Appendix A

Stream Routing Methods

Channel routing in HEC-5 is accomplished by hydrologic routing methods which solve the continuity equation using some relationship between outflow and storage in the reach. The variety of routing options can be grouped into two categories: **Storage Methods** and **Coefficient Methods**.

Storage Methods are also referred to as reservoir routing because they route the inflow through a non-linear representation of reach storage. The options include: the modified Puls, modified Puls as a function of Inflow, the Working R&D method, and SSARR Time of Storage.

Coefficient options include: the Muskingum method, the Average-lag methods: Simple Lag, Successive Average-lag (Tatum) and Progressive Average-lag (Straddle-Stagger), plus the direct input of routing coefficients.

Equations used for each of these methods are given below. *Flood-Runoff Analysis*, EM 1110-2-1417, Chapter 9 is the primary Corps' reference for stream routing; however, the topic is covered in most hydrology texts.

A.1 Storage Methods

The Storage Methods operate with the basic reservoir mass balance applied to the routing reach (i.e., Inflow - Outflow = Change in Storage). The primary difference among the methods is how the storage-outflow relationship is defined. An added consideration is the number of increments (Steps) to use in the routing reach. Generally, the travel time through the Step should be equal to the computational time interval. The program will divide the reach storage by the number of Steps and route the inflow sequentially through the storage in each step. The maximum attenuation occurs with one-step routing (reservoir condition).

A.1.1 Modified Puls Routing

In this method, outflow from a routing reach is a unique function of storage. The input is a storage-outflow relationship for the routing reach. The program solves the routing problem by rearranging the terms to have future values equal to known values. The result is called the storage indication, ($S/\Delta t + O/2$). The following equations are used:

$$STRI(2) = STRI(1) + QH - O(1) \quad \text{(A-1)}$$

where:

S	=	volume of storage in routing reach
STRI(1)	=	storage indication at beginning of time
STRI(2)	=	storage indication at end of time interval
QH	=	average inflow during time interval
O(1)	=	outflow at start of time interval
O(2)	=	outflow at the end of time interval, function of STR(2) from the Storage Indication.vs.Outflow

A.1.2 Modified Puls as a Function of Inflow

This is a variation of modified Puls which provides for multiple storage-outflow relations that are dependent on the inflow to the reach. This was added to HEC-5 because it is used in some Corps offices. However, there is a potential that continuity will not be maintained when using the procedure. Therefore, when routing with this procedure you should carefully review the results to ensure that flow is not gained or lost due to routing. An alternative is the **Working R&D Method** which provides a method to include the effect of inflow on the reach storage and maintains continuity.

A.1.3 Working R&D Method

The Working R&D uses a nonlinear storage-outflow relation, like the modified Puls method; and it permits use of wedge storage, like the Muskingum method (described under Coefficient Methods). Indeed for a linear storage-outflow relation, the Working R&D method produces results identical to the Muskingum method. For routing with no wedge storage (Muskingum X = 0), the Working R&D method produces results identical to the modified Puls method. The Working R&D method uses a "working" storage as indicated in the following equations:

$$\frac{R(2)}{\Delta t} = \frac{R(1)}{\Delta t} + QH - D(1) \qquad \text{(A-2)}$$

where:

R(2)	=	"working" storage at end of a time interval
R(1)	=	"working" storage at the beginning of a time interval
QH	=	average inflow during routing interval
D(1)	=	"working" discharge at beginning of time
D(2)	=	“working” discharge at the end of time interval, function of R(2) from the Working-storage Indication.vs.Outflow

The functional relationship described of D(2) is analogous to the storage indication-outflow relation used in the modified Puls method. Outflow is a function of "working" discharge and is determined from the following equation:

$$O(2) = D(2) - \frac{X}{(1-X)} \bullet (I(2) - D(2)) \qquad \text{(A-3)}$$

where:

X	=	Muskingum X (dimensionless)
O(2)	=	outflow at end of time interval
I(2)	=	inflow at end of time interval

A.1.4 SSARR Time of Storage Routing

This channel routing method is used in the computer program *Streamflow Synthesis & Reservoir Regulation (SSARR)*, from the Corps' North Pacific Division. The storage in the routing reach is defined by Time of Storage values (T_S), in units of hours. Therefore, the storage is defined like Muskingum K values (described below). Instead of using modified Puls storage-outflow, this method uses T_S versus outflow.

An alternative method for defining T_S is by equation 4, which defines Ts as a power function of flow.

$$T_s = \frac{KTS}{Q^n} \qquad \text{(A-4)}$$

where:

T_S	=	Time of storage per increment in hours
Q	=	Discharge in cubic meters (or feet) per hour
n	=	A coefficient usually between -1 and 1

As evident from the equation, T_S is a nonlinear function of discharge except when n = 1. It is possible to use a negative value of n if time of storage increases as discharge increases. According to the SSARR User's Manual, a value of n = 0.2 is reasonable for most streams in the Columbia River Basin.

A.2 Coefficient Methods

All coefficient methods compute outflow from a routing reach as a linear function. Equation 6 is the basic routing equation. For the direct input of coefficients, the series of 'C' values are input and their sum should equal one, to maintain continuity.

$$O_n = C_1 I_n + C_2 I_{n-1} + C_3 I_{n-2} \cdots \qquad \text{(A-5)}$$

where:

O_n	=	ordinate of outflow hydrograph at time n
I_n, I_{n-1}, etc.	=	ordinates of inflow hydrograph at times n, -1, etc.
C_1, C_2, etc.	=	routing coefficients, as coefficients of inflow

A.2.1 Muskingum Routing

The Muskingum method can be considered a storage method with the reach storage defined by the sum of *prism* and *wedge* storage. However, the storage-outflow relationship is linear and the method provides a set of constant coefficients. The equations used to determine the coefficients C_1, C_2, etc., are as follows:

$$C_1 = \frac{(\Delta t - 2XK)}{(2K(1-X) + \Delta t)} \qquad \text{(A-6)}$$

$$CC = \frac{((2K(1-X) + \Delta t) - 2\Delta t)}{(2K(1-X) + \Delta t)} \qquad \text{(A-7)}$$

$$C_2 = C_1 \bullet CC + \frac{(\Delta t + 2KX)}{(2K(1-X) + \Delta t)} \qquad \text{(A-8)}$$

$$C_i = C_{i-1} \bullet CC \quad (\text{for } i > 2) \qquad \text{(A-9)}$$

where:

Δt = routing time increment

K = Travel Time in units of time (hours)

X = Dimensionless routing parameter between 0 and .5

To avoid negative coefficients in Equation 7, the Muskingum K should be greater than or equal to (Δt) / [2 •(1-X)] and less than or equal to (Δt) / 2X.

The reach storage-outflow relationship is defined by the coefficient **K**, which can be considered the slope of the storage-outflow relationship. The *prism* storage in a reach is the product of **K** times outflow. The *wedge* storage is computed from the difference between inflow and outflow. The weighting factor on inflow is the coefficient **X**. If **X** = 0, inflow does not effect storage (like reservoir routing) and there is the maximum attenuation of outflow. If **X** =0.5, inflow has the maximum effect, and there is no attenuation of outflow (just translation based on **K** hours).

Typically, the reach is divided into steps in order to keep the step travel time approximately equal to the computational time step. For travel times significantly less than the computational time step, the reach can be treated as having no routing to avoid the computation of negative coefficients.

A.2.2 Average-Lag Methods

These methods route floods by time displacement of average inflow. The methods were developed from intuitive processes rather than mathematical equations. As such, they remain in HEC-5 because they are used in some Corps offices; however, they are not presented in the EM and their use is not encouraged. The three methods are: **Lag**, **Successive Average-Lag** (Tatum Method) and **Progressive Average-Lag** (Straddle-Stagger Method).

Lag. The inflow hydrograph can be lagged an integer number of computational time steps. This approach displaces the hydrograph in time, but does not change the shape of the routed hydrograph (no attenuation).

Successive Average-Lag averages the inflow successively through a number of subreaches (n). If there were two subreaches, the outflow for period 3 would be:

$$O_3 = \frac{1}{2}\left[\frac{I_1+I_2}{2}+\frac{I_2+I_3}{2}\right] \qquad \text{(A-10)}$$

where:

I = Inflow in increments of time

O = Outflow in increments of time

The coefficients are computed based on the number of subreaches for the successive average.

$$C_1 = \frac{1}{2^n} \tag{A-11}$$

$$C_2 = \frac{n}{2^n} \tag{A-12}$$

$$C_3 = \frac{n(n-1)}{2^n \ 2!} \tag{A-13}$$

$$C_4 = \frac{n(n-1)(n-2)}{2^n 3!} \tag{A-14}$$

$$C_n = \frac{n(n-1)(n-2)\ .\ .\ .}{2^n (n-1)!} \tag{A-15}$$

$$C_{(n+1)} = \frac{n!}{2^n n!} = \frac{1}{2^n} \tag{A-16}$$

Progressive Average-Lag Routing. This routing method is defined by the number of periods of inflow to average (Straddle) and the number of periods the average value then is lagged (Stagger). The method differs from the successive average-lag method in two respects:

(1) Equal rather than variable weight is given each inflow value in deriving an outflow, and

(2) the length of period for which inflow values are averaged to obtain an outflow value does not necessarily have any relation to the flood-wave travel time.

Generally, the length of the inflow period is determined by trial until a satisfactory agreement is obtained between the computed and actual peak outflows.

The method uses Coefficient routing equation (5) to route the inflow, with the routing coefficients determined as follows:

$$NCOEF = STAG + \frac{(STRAD+1)}{2} \qquad \text{(A-17)}$$

$$M = NCOEF - STRAD \qquad \text{(A-18)}$$

$$C_i = 0 \text{ for } 1 \leq i \leq m \qquad \text{(A-19)}$$

$$C_i = \frac{1}{STRAD} \text{ for } (M+1) \leq i \leq NCOEF \qquad \text{(A-20)}$$

where:

NCOEF	=	number of routing coefficients in equation (5)
STRAD	=	number of inflow ordinates to be averaged
STAG	=	number of ordinates the average value is to be lagged, from the mid-ordinate of the averaged ordinates