a problem as might appear at first. As shown in Figure 11, a triangle with side slope of to 1:1 matches reasonably well with the area-discharge relationships for circular conduits for a given slope and roughness. HEC-1 assumes a triangular shape if the channel shape is not specified in the input data. The selection of channel shape is discussed in more detail in Section 2.10.

If the representation is by a circular or trapezoidal shape, the chosen channel dimensions should represent the most commonly used size of channel in the system.

The channel slope can be estimated from a topographic map by taking the difference in elevation between the upstream and downstream ends and dividing by the length. If drop structures are used in the storm drains, the slopes should be adjusted accordingly.

A Manning's 'n' which best represents the roughness of the major portion of the channel should be used. Tables of 'n' for various types of channels, such as concrete pipe, line or unlined open channels, etc., are available in hydraulic handbooks and other sources.

2.10 Main Channel

The main channel can carry both inflows from upstream subbasins as well as flows supplied by the collector channels within the subbasin. The inflow from the collector channel is

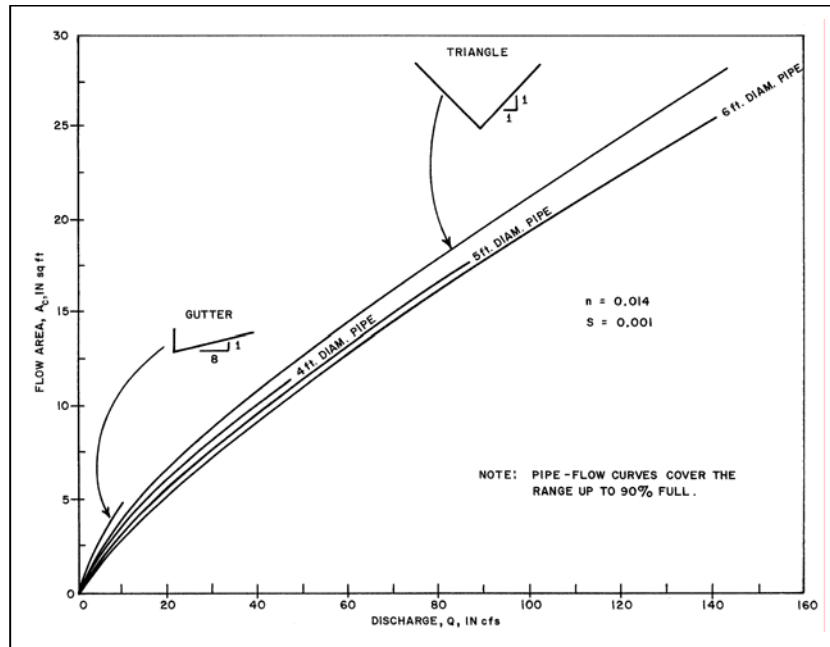


Figure 11 Discharge Versus Flow Area for Various Cross Sections

taken to be uniformly distributed along the length of the main channel. This is assumed to reasonably approximate the actual situation where the flow enters the channel from various collectors at a number of discrete points at various spacings. In Equation 39, the lateral inflow, q_c , is determined by scaling up the collector channel flow to match the total subbasin areas and then dividing the flow by the total main channel length.

An **example** will help clarify this point:

Suppose that the subbasin area is 1.0 square mile, while the collector channel area is 0.30 square mile. The length of the main channel is 2,000 feet. If the collector channel flow is designated as Q_c , the inflow per foot of main channel is:

$$q_c = \left(\frac{1.0}{0.30} \right) \left(\frac{Q_c}{2000} \right) \quad [cfs/ft]$$

The channel routing element can also be used independently for routing a hydrograph through a channel reach. If desired, the subbasin flow can be computed separately and combined with a routed flow at the subbasin outlet. Any of the routing methods available in HEC-1 can be used for channel routing (Muskingum, modified Puls, Tatum, etc.) if desired.

The channel routine procedure requires the following data:

- Channel or stream length, L_m
- Slope, S_m
- Manning's 'n'
- Area of subbasin, A_{subbasin}
- Channel shape (trapezoidal or circular)
- Channel dimensions (e.g., width (w), side slopes (z), or diameter (D_c))
- The upstream hydrograph to be routed through the reach if desired.

2.11 Selection of Main Channel Parameters

Most of the channel data can be obtained from physically measurable parameters for the channel and subbasin, with the exception of Manning's 'n'. The following procedure can be used to determine these data:

1. The channel length can be scaled from a drainage map of the basin.
2. The mean channel slope can be obtained from field measurements or estimated using topographic maps.
3. Selection of Manning's 'n' should be based on the average channel conditions.
4. The subbasin area can be measured from topographic maps.
5. The selection of channel cross section is discussed in Section 2.12.
6. The channel dimensions follow from the preceding item.
7. An upstream hydrograph will not be routed through the channel reach unless the user specifically requests the HEC-1 program to do so (RK Record, Field 8)

2.12 Selection of Channel Cross Section

The channel cross section for either the main channel or the collector channel can be defined as one of two simple shapes to permit modeling a variety of natural channels. The kinematic wave model is not especially sensitive to channel cross-sectional shape in the simulation of discharge, and therefore, it is not necessary to use complex channel shapes. The shapes, which can be used in HEC-1, are trapezoidal, deep rectangular and circular. A triangular shape can best simulate most main channels. For a main channel, specifying the base width for the trapezoid as zero can do this. For a collector channel, the trapezoid base width defaults to zero. When the channel is small, flood flows generally require overbank areas to carry the flows, and a triangular shape usually represents this situation quite well. As seen by a plot of area versus flow, Figure 11, a triangle can be used to approximate circular sections as well as street gutters.

In the downstream reaches of the basin, the trapezoidal section is usually the most appropriate choice for open channels. The chosen shape should provide the best fit to the channel shape for the given flood flow, however. For example, in some situations a trapezoid might be the best fit for low flows, while a triangle might be more appropriate for high flows. An illustration of this is shown in Figure 12.

The circular section allows modeling of storm sewers. The flow behavior of the conduit is simulated properly up to a point where the conduit is approximately ninety percent full. The program does not handle the effects of pressure flow, so for flows greater than about ninety percent of pipe capacity, HEC-1 assumes that the capacity of the element increases as required and has no upper limit. In many cases this approach adequately represents what is

happening in nature, because water that does not enter the storm drains flows over the surface or along a roadway until it finds another location to enter the drainage system.

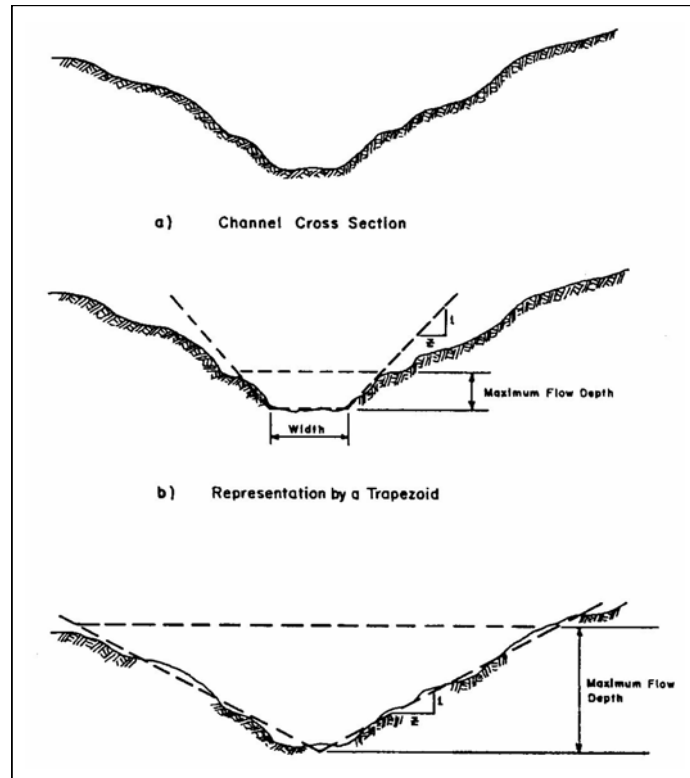


Figure 12 Fitting Channel Cross-Section Shape

2.13 Summary and Conclusions

The material presented in this chapter provides the user of HEC-1 with background information for modeling watershed basins using kinematic wave routing. The example problem in Appendix A gives an illustration of the use of the method and highlights the internal processes used in HEC-1 to minimize errors. Guidance on selection of parameters is provided, along with a brief sensitivity analysis to show effects of varying the parameters.

It is important for the user of this method to verify results by comparing them to measured rainfall-runoff events. This comparison assesses the performance of the model with the selected parameters. Without such a check, the inexperienced modeler should interpret the results with a great deal of caution. However, kinematic wave models have been used successfully in urban hydrology in a large number of applications, and when the models are properly formulated, good simulations result.

2.14 References

Crawford, N.H., and R.K. Linsley, **Digital Simulation in Hydrology - Stanford Watershed Model IV**, Stanford University, Department of Civil Engineering, Technical Report No. 39, Stanford, California, 1966.

Harley, B.M., F.E. Perkins, and P.S. Eagleson, A Modular Distributed Model of Catchment Dynamics, Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics, Technical Report No. 133, M.I.T., Cambridge, Massachusetts, 1972.

Hydrologic Engineering Center (HEC), **HEC-1 Flood Hydrograph Package, User's Manual**, U.S. Army Corps of Engineers, Davis, California, 1990.

Lighthill, M.J., and G.B. Whitman, "Kinematic waves," **Proceedings Royal Society London**, Volume 229, 1955, pages 281-316.

McCuen, R. H., **Hydrologic Analysis and Design**, Prentice Hall, Englewood Cliffs, New Jersey, 1989.

Resource Analysis, Incorporated, **MITCAT Catchment Simulation Model, Description and User's Manual**, Version 6, Cambridge, Massachusetts, 1975.

Soil Conservation Service (SCS), **Urban Hydrology for Small Watersheds**, Technical Release No. 55, U.S. Department of Agriculture, Washington D.C., 1975.

Woolhiser, D.A., "Simulation of unsteady overland flow," in **Unsteady Flow in Open Channels**, K. Mahmood and V. Yevjevich, eds., Water Resources Publications, Fort Collins Colorado, 1975.

Appendix A

An Example Application of Kinematic Wave Methods

The small partially urban basin shown in Figure A-1 is to be modeled using kinematic wave runoff and routing options of HEC-1. The hydrologic characteristics of this basin (obtained by previous calibration) are as follows:

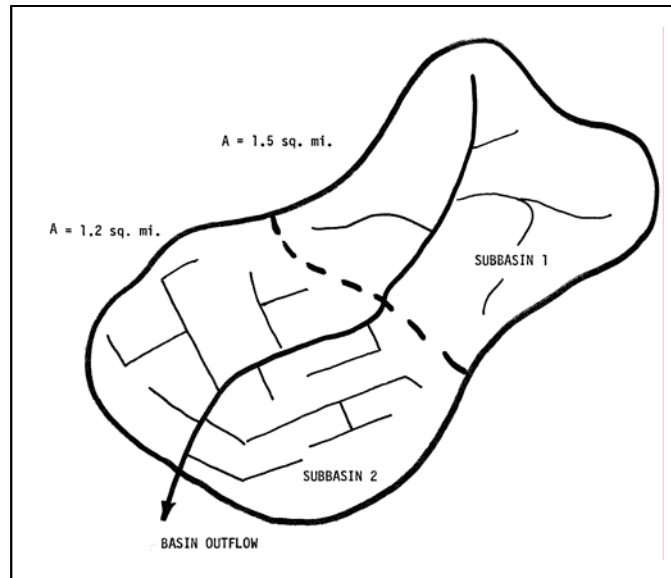


Figure A-1 Basin for Example Problem

Subbasin 1: The upper of the two subareas making up the basin is presently not urbanized and is primarily rolling pastureland with few trees. The typical distance L_o for flow to travel to tributary stream channels is 500 feet. The overland flow roughness coefficient, N , is 0.4. The representative ground slope S_o is 0.04. The amount of impervious area is assumed to be negligible, and the subbasin area A_{o1} is 1.5 square miles.

The collector or tributary channels have a typical slope S_c of 0.025 and an 'n' value of 0.10, with a typical channel length, L_c , of 1,500 feet. The most representative section is a triangle. The area, A_c , contributing to a typical collector stream is 0.4 square mile. The main channel is approximately triangular in cross section with side slopes, z , of 1 in 4. The mean channel slope S_m is 0.01 and Manning's 'n' is 0.05. Its length, L_m , is 3,500 feet.

Subbasin 2: The lower subbasin is completely urbanized, and twenty percent of the subbasin surfaces are impervious. In this subbasin the impervious runoff areas have the following characteristics: $L_{o1} = 50$ feet, $S_{o1} = 0.06$, $N = 0.15$. The pervious areas can be represented by the following parameters: $L_{o2} = 130$ feet, $S_{o2} = 0.01$, $N = 0.30$. The subbasin area, A_{o2} , is 1.2 square miles. The total basin area is 2.7 square miles. The collector channel system involves three hundred feet of gutter plus an additional 1,800 feet of pipe storm drain ranging up to four feet in diameter. A triangular section is used to represent the various channel components (the program default value with one to one side slopes). The average

slope, S_c , is 0.008, and the Manning's 'n', which accounts for friction and other channel head losses is 0.020. The area, A, contributing to the collector channel system is 0.35 square mile.

The parameters describing this basin are given in Table A-1. Also, a listing of the program input for modeling this basin is provided in Figure A-2 and hydrographs from the run output are shown in Figure A-3. A listing of the program output is provided in Figure A-4.

**Table A-1
Parameters for Kinematic Wave Example Problem**

	Percent Total Subbasin Area	L (ft)	S	Roughness	Channel Shape	Channel Size (ft)	Z	Collector System Area (mi ²)	Loss Rate (in/hr)
Subbasin 1 (1.5 mi²)									
1. Overland Flow Strip	100	500	0.04	0.40	--	--	--	--	0.50
2. Collector Channel	--	1,500	0.025	0.10	Triangle	--	1	0.40	--
3. Main Channel	--	3,500	0.010	0.05	Triangle	--	4	--	--
Subbasin 2 (1.2 mi²)									
1. Overland Flow Strip 1	20	50	0.06	0.30	--	--	--	--	0.02
2. Overland Flow Strip 2	80	180	0.01	0.40	--	--	--	--	0.20
3. Collector Channel	--	2,100	0.008	0.020	Triangle	--	1	0.35	--
4. Main Channel	--	4,000	0.003	0.025	Trapezoid	2	2	--	--

```

ID          Kinematic Wave Routing in HEC-1
ID          Example Basin Model Used in Training Document 10
ID
ID          Original Basin Conditions
IT          5 1MAY79 1200 100
IO          3
KK          SUB1
KM          Subarea 1 A=1.5 Sq Mi L=3500 ft
KO          1 0
PB          2.0
ZR          =PI A=TEST B=EXAMPLE C=PRECIP-INC F=OBS
LU          0.0 0.5 0.0
BA          1.5
UK          500 .04 .4 100
RK          1500 .025 .1 .4
RK          3500 .01 .05 2.17 TRAP 0 4
ZW
KK          SUB2
KM          Subarea 2 A=1.2 Sq Mi L=400 ft
KO          1 0
LU          0.0 0.02 0.0 0.0 0.20 0.0
BA          1.2
UK          50 .06 .3 20
UK          180 .01 .4 80
RK          2100 .008 .02 .35
RK          4000 .003 .025 0 TRAP 2 2 YES
ZW          C=FLOW
ZZ
  
```

Figure A-2 HEC-1 Input File Using Kinematic Wave Method

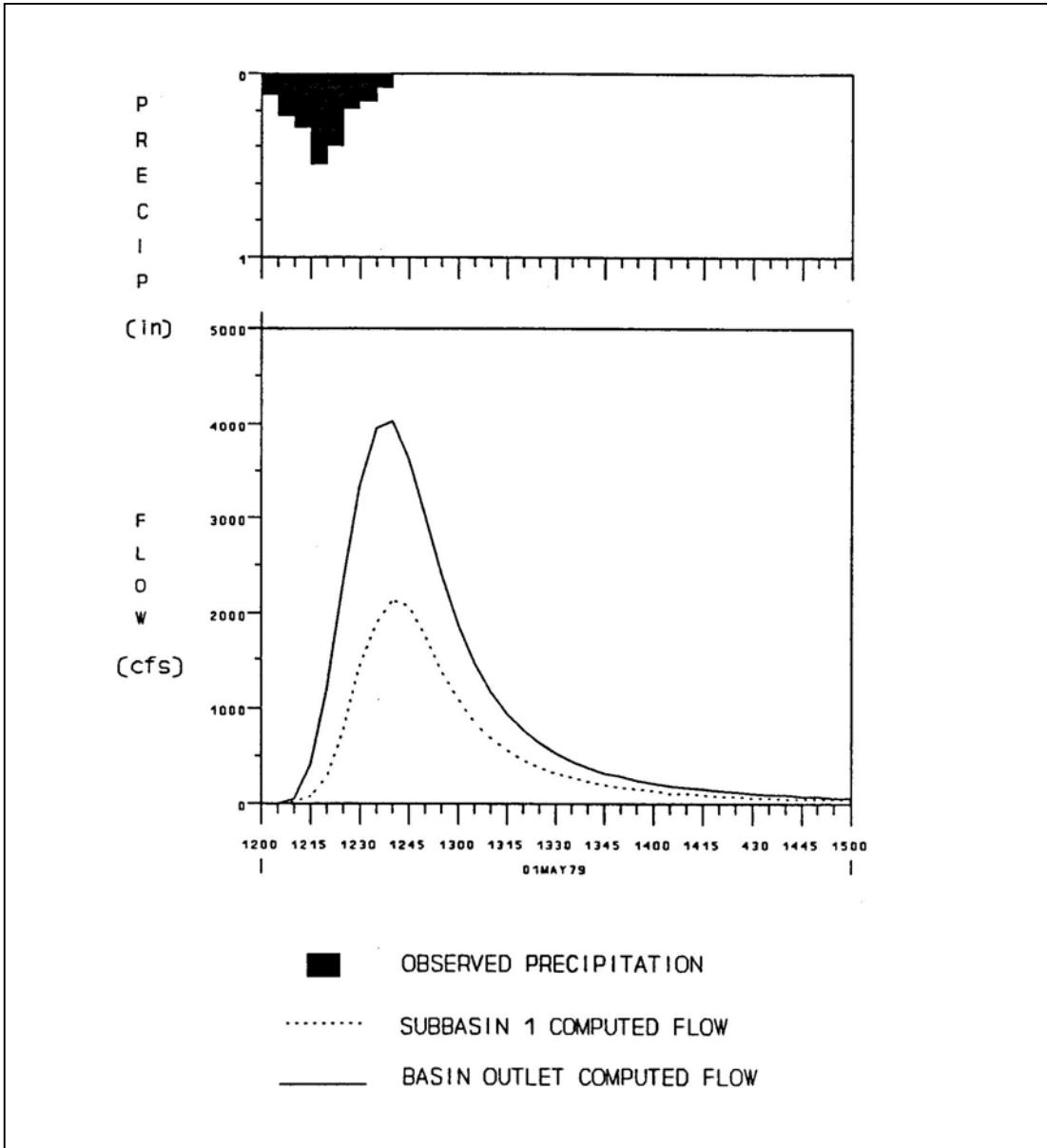


Figure A-3 Computed Hydrograph at Outlet of Subbasin 1 and at Outlet of Entire Basin

```

*****
* FLOOD HYDROGRAPH PACKAGE (HEC-1) *
* SEPTEMBER 1990 *
* VERSION 4.0 *
* RUN DATE 04/30/1991 TIME 14:32:49 *
*****

```

```

*****
* U.S. ARMY CORPS OF ENGINEERS *
* HYDROLOGIC ENGINEERING CENTER *
* 609 SECOND STREET *
* DAVIS, CALIFORNIA 95616 *
* (916) 756-1104 *
*****

```

```

X X XXXXXXXX XXXXX X
X X X X X XX
X X X X X X
XXXXXXX XXXX X XXXXX X
X X X X X X
X X X X X X
X X XXXXXXXX XXXXX XXX

```

THIS PROGRAM REPLACES ALL PREVIOUS VERSIONS OF HEC-1 KNOWN AS HEC1 (JAN 73), HEC1GS, HEC1DB, AND HEC1KW.

THE DEFINITIONS OF VARIABLES -RTIMP- AND -RTIOR- HAVE CHANGED FROM THOSE USED WITH THE 1973-STYLE INPUT STRUCTURE. THE DEFINITION OF -AMSK- ON RM-CARD WAS CHANGED WITH REVISIONS DATED 28 SEP 81. THIS IS THE FORTRAN77 VERSION

NEW OPTIONS: DAMBREAK OUTFLOW SUBMERGENCE , SINGLE EVENT DAMAGE CALCULATION, DSS:WRITE STAGE FREQUENCY, DSS:READ TIME SERIES AT DESIRED CALCULATION INTERVAL LOSS RATE:GREEN AND AMPT INFILTRATION KINEMATIC WAVE: NEW FINITE DIFFERENCE ALGORITHM

HEC-1 INPUT PAGE 1

```

LINE ID.....1.....2.....3.....4.....5.....6.....7.....8.....9.....10
1 ID Kinematic Wave Routing in HEC-1
2 ID Example Basin Model Used in Training Document 10
3 ID
4 ID Original Basin Conditions
5 IT 5 1MAY79 1200 100
6 IO 4
7 KK SUB1
8 KM Subarea 1 A=1.5 Sq Mi L=3500 ft
9 KO 3 2
10 PB 2.0
11 PI 6 12 15 25 20 10 8 4
12 LU 0.0 0.5 0.0
13 BA 1.5
14 UK 500 .04 .4 100
15 RK 1500 .025 .1 .4
16 RK 3500 .01 .05 2.17 TRAP 0 4
17 KK SUB2
18 KM Subarea 2 A=1.2 Sq Mi L=400 ft
19 KO 1 2
20 LU 0.0 0.02 0.0 0.0 0.20 0.0
21 BA 1.2
22 UK 50 .06 .3 20
23 UK 180 .01 .4 80
24 RK 2100 .008 .02 .35
25 RK 4000 .003 .025 0 TRAP 2 2 YES
26 ZZ

```

```

*****
* FLOOD HYDROGRAPH PACKAGE (HEC-1) *
* SEPTEMBER 1990 *
* VERSION 4.0 *
* RUN DATE 04/30/1991 TIME 14:32:49 *
*****

```

```

*****
* U.S. ARMY CORPS OF ENGINEERS *
* HYDROLOGIC ENGINEERING CENTER *
* 609 SECOND STREET *
* DAVIS, CALIFORNIA 95616 *
* (916) 756-1104 *
*****

```

```

Kinematic Wave Routing in HEC-1
Example Basin Model Used in Training Document 10
Original Basin Conditions
6 IO OUTPUT CONTROL VARIABLES
IPRNT 4 PRINT CONTROL
IPLOT 0 PLOT CONTROL
QSCAL 0. HYDROGRAPH PLOT SCALE
IT HYDROGRAPH TIME DATA
NMIN 5 MINUTES IN COMPUTATION INTERVAL
IDATE 1MAY79 STARTING DATE
ITIME 1200 STARTING TIME
NQ 100 NUMBER OF HYDROGRAPH ORDINATES
NDDATE 1MAY79 ENDING DATE
NDTIME 2015 ENDING TIME
ICENT 19 CENTURY MARK
COMPUTATION INTERVAL .08 HOURS
TOTAL TIME BASE 8.25 HOURS
ENGLISH UNITS
DRAINAGE AREA SQUARE MILES
PRECIPITATION DEPTH INCHES
LENGTH, ELEVATION FEET
FLOW CUBIC FEET PER SECOND
STORAGE VOLUME ACRE-FEET
SURFACE AREA ACRES
TEMPERATURE DEGREES FAHRENHEIT

```

Figure A-4 HEC-1 Output File for Example Problem


```

*****
*          *
7 KK      SUB1 *
*          *
*****

          Subarea 1 A=1.5 Sq Mi L=3500 ft

9 KO      OUTPUT CONTROL VARIABLES
          IPRNT      3 PRINT CONTROL
          IPLOT      2 PLOT CONTROL
          QSCAL      0. HYDROGRAPH PLOT SCALE

SUBBASIN RUNOFF DATA

13 BA     SUBBASIN CHARACTERISTICS
          TAREA      1.50 SUBBASIN AREA

PRECIPITATION DATA

10 PB     STORM      2.00 BASIN TOTAL PRECIPITATION

11 PI     INCREMENTAL PRECIPITATION PATTERN
          6.00      12.00      15.00      20.00      10.00      8.00      4.00

12 LU     UNIFORM LOSS RATE
          STRTL      .00 INITIAL LOSS
          CNSTL      .50 UNIFORM LOSS RATE
          RTIMP      .00 PERCENT IMPERVIOUS AREA

KINEMATIC WAVE

14 UK     OVERLAND-FLOW ELEMENT NO. 1
          L          500. OVERLAND FLOW LENGTH
          S          .0400 SLOPE
          N          .400 ROUGHNESS COEFFICIENT
          PA         100.0 PERCENT OF SUBBASIN
          DXMIN      5 MINIMUM NUMBER OF DX INTERVALS

KINEMATIC WAVE

15 RK     COLLECTOR CHANNEL
          L          1500. CHANNEL LENGTH
          S          .0250 SLOPE
          N          .100 CHANNEL ROUGHNESS COEFFICIENT
          CA         .40 CONTRIBUTING AREA
          SHAPE      TRAP CHANNEL SHAPE
          WD         .00 BOTTOM WIDTH OR DIAMETER
          Z          1.00 SIDE SLOPE
          NDXMIN     2 MINIMUM NUMBER OF DX INTERVALS

16 RK     MAIN CHANNEL
          L          3500. CHANNEL LENGTH
          S          .0100 SLOPE
          N          .050 CHANNEL ROUGHNESS COEFFICIENT
          CA         1.50 CONTRIBUTING AREA
          SHAPE      TRAP CHANNEL SHAPE
          WD         .00 BOTTOM WIDTH OR DIAMETER
          Z          4.00 SIDE SLOPE
          NDXMIN     2 MINIMUM NUMBER OF DX INTERVALS
          RUPSTQ     NO ROUTE UPSTREAM HYDROGRAPH

          ***
          COMPUTED KINEMATIC PARAMETERS
          VARIABLE TIME STEP
          (DT SHOWN IS A MINIMUM)

          ELEMENT   ALPHA      M          DT          DX          PEAK      TIME TO      VOLUME      MAXIMUM
                   (MIN)      (FT)      (CFS)      (MIN)      (IN)      CELERITY
                   (MIN)      (FT)      (CFS)      (MIN)      (IN)      (FPS)

          PLANE1    .75      1.67      5.01      100.00      2250.81      36.15      1.63      .33
          COLLECTOR1 1.18      1.33      1.21      500.00      2181.44      40.81      1.63      8.02
          MAIN      1.16      1.33      1.91      1166.67      2151.60      41.02      1.63      10.92

CONTINUITY SUMMARY (AC-FT) - INFLOW= .0000E+00 EXCESS= .1333E+03 OUTFLOW= .1304E+03 BASIN STORAGE= .3689E+00 PERCENT ERROR= 1.9

          INTERPOLATED TO SPECIFIED COMPUTATION INTERVAL

          MAIN      1.16      1.33      5.00      2137.53      40.00      1.63

          ***          ***          ***          ***          ***

          HYDROGRAPH AT STATION SUB1

TOTAL RAINFALL = 2.00, TOTAL LOSS = .33, TOTAL EXCESS = 1.67

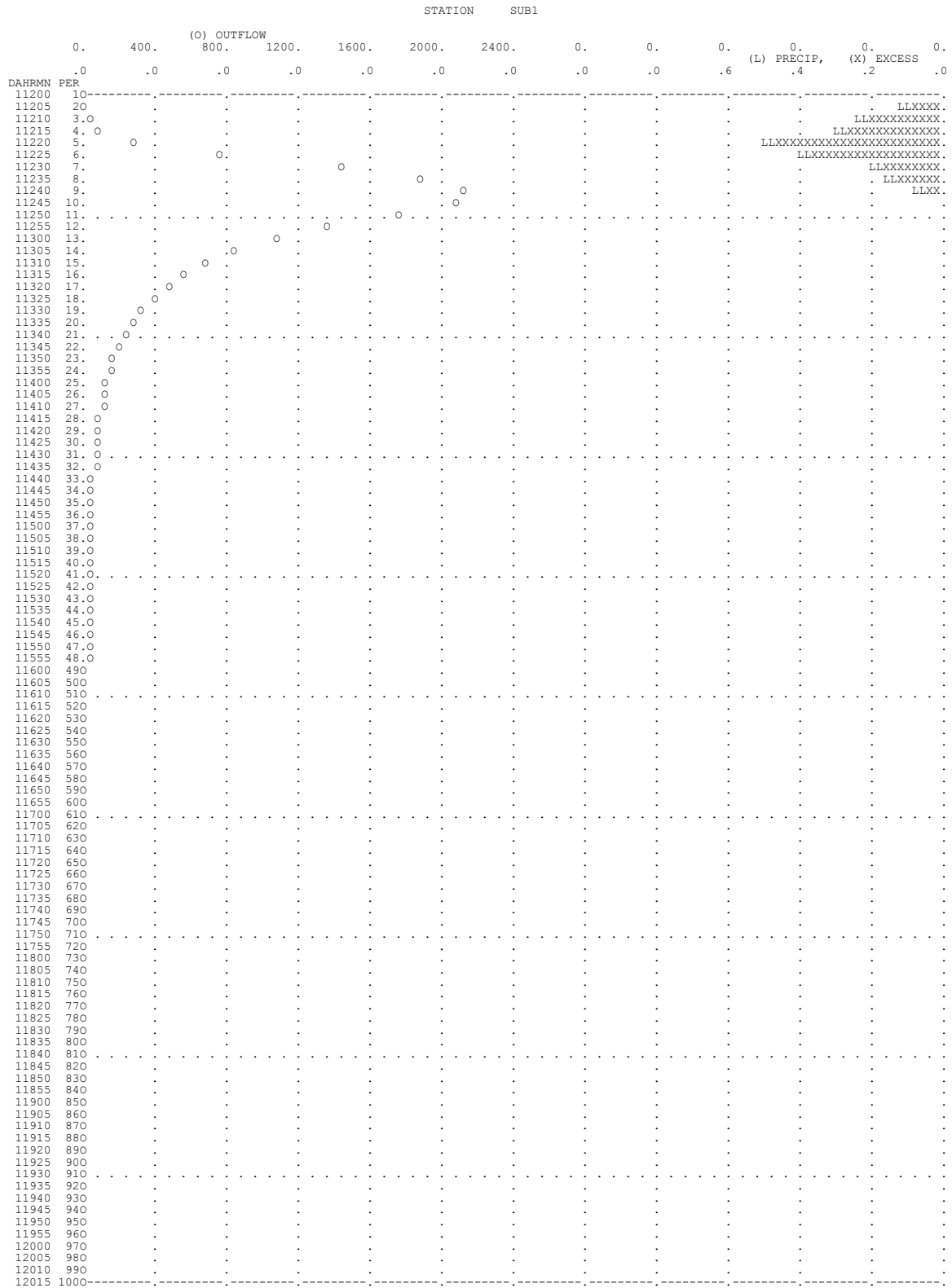
PEAK FLOW      TIME      MAXIMUM AVERAGE FLOW
(CFS)          (HR)          6-HR      24-HR      72-HR      8.25-HR

2138.          .67 (CFS)          261.          191.          191.          191.
              (INCHES) 1.620          1.630          1.630          1.630
              (AC-FT) 130.          130.          130.          130.

CUMULATIVE AREA = 1.50 SQ MI

```

Figure A-4 HEC-1 Output File for Example Problem (continued)



*** **

Figure A-4 HEC-1 Output File for Example Problem (continued)

```

*****
*           *
*   SUB2   *
*           *
*****
Subaea 2  A=1.2 Sq Mi  L=400 ft

19 KO      OUTPUT CONTROL VARIABLES
          IFRNT      1      PRINT CONTROL
          IPLOT      2      PLOT CONTROL
          QSCAL      0.     HYDROGRAPH PLOT SCALE

SUBBASIN RUNOFF DATA

21 BA      SUBBASIN CHARACTERISTICS
          TAREA      1.20  SUBBASIN AREA

PRECIPITATION DATA

10 PB      STORM      2.00  BASIN TOTAL PRECIPITATION

11 PI      INCREMENTAL PRECIPITATION PATTERN
          6.00      12.00      15.00      25.00      20.00      10.00      8.00      4.00

20 LU      UNIFORM LOSS RATE
          STRTL      .00    INITIAL LOSS
          CNSTL      .02    UNIFORM LOSS RATE
          RTIMP      .00    PERCENT IMPERVIOUS AREA

          LOSS RATE VARIABLES FOR SECOND OVERLAND FLOW ELEMENT
          STRTL      .00    INITIAL LOSS
          CNSTL      .20    UNIFORM LOSS RATE
          RTIMP      .00    PERCENT IMPERVIOUS AREA

KINEMATIC WAVE

22 UK      OVERLAND-FLOW ELEMENT NO. 1
          L          50.    OVERLAND FLOW LENGTH
          S          .0600  SLOPE
          N          .300   ROUGHNESS COEFFICIENT
          PA         20.0   PERCENT OF SUBBASIN
          DXMIN      5      MINIMUM NUMBER OF DX INTERVALS

23 UK      OVERLAND-FLOW ELEMENT NO. 2
          L          180.   OVERLAND FLOW LENGTH
          S          .0100  SLOPE
          N          .400   ROUGHNESS COEFFICIENT
          PA         80.0   PERCENT OF SUBBASIN
          DXMIN      5      MINIMUM NUMBER OF DX INTERVALS

KINEMATIC WAVE

24 RK      COLLECTOR CHANNEL
          L          2100.  CHANNEL LENGTH
          S          .0080  SLOPE
          N          .020   CHANNEL ROUGHNESS COEFFICIENT
          CA         .35    CONTRIBUTING AREA
          SHAPE      TRAP   CHANNEL SHAPE
          WD         .00    BOTTOM WIDTH OR DIAMETER
          Z          1.00   SIDE SLOPE
          NDXMIN     2      MINIMUM NUMBER OF DX INTERVALS

25 RK      MAIN CHANNEL
          L          4000.  CHANNEL LENGTH
          S          .0030  SLOPE
          N          .025   CHANNEL ROUGHNESS COEFFICIENT
          CA         1.20   CONTRIBUTING AREA
          SHAPE      TRAP   CHANNEL SHAPE
          WD         2.00   BOTTOM WIDTH OR DIAMETER
          Z          2.00   SIDE SLOPE
          NDXMIN     2      MINIMUM NUMBER OF DX INTERVALS
          RUPSTQ     YES    ROUTE UPSTREAM HYDROGRAPH

          ***
          COMPUTED KINEMATIC PARAMETERS
          VARIABLE TIME STEP
          (DT SHOWN IS A MINIMUM)

          ELEMENT   ALPHA      M          DT          DX          PEAK      TIME TO      VOLUME      MAXIMUM
                   (MIN)      (FT)      (CFS)      (MIN)      (IN)      (FPS)
          PLANE1    1.22      1.67      .62      10.00      910.43      19.86      1.98      .27
          PLANE2    .37       1.67      3.52      36.00      2047.23      32.29      1.84      .17
          COLLECTOR1 3.34      1.33      .68      700.00      2398.00      30.41      1.87      18.21
          MAIN      1.42      1.35      1.47      1333.33      4086.93      37.47      1.73      15.75

CONTINUITY SUMMARY (AC-FT) - INFLOW= .1304E+03  EXCESS= .1210E+03  OUTFLOW= .2493E+03  BASIN STORAGE= .2237E+00  PERCENT ERROR= .7

```

Figure A-4 HEC-1 Output File for Example Problem (continued)

INTERPOLATED TO SPECIFIED COMPUTATION INTERVAL

MAIN 1.42 1.35 5.00 4030.04 40.00 1.73

HYDROGRAPH AT STATION SUB2

DA	MON	HRMN	ORD	RAIN	LOSS	EXCESS	COMP Q	*	DA	MON	HRMN	ORD	RAIN	LOSS	EXCESS	COMP Q
1	MAY	1200	1	.00	.00	.00	0.	*	1	MAY	1610	51	.00	.00	.00	26.
1	MAY	1205	2	.12	.01	.11	3.	*	1	MAY	1615	52	.00	.00	.00	24.
1	MAY	1210	3	.24	.01	.23	57.	*	1	MAY	1620	53	.00	.00	.00	23.
1	MAY	1215	4	.30	.01	.29	413.	*	1	MAY	1625	54	.00	.00	.00	22.
1	MAY	1220	5	.50	.01	.49	1193.	*	1	MAY	1630	55	.00	.00	.00	22.
1	MAY	1225	6	.40	.01	.39	2332.	*	1	MAY	1635	56	.00	.00	.00	21.
1	MAY	1230	7	.20	.01	.19	3320.	*	1	MAY	1640	57	.00	.00	.00	20.
1	MAY	1235	8	.16	.01	.15	3954.	*	1	MAY	1645	58	.00	.00	.00	18.
1	MAY	1240	9	.08	.01	.07	4030.	*	1	MAY	1650	59	.00	.00	.00	17.
1	MAY	1245	10	.00	.00	.00	3625.	*	1	MAY	1655	60	.00	.00	.00	16.
1	MAY	1250	11	.00	.00	.00	3015.	*	1	MAY	1700	61	.00	.00	.00	16.
1	MAY	1255	12	.00	.00	.00	2405.	*	1	MAY	1705	62	.00	.00	.00	15.
1	MAY	1300	13	.00	.00	.00	1876.	*	1	MAY	1710	63	.00	.00	.00	15.
1	MAY	1305	14	.00	.00	.00	1473.	*	1	MAY	1715	64	.00	.00	.00	14.
1	MAY	1310	15	.00	.00	.00	1169.	*	1	MAY	1720	65	.00	.00	.00	14.
1	MAY	1315	16	.00	.00	.00	942.	*	1	MAY	1725	66	.00	.00	.00	13.
1	MAY	1320	17	.00	.00	.00	770.	*	1	MAY	1730	67	.00	.00	.00	12.
1	MAY	1325	18	.00	.00	.00	640.	*	1	MAY	1735	68	.00	.00	.00	11.
1	MAY	1330	19	.00	.00	.00	530.	*	1	MAY	1740	69	.00	.00	.00	11.
1	MAY	1335	20	.00	.00	.00	450.	*	1	MAY	1745	70	.00	.00	.00	11.
1	MAY	1340	21	.00	.00	.00	382.	*	1	MAY	1750	71	.00	.00	.00	10.
1	MAY	1345	22	.00	.00	.00	330.	*	1	MAY	1755	72	.00	.00	.00	10.
1	MAY	1350	23	.00	.00	.00	286.	*	1	MAY	1800	73	.00	.00	.00	10.
1	MAY	1355	24	.00	.00	.00	248.	*	1	MAY	1805	74	.00	.00	.00	10.
1	MAY	1400	25	.00	.00	.00	221.	*	1	MAY	1810	75	.00	.00	.00	9.
1	MAY	1405	26	.00	.00	.00	195.	*	1	MAY	1815	76	.00	.00	.00	9.
1	MAY	1410	27	.00	.00	.00	172.	*	1	MAY	1820	77	.00	.00	.00	8.
1	MAY	1415	28	.00	.00	.00	155.	*	1	MAY	1825	78	.00	.00	.00	8.
1	MAY	1420	29	.00	.00	.00	140.	*	1	MAY	1830	79	.00	.00	.00	8.
1	MAY	1425	30	.00	.00	.00	124.	*	1	MAY	1835	80	.00	.00	.00	7.
1	MAY	1430	31	.00	.00	.00	113.	*	1	MAY	1840	81	.00	.00	.00	7.
1	MAY	1435	32	.00	.00	.00	104.	*	1	MAY	1845	82	.00	.00	.00	7.
1	MAY	1440	33	.00	.00	.00	95.	*	1	MAY	1850	83	.00	.00	.00	7.
1	MAY	1445	34	.00	.00	.00	85.	*	1	MAY	1855	84	.00	.00	.00	7.
1	MAY	1450	35	.00	.00	.00	78.	*	1	MAY	1900	85	.00	.00	.00	7.
1	MAY	1455	36	.00	.00	.00	72.	*	1	MAY	1905	86	.00	.00	.00	6.
1	MAY	1500	37	.00	.00	.00	67.	*	1	MAY	1910	87	.00	.00	.00	6.
1	MAY	1505	38	.00	.00	.00	62.	*	1	MAY	1915	88	.00	.00	.00	6.
1	MAY	1510	39	.00	.00	.00	57.	*	1	MAY	1920	89	.00	.00	.00	6.
1	MAY	1515	40	.00	.00	.00	52.	*	1	MAY	1925	90	.00	.00	.00	5.
1	MAY	1520	41	.00	.00	.00	49.	*	1	MAY	1930	91	.00	.00	.00	5.
1	MAY	1525	42	.00	.00	.00	46.	*	1	MAY	1935	92	.00	.00	.00	5.
1	MAY	1530	43	.00	.00	.00	44.	*	1	MAY	1940	93	.00	.00	.00	5.
1	MAY	1535	44	.00	.00	.00	40.	*	1	MAY	1945	94	.00	.00	.00	5.
1	MAY	1540	45	.00	.00	.00	37.	*	1	MAY	1950	95	.00	.00	.00	5.
1	MAY	1545	46	.00	.00	.00	35.	*	1	MAY	1955	96	.00	.00	.00	5.
1	MAY	1550	47	.00	.00	.00	33.	*	1	MAY	2000	97	.00	.00	.00	5.
1	MAY	1555	48	.00	.00	.00	32.	*	1	MAY	2005	98	.00	.00	.00	5.
1	MAY	1600	49	.00	.00	.00	30.	*	1	MAY	2010	99	.00	.00	.00	4.
1	MAY	1605	50	.00	.00	.00	28.	*	1	MAY	2015	100	.00	.00	.00	4.

TOTAL RAINFALL = 2.00, TOTAL LOSS = .11, TOTAL EXCESS = 1.89

PEAK FLOW (CFS)	TIME (HR)	MAXIMUM AVERAGE FLOW			
		6-HR	24-HR	72-HR	8.25-HR
4030.	.67	500. (INCHES) 248.	365. 1,729 249.	365. 1,729 249.	365. 1,729 249.

CUMULATIVE AREA = 2.70 SQ MI

Figure A-4 HEC-1 Output File for Example Problem (continued)

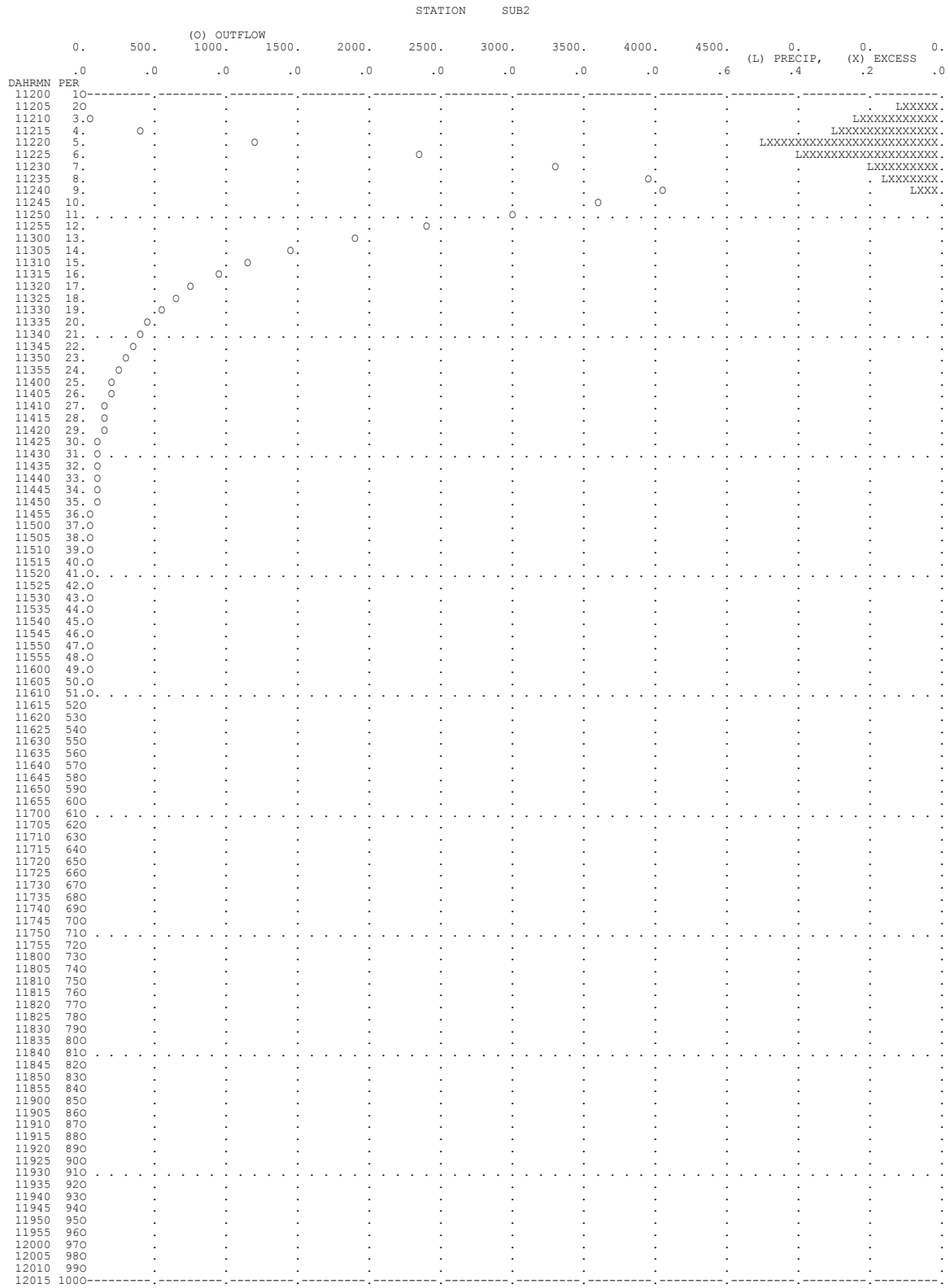


Figure A-4 HEC-1 Output File for Example Problem (continued)

```

                                RUNOFF SUMMARY
                                FLOW IN CUBIC FEET PER SECOND
                                TIME IN HOURS, AREA IN SQUARE MILES
OPERATION      STATION      PEAK FLOW      TIME OF PEAK      AVERAGE FLOW FOR MAXIMUM PERIOD
HYDROGRAPH AT SUB1      2138.         .67             261.             6-HOUR           24-HOUR           72-HOUR
HYDROGRAPH AT SUB2      4030.         .67             500.             365.             365.             365.
                                BASIN AREA      MAXIMUM STAGE      TIME OF MAX STAGE
                                1.50
                                2.70

                                SUMMARY OF KINEMATIC WAVE - MUSKINGUM-CUNGE ROUTING
                                (FLOW IS DIRECT RUNOFF WITHOUT BASE FLOW)
                                INTERPOLATED TO COMPUTATION INTERVAL
                                PEAK TIME TO PEAK
                                DT (MIN)      VOLUME (IN)      DT (MIN)      PEAK (CFS)      TIME TO PEAK (MIN)      VOLUME (IN)
SUB1  MANE      1.91      2151.60      41.02      1.63      5.00      2137.53      40.00      1.63

CONTINUITY SUMMARY (AC-FT) - INFLOW= .0000E+00 EXCESS= .1333E+03 OUTFLOW= .1304E+03 BASIN STORAGE= .3689E+00 PERCENT ERROR= 1.9

SUB2  MANE      1.47      4086.93      37.47      1.73      5.00      4030.04      40.00      1.73

CONTINUITY SUMMARY (AC-FT) - INFLOW= .1304E+03 EXCESS= .1210E+03 OUTFLOW= .2493E+03 BASIN STORAGE= .2237E+00 PERCENT ERROR= .7

*** NORMAL END OF HEC-1 ***

```

Figure A-4 HEC-1 Output File for Example Problem (continued)

The table entitled "SUMMARY OF KINEMATIC WAVE - MUSKINGUM-CUNGE ROUTING", found at the end of the output listing, and is especially important for kinematic wave modeling. The user should always examine the difference between the peak discharges, time to peak and hydrograph volume computed with an internal Δt with that interpolated to the user specified Δt .

Consider, for example, the results summarized for subbasin 2 in Figure A-4. The computed peak flow is 4,086.93 cfs, the time to peak is 27.47 minutes and the hydrograph volume is 1.73 inches. When interpolated to the specified computation interval (5 minutes, NMIN on the IT record), these values become 4,030.04 cfs, 40.00 minutes and 1.73 inches. This corresponds to an overall interpolation error of 0.7 percent, which is acceptable for most purposes. However, if the interpolation error becomes too large, reducing the computation interval, NMIN, may reduce it.

As an example of the way the program can be used to evaluate the effects of future urbanization, the following changes were made to the parameters describing subbasin 1. Two overland flow strips were used instead of one; the impervious overland flow element was sized as $l_{01}=50$ feet; $N=0.3$; and this impervious flow strip represented twenty percent of the subbasin area. The pervious element's representative flow length was reduced from 500 to 400 feet, and it represented 80 percent of the basin. The other parameters were held constant. The results from this run are shown in Figure A-5. The peak flow at the basin outlet increased from 4,030 cfs for the initial basin condition to 4,803 cfs for the increased urbanization in subbasin 1. The peak flow occurs five minutes earlier due to the increased urbanization.

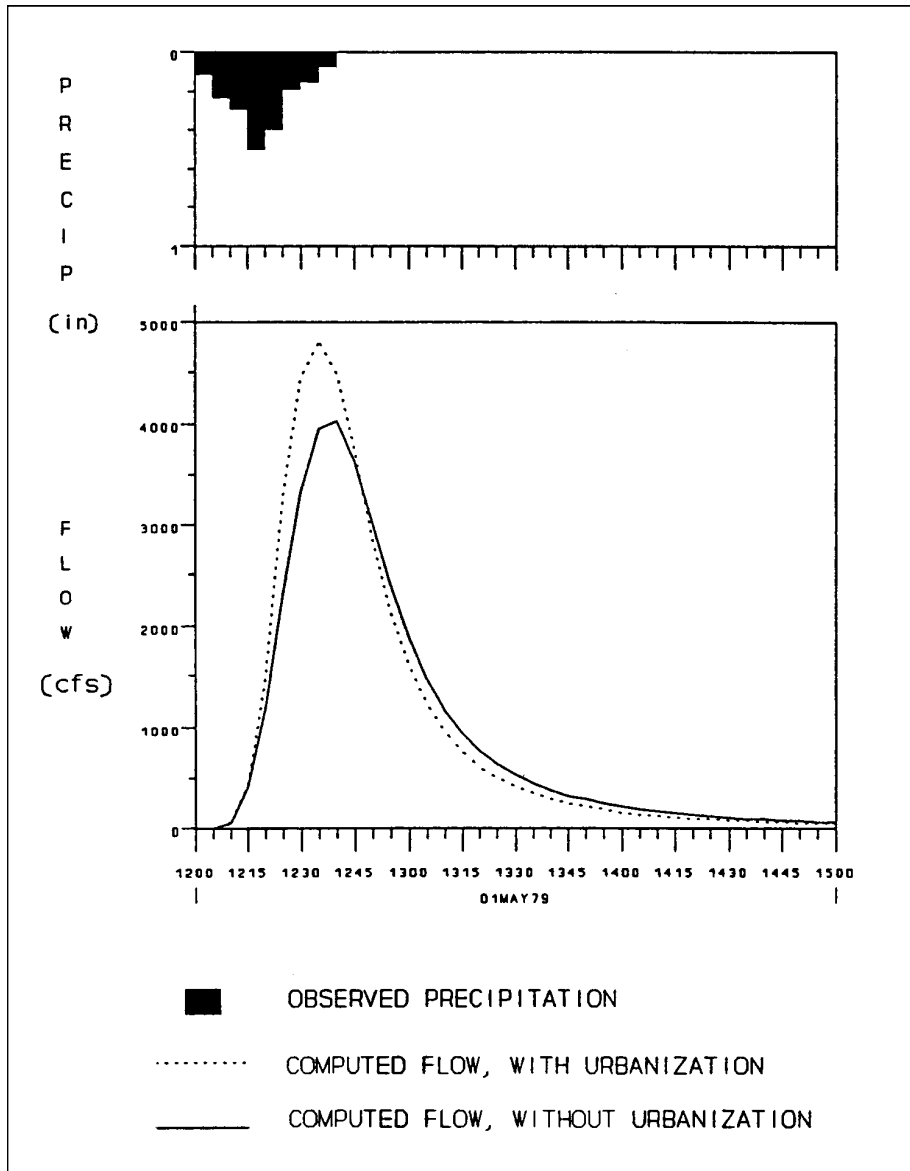


Figure A-5 Computed Hydrographs at Outlet with and without Urbanization

Parameter Sensitivity

Additional HEC-1 runs were made with changes to some of the watershed parameters to give an indication of the sensitivity of the modeling process to changes of various magnitudes. Parameters were only adjusted for subbasin 2. The flow output, however, includes routed flows from the unmodified subbasin 1.

Analysis of Q Versus Surface Roughness

The effects of variation of the surface roughness parameter, N , for overland flow is shown in Table A-2. Low values of N produce high flows and low times for the hydrograph peak. High N values produce lower peak flows and retard the peak. Note that the high values of N are outside the acceptable range shown in Figure 3 (page 6), but are used here for trend

extrapolation only. An order of magnitude change in N values results in a 27 percent change in the flow rate. This variation of results with changes in N is shown graphically in Figure A-6.

Table A-2
Sensitivity to Changes in Surface Roughness

Case	Subbasin 2		Percent Original	Q _{peak} Total (cfs)	TP (min)
	N ₁	N ₂			
1	.075	.10	25	4218	25
2	.15	.20	50	4173	30
3	.23	.30	75	4110	35
4	.30	.40	100	4030	40
5	.45	.60	150	3826	40
6	.60	.80	200	3534	45
7	.75	1.0	250	3308	45

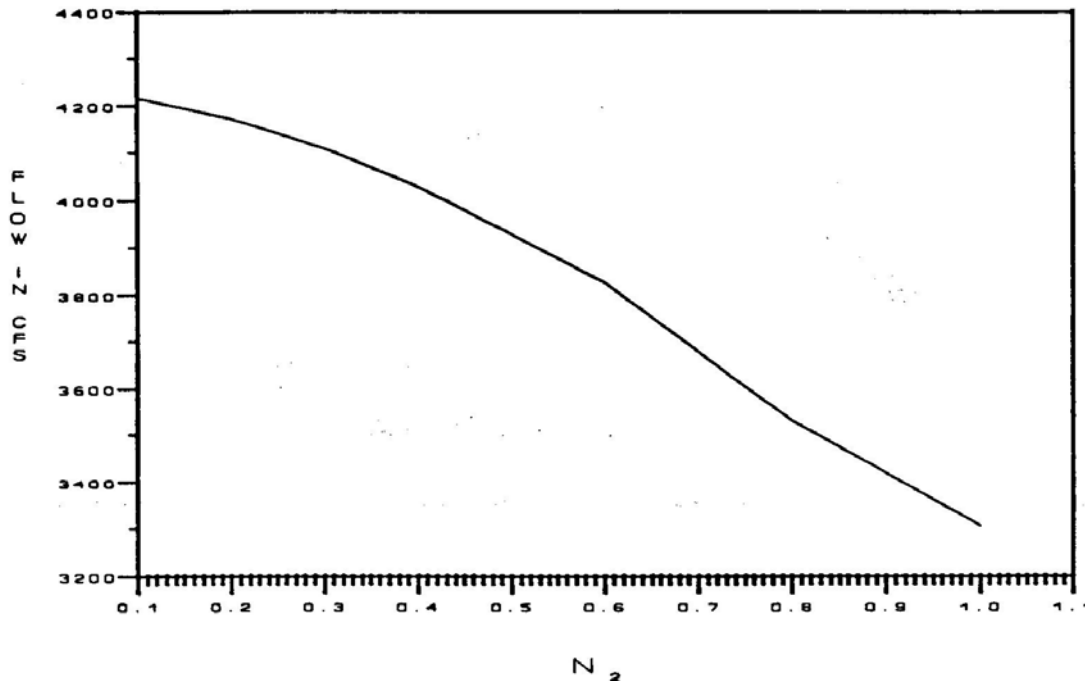


Figure A-6 Graphical Representation of Sensitivity to Changes in Surface Roughness

Analysis of Q Versus Overland Flow Length

As shown in Table A-3, increasing the overland flow plane reduces the computed peak flow and delays the hydrograph peak. Using short lengths has the opposite effect. The results are shown graphically in Figure A-7.

Table A-3
Sensitivity to Changes in Overland Flow Length

Case	L-strip 2 in Subbasin 2 (ft)	Q _{peak} Total (cfs)	TP (min)
1	80	4248	30
2	120	4129	35
3	160	4068	35
4	180	4030	40
5	220	3996	40
6	260	3858	40
7	300	3696	45

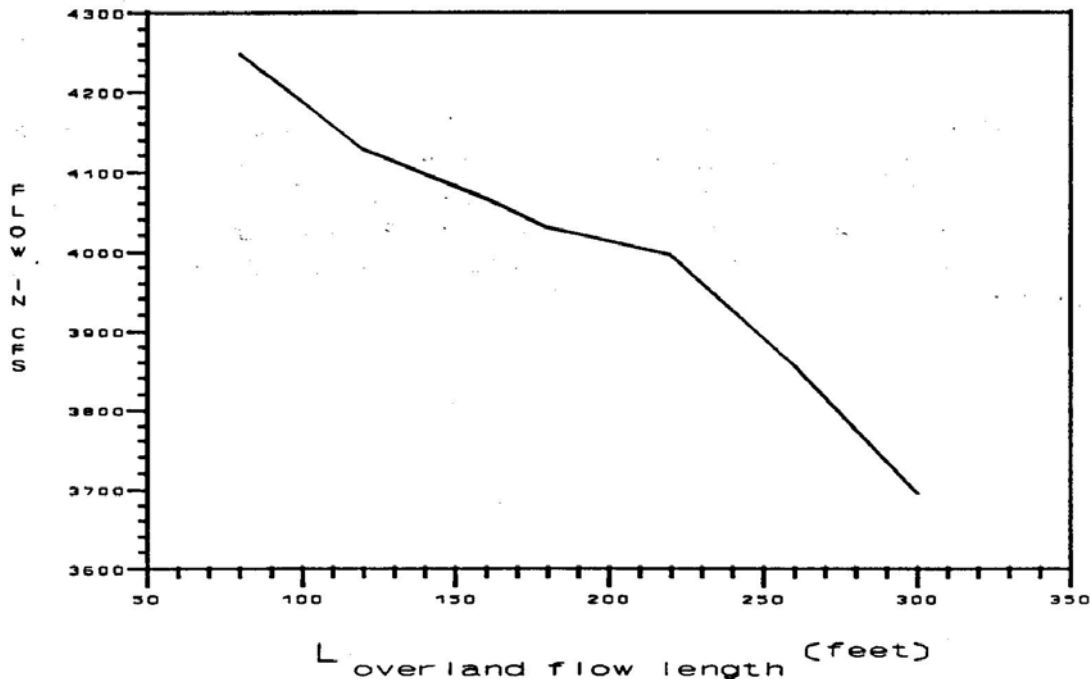


Figure A-7 Graphical Representation of Sensitivity to Changes in Overland Flow Length

Analysis of Q Versus NDXMIN

As mentioned in Chapter 1, NDXMIN is an optional parameter used to force the HEC-1 program to use a smaller distance step. The distance step used in the analysis, Δx , is defined as the minimum of the calculated Δx and $L/NDXMIN$. This NDXMIN parameter was varied between acceptable bounds ($5 \leq NDXMIN \leq 50$ for overland flow planes) for the overland flow plane in subbasin 1 to determine its effect on the resulting peak flow rate. The results are shown in Table A-4.

The first case in the table is from an HEC-1 run with NDXMIN unspecified. Note how the peak flow rate increases with NDXMIN when NDXMIN is less than twenty. However, any increase of NDXMIN above this amount does not appreciably increase the peak flow rate. Thus, the solution with NDXMIN equal to twenty is a good compromise between accuracy and computational efficiency. Although some accuracy was gained by increasing NDXMIN, the

interpolation errors reported in Table A-4 are relatively low. Thus, the default Δx selection would most likely be adequate.

Table A-4
Sensitivity to Changes in NDXMIN for Subbasin 1

Case	DXMIN	Q ₁ (cfs)	%E ₁
1	5	2138	1.9
2	10	2170	1.1
3	15	2190	0.8
4	20	2199	0.7
5	25	2205	0.6
6	50	2206	0.6

Another kinematic wave routing parameter which can be adjusted is the slope. However, both the slope, S_o , and the roughness, N , are combined in the parameter α_o , and either may be adjusted to produce the desired effect. However, as indicated by Equation 4 (page 34), α_o is a function of $S_o^{1/2}$; therefore, changing S_o has less relative effect than changing the roughness. For example, to produce the same change in α_o as caused by reducing N by a factor of two, the slope, S_o , would have to be four times as large.

Appendix B Sample Data Tabulation Form

	Percent Total Subbasin Area	L (ft)	S	Roughness	Channel Shape	Channel Size (ft)	Z	Collector System Area (mi ²)	Loss Rate (in/hr)
Subbasin 1 1. Overland Flow Strip 1 2. Overland Flow Strip 2 3. Collector Channel 4. Main Channel	-- --				-- --	-- --	-- --	-- --	-- --
Subbasin 2 1. Overland Flow Strip 1 2. Overland Flow Strip 2 3. Collector Channel 4. Main Channel	-- --				-- --	-- --	-- --	-- --	-- --
Subbasin 3 1. Overland Flow Strip 1 2. Overland Flow Strip 2 3. Collector Channel 4. Main Channel	-- --				-- --	-- --	-- --	-- --	-- --
Subbasin 4 1. Overland Flow Strip 1 2. Overland Flow Strip 2 3. Collector Channel 4. Main Channel	-- --				-- --	-- --	-- --	-- --	-- --
Subbasin 5 1. Overland Flow Strip 1 2. Overland Flow Strip 2 3. Collector Channel 4. Main Channel	-- --				-- --	-- --	-- --	-- --	-- --
Subbasin 6 1. Overland Flow Strip 1 2. Overland Flow Strip 2 3. Collector Channel 4. Main Channel	-- --				-- --	-- --	-- --	-- --	-- --

