APPENDIX B

Flow Transitions in Bridge Backwater Analysis

Bridges across floodplains may require special attention in one-dimensional hydraulic modeling if they cause severe contraction and expansion of the flow. The accurate prediction of the energy losses in the contraction reach upstream from the bridge and the expansion reach downstream from the bridge, using one-dimensional models, presents particular difficulty. Modeling these reaches requires the accurate evaluation of four parameters: the expansion reach length, L_e; the contraction reach length, L_c; the expansion coefficient, C_e; and the contraction coefficient, C_c. Research was conducted at the Hydrologic Engineering Center to investigate these four parameters through the use of field data, two-dimensional hydraulic modeling, and one-dimensional modeling. The conclusions and recommendations from that study are reported in this appendix. For further information regarding this study, the reader should obtain a copy of Research Document 42 (HEC,1995).

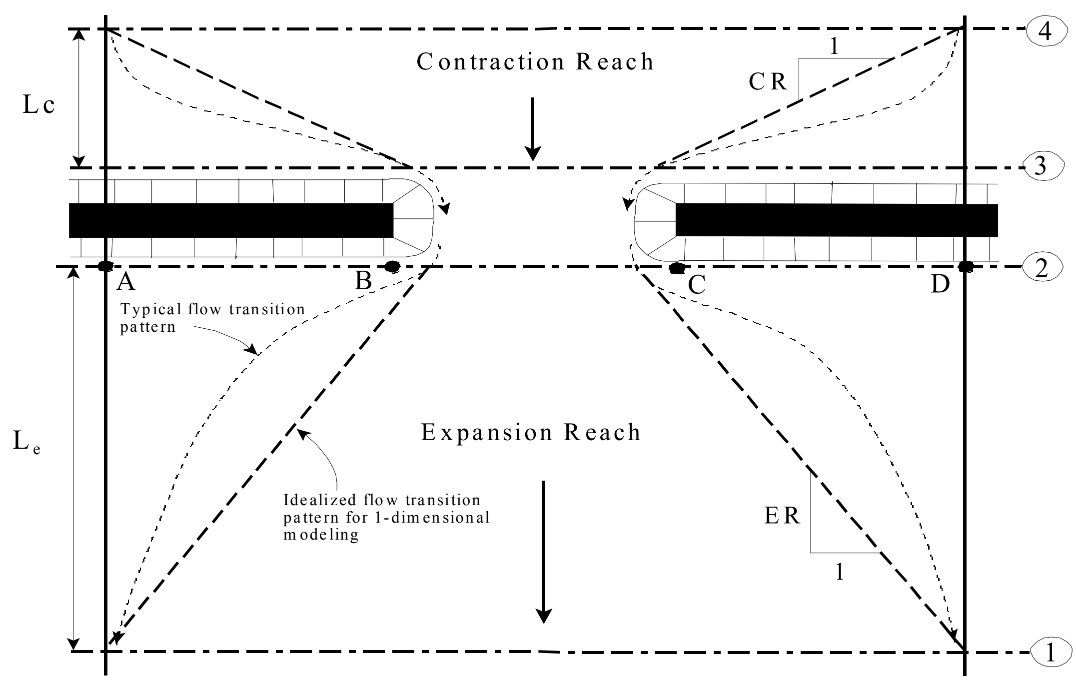

Figure B-1 Typical Cross Section Layout for Bridge Modeling

The data used in this study consisted of 3 actual bridge sites and 76 idealized bridge sites. The field data had certain hydraulic characteristics in common. All had wide, heavily vegetated overbanks, with Manning's *n* values from 0.07 to 0.24, and slopes between 2.5 feet/mile and 8.0 feet/mile. To extend the scope and general applicability of the study, it was decided to create a large number of two-dimensional models (using RMA-2, King, 1994) of idealized floodplain and bridge geometries. Figure 2 shows a typical cross section for the idealized cases. The overall floodplain width was constant at 1000 feet. The main channel *n* value was constant at 0.04. The other pertinent parameters were systematically varied as follows:

Bridge opening width, b	100, 250, and 500 feet
Discharge, Q	5000, 10000, 20000, and 30000 cfs
Overbank Manning coef., n_{ob}	0.04, 0.08, and 0.16
Bed slope, S	1, 5, and 10 feet/mile

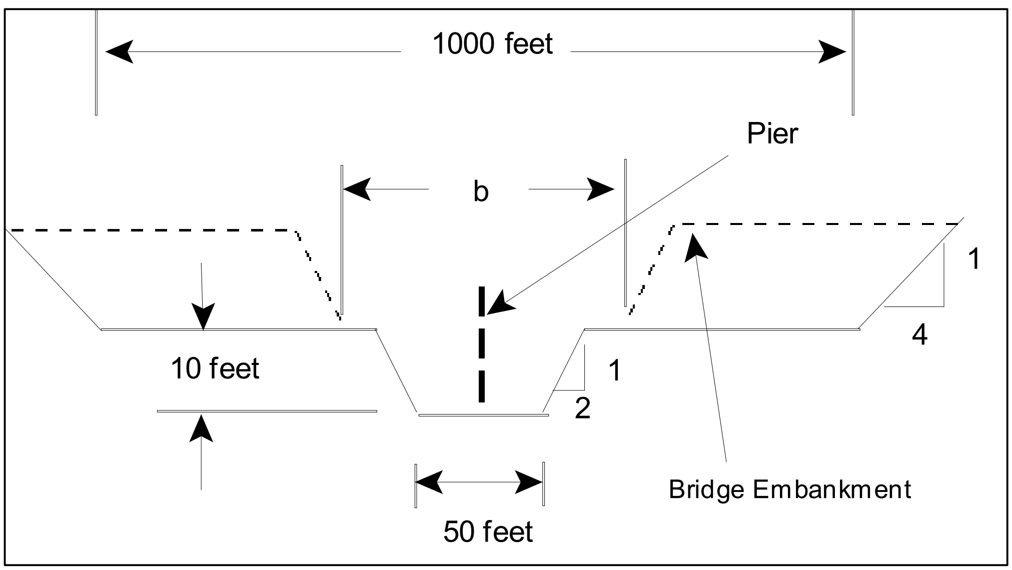

Figure B-2   Idealized Case Cross Section

In addition to the systematic variation of these parameters, eleven additional cases were created which had vertical abutments rather than spill-through abutments, six cases were developed which had asymmetric rather than symmetric bridge obstructions, and four more cases were studied which were enlarged-scale and reduced-scale versions of four of the standard cases. A total of 97 idealized models were created.

Once the data were collected for all of the idealized models, they were analyzed with the aid of the statistical analysis program STATGRAPHICS (STSC, 1991). The goals of the statistical analysis were to compile summary statistics and develop regression relationships for the parameters of interest where possible. Table B.1 lists the summary statistics for the four parameters of interest.

Table B.1
Summary Statistics

Variable	L_e	L_c	C_e	C_c
Sample size	76	76	76	76
Average	564 feet	386 feet	0.27	0.11
Median	510 feet	360 feet	0.30	0.10
Standard deviation	249 feet	86 feet	0.15	0.06
Minimum	260 feet	275 feet	0.10	0.10
Maximum	1600 feet	655 feet	0.65	0.50
Range	1340 feet	380 feet	0.55	0.40

The regression relationships were required to express L_e, L_c, C_e, and C_c as functions of independent hydraulic variables which could be easily evaluated by the users of a one-dimensional model such as HEC-RAS. Some of the independent variables used in the regression analysis, such as discharge, slope, and roughness, had been set in defining each case. The other variables, such as Froude numbers, discharge distributions, velocities, depths, and conveyances, were evaluated from the HEC-RAS models, which had been developed for each case. The raw independent variables were then entered into a spreadsheet. In the spreadsheet other variables were created as ratios and multiples of some of the raw variables.

After the spreadsheet of independent variables was complete, it was saved as an ASCII text file, which was in turn converted into a STATGRAPHICS data file. Only the cases with symmetric openings and spill-through abutments were included in the regression analyses. Those cases which had asymmetric openings or vertical abutments, were later compared with the corresponding symmetric, spill-through cases.

Conclusions From The Study

The research has successfully provided valuable insight with regard to all four parameters of concern. Also, strong relationships between the expansion reach length, the contraction reach length and the expansion coefficient and the independent variables that affect them have emerged from the analysis of the idealized two-dimensional models. The insights gained and relationships determined from this study provide a basis for improved guidance in the bridge-related application of one-dimensional models such as HEC-RAS and HEC-2.

Expansion Reach Lengths (L_e on Figure B-1)

Of all of the two-dimensional cases created for this study, which included a wide range of hydraulic and geometric conditions, none of the cases had an expansion ratio (ER on Figure B-1) as great as 4:1. Most of the cases had expansion ratios between 1:1 and 2:1. This indicates that a dogmatic use of the traditional 4:1 rule of thumb for the expansion ratio leads to a consistent over prediction of the energy losses in the expansion reach in most cases. The accompanying over prediction of the water surface elevation at the downstream face of the bridge may be conservative for flood stage prediction studies. For bridge scour studies, however, this overestimation of the tailwater elevation could in some circumstances lead to an underestimation of the scour potential.

The results from the two-dimensional flow models did not always indicate the presence of large-scale flow separations or eddy zones downstream of the bridge. Their presence corresponded with the larger values of L_e. For many of the cases there was no significant separation evident in the results. In sensitivity tests, the presence or absence of eddy zones was not sensitive to the eddy viscosity coefficient value. Likewise, eddy viscosity settings did not have an appreciable effect on L_e.

It was found that the ratio of the channel Froude number at Section 2 to that at Section 1 (F_{c2}/F_{c1}) correlated strongly with the length of the expansion reach. Regression equations were developed for both the expansion reach length and the expansion ratio. The equations are presented later in this appendix. Both equations are linear and contain terms involving the Froude number ratio and the discharge. The equation for expansion length also includes the average obstruction length in one term. To use these regression equations in the application of a one-dimensional model will usually require an iterative process since the hydraulic properties at Section 2 will not be known in advance. The effort involved in this process will not be large, however, because the method will usually converge rapidly.

The value of the Froude number ratio reflects important information about the relationship between the constricted flow and the normal flow conditions. It is in effect a measure of the degree of flow constriction since it compares the intensity of flow at the two locations. Since these Froude numbers are for the main channel only, the value of F_{c1} also happens to reflect to some extent the distribution of flow between the overbanks and main channel.

There was no support from these investigations for the WSPRO concept of the expansion reach length being proportional to or equal to the bridge opening width.

Contraction Reach Lengths (L_c on Figure B-1)

While the apparent contraction ratios of the five field prototype cases were all below 1:1, the contraction ratios (CR on Figure B-1) for the idealized cases ranged from 0.7:1 to 2.3:1. As with the expansion reach lengths, these values correlated strongly with the same Froude number ratio. A more important independent variable, however, is the decimal fraction of the total discharge conveyed in the overbanks (Q_{ob} / Q) at the approach section. A strong regression equation was developed for the contraction length and is presented later in this appendix.

Because the mean and median values of the contraction ratios were both around 1:1, there is some support from this study for the rule of thumb which suggests the use of a 1:1 contraction ratio. There is no support, however, for the concept of the contraction reach length being equal to or proportional to the bridge opening width.

Expansion Coefficients

Regression analysis for this parameter was only marginally successful. The resulting relationship is a function of the ratio of hydraulic depth in the overbank to that in the main channel for undisturbed conditions (evaluated at Section 1). Perhaps more interesting are the summary statistics, which indicate lower values for this coefficient than the traditional standard values for bridges.

Contraction Coefficients

Owing to the nature of this data (69 out of 76 cases had the minimum value of 0.10), a regression analysis was not fruitful. Like the expansion coefficients, the prevailing values are significantly lower than the standard recommended values.

Asymmetric Bridge Openings

For these data the averages of the reach length values for the two corresponding symmetric cases closely approximated the values determined for the asymmetric cases. When the regression equations for L_e, ER, and L_c were applied to the asymmetric cases, the predicted values were near the observed values. This indicates that the regression relationships for the transition reach lengths can also be applied to asymmetric cases (that is, most real-world cases).

Vertical-Abutment Cases

For these data there was no major effect on the transition lengths or the coefficients due to the use of vertical rather than spill-through abutments. The exceptions to this statement were three vertical-abutment cases in the narrow-opening class for which square corners were used. The square-cornered abutments were a deliberate attempt to model a very severe situation. Because the RMA-2 program, or any two-dimensional numerical model for that matter, is not well-formulated to handle such drastic boundary conditions, no general conclusions should be drawn from these cases about actual field sites having such a configuration.

Recommendations From The Study

The remainder of this appendix presents recommendations arising from the results documented in RD-42 (HEC,1995). These recommendations are intended to provide the users of one-dimensional water surface profile programs, such as HEC-RAS, with guidance on modeling the flow transitions in bridge hydraulics problems.

In applying these recommendations, the modeler should always consider the range of hydraulic and geometric conditions included in the data. Wherever possible, the transition reach lengths used in the model should be validated by field observations of the site in question, preferably under conditions of high discharge. The evaluation of contraction and expansion coefficients should ideally be substantiated by site-specific calibration data, such as stage-discharge measurements just upstream of the bridge. The following recommendations are given in recognition of the fact that site-specific field information is often unavailable or very expensive to obtain.

Expansion Reach Lengths

In some types of studies, a high level of sophistication in the evaluation of the transition reach lengths is not justified. For such studies, and for a starting point in more detailed studies, Table B.2 offers ranges of expansion ratios, which can be used for different degrees of constriction, different slopes, and different ratios of overbank roughness to main channel roughness. Once an expansion ratio is selected, the distance to the downstream end of the expansion reach (the distance L_e on Figure B-1) is found by multiplying the expansion ratio by the average obstruction length (the average of the distances A to B and C to D from Figure B-1). The average obstruction length is half of the total reduction in floodplain width caused by the two bridge approach embankments. In Table B.2, b/B is the ratio of the bridge opening width to the total floodplain width, n_{ob} is the Manning *n* value for the overbank, n_c is the *n* value for the main channel, and S is the longitudinal slope. The values in the interior of the table are the ranges of the expansion ratio. For each range, the higher value is typically associated with a higher discharge.

Table B.2
Ranges of Expansion Ratios

		$n_{ob} / n_{c} = 1$	$n_{ob} / n_{c} = 2$	$n_{ob} / n_{c} = 4$
b/B = 0.10	S = 1 ft/mile	1.4 – 3.6	1.3 – 3.0	1.2 – 2.1
	5 ft/mile	1.0 – 2.5	0.8 – 2.0	0.8 – 2.0
	10 ft/mile	1.0 – 2.2	0.8 – 2.0	0.8 – 2.0
b/B = 0.25	S = 1 ft/mile	1.6 – 3.0	1.4 – 2.5	1.2 – 2.0
	5 ft/mile	1.5 – 2.5	1.3 – 2.0	1.3 – 2.0
	10 ft/mile	1.5 – 2.0	1.3 – 2.0	1.3 – 2.0
b/B = 0.50	S = 1 ft/mile	1.4 – 2.6	1.3 – 1.9	1.2 – 1.4
	5 ft/mile	1.3 – 2.1	1.2 – 1.6	1.0 – 1.4
	10 ft/mile	1.3 – 2.0	1.2 – 1.5	1.0 – 1.4

The ranges in Table B.2, as well as the ranges of other parameters to be presented later in this appendix, capture the ranges of the idealized model data from this study. Another way of establishing reasonable ranges would be to compute statistical confidence limits (such as 95% confidence limits) for the regression equations. Confidence limits in multiple linear regression equations have a different value for every combination of values of the independent variables (Haan, 1977). The computation of these limits entails much more work and has a more restricted range of applicability than the corresponding limits for a regression, which is based on only one independent variable. The confidence limits were, therefore, not computed in this study.

Extrapolation of expansion ratios for constriction ratios, slopes or roughness ratios outside of the ranges used in this table should be done with care. The expansion ratio should not exceed 4:1, nor should it be less than 0.5:1 unless there is site-specific field information to substantiate such values. The ratio of overbank roughness to main-channel roughness provides information about the relative conveyances of the overbank and main channel. The user should note that in the data used to develop these recommendations, all cases had a main-channel *n* value of 0.04. For significantly higher or lower main-channel *n* values, the *n* value ratios will have a different meaning with respect to overbank roughness. It is impossible to determine from the data of this study whether this would introduce significant error in the use of these recommendations.

When modeling situations which are similar to those used in the regression analysis (floodplain widths near 1000 feet; bridge openings between 100 and 500 feet wide; flows ranging from 5000 to 30000 cfs; and slopes between one and ten feet per mile), the regression equation for the expansion reach length can be used with confidence. The equation developed for the expansion reach length is as follows:

$$L_e = -298 + 257\left(\frac{F_{c2}}{F_{c1}}\right) + 0.918\,\overline{L}_{obs} + 0.00479\,Q \qquad \text{(B-1)}$$

Where: L_e = length of the expansion reach, in feet

F_{c2} = main channel Froude number at Section 2

F_{c1} = main channel Froude number at Section 1

$\overline{L}_{obs}$ = average length of obstruction caused by the two bridge approaches, in feet, and

Q = total discharge, cfs

When the width of the floodplain and the discharge are smaller than those of the regression data (1000 ft wide floodplain and 5000 cfs discharge), the expansion ratio can be estimated by Equation B-2. The computed value should be checked against ranges in Table B-1. Equation B-2 is:

$$ER = \frac{L_e}{\overline{L}_{obs}} = 0.421 + 0.485\left(\frac{F_{c2}}{F_{c1}}\right) + 0.000018\,Q \qquad \text{(B-2)}$$

When the scale of the floodplain is significantly larger than that of the data, particularly when the discharge is much higher than 30,000 cfs, Equations B-1 and B-2 will overestimate the expansion reach length. Equation B-3 should be used in such cases, but again the resulting value should be checked against the ranges given in Table B.1:

$$ER = \frac{L_e}{\overline{L}_{obs}} = 0.489 + 0.608\left(\frac{F_{c2}}{F_{c1}}\right) \qquad \text{(B-3)}$$

The depth at Section 2 is dependent upon the expansion reach length, and the Froude number at the same section is a function of the depth. This means that an iterative process is required to use the three equations above, as well as the equations presented later in this chapter for contraction reach lengths and expansion coefficients. It is recommended that the user start with an expansion ratio from Table B.1, locate Section 1 according to that expansion ratio, set the main channel and overbank reach lengths as appropriate, and limit the effective flow area at Section 2 to the approximate bridge opening width. The program should then be run and the main channel Froude numbers at Sections 2 and 1 read from the model output. Use these Froude number values to determine a new expansion length from the appropriate equation, move Section 1 as appropriate and recompute. Unless the geometry is changing rapidly in the vicinity of Section 1, no more than two iterations after the initial run should be required.

When the expansion ratio is large, say greater than 3:1, the resulting reach length may be so long as to require intermediate cross sections, which reflect the changing width of the effective flow area. These intermediate sections are necessary to reduce the reach lengths when they would otherwise be too long for the linear approximation of energy loss that is incorporated in the standard step method. These interpolated sections are easy to create in the HEC-RAS program, because it has a graphical cross section interpolation feature. The importance of interpolated sections in a given reach can be tested by first inserting one interpolated section and seeing the effect on the results. If the effect is significant, the subreaches should be subdivided into smaller units until the effect of further subdivision is inconsequential.

Contraction Reach Lengths

Ranges of contraction ratios (CR) for different conditions are presented in Table B.3. These values should be used as starting values and for studies which do not justify a sophisticated evaluation of the contraction reach length. Note that this table does not differentiate the ranges on the basis of the degree of constriction. For each range the higher values are typically associated with higher discharges and the lower values with lower discharges.

Table B.3
Ranges of Contraction Ratios (CR)

	$n_{ob} / n_c = 1$	$n_{ob} / n_c = 2$	$n_{ob} / n_c = 4$
S = 1 ft/mile	1.0 - 2.3	0.8 - 1.7	0.7 - 1.3
5 ft/mile	1.0 - 1.9	0.8 - 1.5	0.7 - 1.2
10 ft/mile	1.0 - 1.9	0.8 - 1.4	0.7 - 1.2

When the conditions are within or near those of the data, the contraction reach length regression equation (Equation B-4) may be used with confidence:

$$L_c = 263 + 38.8\left(\frac{F_{c2}}{F_{c1}}\right) + 257\left(\frac{Q_{ob}}{Q}\right)^2 - 58.7\left(\frac{n_{ob}}{n_c}\right)^{0.5} + 0.161\overline{L}_{obs} \qquad \text{(B-4)}$$

Where: $\underline{L}_{obs}$ = average length of obstruction as described earlier in this chapter, in feet
Q_{ob} = the discharge conveyed by the two overbanks, in cfs, at the approach section (Section 4)
n_{ob} = the Manning n value for the overbanks at Section 4, and
n_c = the Manning n value for the main channel at Section 4

In cases where the floodplain scale and discharge are significantly larger or smaller than those that were used in developing the regression formulae, Equation B-4 should not be used. The recommended approach for estimating the contraction ratio at this time is to compute a value from Equation B-5 and check it against the values in Table B.3:

$$CR = 1.4 - 0.333\left(\frac{F_{c2}}{F_{c1}}\right) + 1.86\left(\frac{Q_{ob}}{Q}\right)^2 - 0.19\left(\frac{n_{ob}}{n_c}\right)^{0.5} \qquad \text{(B-5)}$$

As with the expansion reach lengths, the modeler must use Equations B-4 and B-5 and the values from Table B.2 with extreme caution when the prototype is outside of the range of data used in this study. The contraction ratio should not exceed 2.5:1 nor should it be less than 0.3:1.

Expansion Coefficients

The analysis of the data with regard to the expansion coefficients did not yield a regression equation, which fit the data well. Equation B-6 was the best equation obtained for predicting the value of this coefficient:

$$C_e = -0.09 + 0.570\left(\frac{D_{ob}}{D_c}\right) + 0.075\left(\frac{F_{c2}}{F_{c1}}\right) \quad \text{(B-6)}$$

Where: D_{ob} = hydraulic depth (flow area divided by top width) for the overbank at the fully- expanded flow section (Section 1), in feet, and

D_c = hydraulic depth for the main channel at the fully-expanded flow section, in feet

It is recommended that the modeler use Equation B-6 to find an initial value, then perform a sensitivity analysis using values of the coefficient that are 0.2 higher and 0.2 lower than the value from Equation B-6. The plus or minus 0.2 range defines the 95% confidence band for Equation B-6 as a predictor within the domain of the regression data. If the difference in results between the two ends of this range is substantial, then the conservative value should be used. The expansion coefficient should not be higher than 0.80.

Contraction Coefficients

The data of this study did not lend itself to regression of the contraction coefficient values. For nearly all of the cases the value that was determined was 0.1, which was considered to be the minimum acceptable value. The following table presents recommended ranges of the contraction coefficient for various degrees of constriction, for use in the absence of calibration information.

Table B.4
Contraction Coefficient Values

Degree of Constriction	Recommended Contraction Coefficient
$0.0 < b/B < 0.25$	0.3 - 0.5
$0.25 < b/B < 0.50$	0.1 - 0.3
$0.50 < b/B < 1.0$	0.1

The preceding recommendations represent a substantial improvement over the guidance information that was previously available on the evaluation of transition reach lengths and coefficients. They are based on data, which, like all data, have a limited scope of direct application. Certain situations, such as highly skewed bridge crossings and bridges at locations of sharp curvature in the floodplain were not addressed by this study. Even so, these recommendations may be applicable to such situations if proper care is taken and good engineering judgment is employed.