Rheological Parameter Estimation

Stanford Gibson, PhD

\[ \tau = \tau_y + \mu \left( \frac{dv_x}{dz} \right) \]

- \( \tau \): Shear Stress
- \( \tau_y \): Yield Stress
- \( \mu \): Dynamic Viscosity
- \( \frac{dv_x}{dz} \): Shear Rate (Strain)

Graph showing Bingham Plastic and Newtonian Fluid behavior.
1. Yield Strength ($\tau_y$)
2. Viscosity ($\mu$)
3. Coarse/Fine Impacts
4. Sensitivity
5. HB Power ($n$)
Rheological Parameter Estimation

1. Yield Strength ($\tau_y$)
2. Viscosity ($\mu$)
3. Coarse/Fine Impacts
4. Sensitivity
5. HB Power ($n$)
Yield Strength

**Bingham:**
\[ \tau = \tau_y + \mu_m \left( \frac{3 \bar{u}}{h} \right) \]

**O'Brien Quadratic:**
\[ \tau = \tau_y + \mu_m \left( \frac{3 \bar{u}}{h} \right) + \rho_m \left( \frac{3 \bar{u}}{h} \right)^2 + 0.01 \rho_s \left( \frac{0.615}{C_v} \right)^{1/3} \left( 1 - \frac{1}{C_v} \right)^{-2} \left( \frac{3 \bar{u}}{h} \right)^2 \]

**Herschel-Bulkley:**
\[ \tau = \tau_y + K \left( \frac{3 \bar{u}}{h} \right)^n \]

---

**Non-Newtonian Methods and Parameters**

- **Non-Newtonian Method:** Bingham
- **Concentration and Bulking**
  - Volumetric Concentration ($C_v$) (%): 45
  - Select Bulking Method: Bulk: Fluid Volume
- **Shear Components**
  - Yield Strength: Exponential
  - Mixture Dynamic: Use Coulomb
- **Convert Viscosity Units**
  - Representative Grain Size: 0.5 mm
  - Max $C_v$: 61.5
  - Generalized Herschel-Bulkley Parameters:
    - $K$: 0
    - $n$: 0
    - Clastic Methods: Coulomb

---

**Shear Stress Graphs**

- **Linear: Newtonian & Bingham Plastic**
  - \( \frac{d \tau}{dx} \) (Strain)
- **Non-Linear: Quadratic**
  - \( \frac{d \tau}{dx} \) (Strain)
  - Shear Thickening ($\tau \uparrow \Rightarrow \mu \downarrow$) \( n_a > 1 \)
  - Shear Thinning ($\tau \uparrow \Rightarrow \mu \downarrow$) \( n_a < 1 \)
  - Water: NO \( \tau_y \)
Exponential Method

Many investigators have reported an exponential relationship between $\tau_y$ and $C_v$.

$$\tau_y = ae^{b \cdot C_v}$$
HEC-RAS Includes the Exponential Method

Julian (1995) reports coefficient for:

\[ \tau_y = ae^{b \cdot CV} \]

\[ \tau_y = a10^b \cdot CV \]

Table: Yield strength in Pa, Viscosity in Pa.s, and Range (Pa)

<table>
<thead>
<tr>
<th>Material</th>
<th>Liquid limit CV</th>
<th>Yield strength in Pa</th>
<th>Viscosity in Pa.s</th>
<th>Range (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bentonite (montmorillonite)</td>
<td>0.05–0.2</td>
<td>0.002</td>
<td>100</td>
<td>X2.303</td>
</tr>
<tr>
<td>Sensitive clays</td>
<td>0.35–0.6</td>
<td>0.3</td>
<td>10</td>
<td>X2.303</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>0.4–0.5</td>
<td>0.05</td>
<td>8</td>
<td>X2.303</td>
</tr>
<tr>
<td>Typical soils</td>
<td>0.65–0.8</td>
<td>0.005</td>
<td>8</td>
<td>X2.303</td>
</tr>
<tr>
<td>Granular material</td>
<td>–</td>
<td>–</td>
<td>3</td>
<td>–</td>
</tr>
</tbody>
</table>

To Convert Julien’s (1995) Parameters from

\[ \tau_y = a10^b \cdot CV \]

To

\[ \tau_y = ae^{b \cdot CV} \]

leave “a” the same and multiply “b” by 2.303

Julian (1995) reports coefficient for:

\[ \tau_y = a10^b \cdot CV \]

Graphs showing Yield Strength (Pa) vs. Liquid limit CV for Typical Soils, Kaolinite, Sensitive Clay, and Bentonite.

Non-Newtonian Methods and Parameters:

- Non-Newtonian Method: Bingham
- Concentration and Bulking:
  - Volumetric Concentration (CV) (%)
  - Select Bulking Method: Do Not Bulk
- Shear Components:
  - Yield Strength: Exponential
  - Mixture Dynamic Viscosity: User Defined Viscosity
  - Convert Velocity Units:

Julian (1995)
You can still compute and use Julian’s Values.

<table>
<thead>
<tr>
<th>Type</th>
<th>Liquid Limit CV</th>
<th>a</th>
<th>b</th>
<th>Range (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Soil</td>
<td>65-80%</td>
<td>0.005</td>
<td>17.2</td>
<td>375-5,000</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>40-50%</td>
<td>0.05</td>
<td>20.7</td>
<td>200-1,600</td>
</tr>
<tr>
<td>Sensitive Clays</td>
<td>35-60%</td>
<td>0.3</td>
<td>23.0</td>
<td>950-300k</td>
</tr>
<tr>
<td>Bentonite</td>
<td>5-20%</td>
<td>0.002</td>
<td>230.3</td>
<td>200-2E+17</td>
</tr>
</tbody>
</table>
“Very different behavior may be obtained depending on the values of the large number of parameters of each suspension. For example, the behavior may vary greatly with small changes in the clay type, electrolyte concentration, pH, solid concentration. It appears to be a very difficult task to make a systematic description of each of these types of behavior, as a function of the parameters.”

*Note: These values for Kaolonite are much higher than Julian’s
Jong et al. (2010) Meta Analysis

Viscous porosity, $\mu$ (Pa-s)

Yield stress, $\tau_c$ (Pa)
Laboratory Yield Strength Data

Coussot (1995)
Coussot and Piau (1994)
Parsons et al (2001)
Major and Pierson (1992)
Haldenwang
Socio and Crosta (2009)
Laboratory $\tau_c$ data come in in the high 10s to low 100s, which is low compared to other measures.
Phillips and Davies (1991)

Reported Range:
50-2,400 Pa

Median Range
400-1,000 Pa

Recommended Range:
1,000-3,000 Pa

“For the full-scale mix, yield strengths ... A minimum value of 2000 Pa is suggested, but the figure could be as high as 3000 Pa.”
- Phillips and Davies (1991)

<table>
<thead>
<tr>
<th>Location</th>
<th>Rheological parameters</th>
<th>Ref.*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jiangia Ravine, China</td>
<td>[\eta = 150 \text{ Pa·s}]</td>
<td>[1]</td>
</tr>
<tr>
<td></td>
<td>[\tau = 200-300 \text{ Pa·s}]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[\eta_\text{p} = 2-3 \text{ Pa·s}]</td>
<td></td>
</tr>
<tr>
<td>Hunshui Gully, China</td>
<td>[\eta = 1.5-2.0 \text{ Pa·s}]</td>
<td>[2]</td>
</tr>
<tr>
<td></td>
<td>[\tau = 0.3-0.5 \text{ Pa}]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[\eta_\text{N} = 0.01-0.06 \text{ Pa·s}]</td>
<td>[3]</td>
</tr>
<tr>
<td></td>
<td>[\tau_\text{N} = 5-20 \text{ Pa}]</td>
<td></td>
</tr>
<tr>
<td>Wrightwood Canyon, USA</td>
<td>[\eta_\text{B} = 40-100 \text{ Pa·s}]</td>
<td>[4]</td>
</tr>
<tr>
<td></td>
<td>[\eta_\text{N} = 10-600 \text{ Pa·s}]</td>
<td>[5]</td>
</tr>
<tr>
<td></td>
<td>[\eta_\text{N} = 210-600 \text{ Pa·s}]</td>
<td></td>
</tr>
<tr>
<td>Pine Creek Mt St Helens, USA</td>
<td>[\eta_\text{N} = 20-320 \text{ Pa·s}]</td>
<td>[6]</td>
</tr>
<tr>
<td></td>
<td>[\eta_\text{N} = 400-1000 \text{ Pa·s}]</td>
<td></td>
</tr>
<tr>
<td>Mayflower Gulch, USA</td>
<td>[\eta_\text{N} = 3000 \text{ Pa·s}]</td>
<td>[7]</td>
</tr>
<tr>
<td>Dragon Creek, USA</td>
<td>[\eta_\text{N} = 2780 \text{ Pa·s}]</td>
<td>[8]</td>
</tr>
<tr>
<td>Bullock Creek, New Zealand</td>
<td>[\eta_\text{N} = 210-810 \text{ Pa·s}]</td>
<td>[9]</td>
</tr>
<tr>
<td></td>
<td>[\eta_\text{N} = 19-71 \text{ Pa}]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[\tau = 1300-2400 \text{ Pa}]</td>
<td>[10]</td>
</tr>
<tr>
<td>Weihe River, China</td>
<td>[\eta_\text{N} = 51 \text{ Pa}]</td>
<td></td>
</tr>
</tbody>
</table>

\(\tau\) = shear strength of material; \(\eta\) = viscosity; 
\(\tau_\text{B}\) = Bingham yield strength; \(\eta_\text{NB}\) = Newtonian viscosity; 
\(\eta_\text{B}\) = Bingham viscosity.

*References:
[1] Li et al. (1983); [6] Fink et al. (1981); 
[2] Li and Luo (1981); [7] Curry (1966); 
[3] Zhang et al. (1985); [8] Cooley et al. (1977); 
### Calibrated Values

<table>
<thead>
<tr>
<th>Torrent</th>
<th>( \tau_y ) (Pa)</th>
<th>( \mu ) (Pa·s(^n))</th>
<th>( \eta )</th>
<th>Bulk density (kg/m(^3)) (Cv %)</th>
<th>Aspect</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anhui Dam</td>
<td>38</td>
<td>2.1</td>
<td>1</td>
<td>1600 (38%)</td>
<td>Mud/debris flow</td>
<td>Jin &amp; Fread (1999)</td>
</tr>
<tr>
<td>Aberfan Dam</td>
<td>4794</td>
<td>958</td>
<td>1</td>
<td>1800 (50%)</td>
<td>Mud/debris flow</td>
<td>Jin &amp; Fread (1999)</td>
</tr>
<tr>
<td>Rudd Creek</td>
<td>956</td>
<td>958</td>
<td>1</td>
<td>1600 (38%)</td>
<td>Mud/debris flow</td>
<td>Jin &amp; Fread (1999)</td>
</tr>
<tr>
<td>Kamikamihori</td>
<td>200</td>
<td>3200</td>
<td>1</td>
<td>2000 (63%)</td>
<td>Debris flow</td>
<td>Naeff et al. (2006)</td>
</tr>
<tr>
<td>Faucon stream</td>
<td>30/1000</td>
<td>5/100</td>
<td>1</td>
<td>1600/2000</td>
<td>Muddy</td>
<td>Remaître et al. (2005)</td>
</tr>
<tr>
<td>Yosemite valley</td>
<td>150/300</td>
<td>800/1000</td>
<td>1</td>
<td>2200/2600</td>
<td>Debris flow</td>
<td>Bertolo &amp; Wieczorek (2005)</td>
</tr>
<tr>
<td>Pousset</td>
<td>2000/4000(^*)</td>
<td>600/1200(^*)</td>
<td>0.3</td>
<td>2000 (63%)</td>
<td>Muddy</td>
<td>Laigle &amp; Marchi (2000)</td>
</tr>
<tr>
<td>Moscardo</td>
<td>See Table 2</td>
<td>See Table 2</td>
<td>0.3</td>
<td>–</td>
<td>Debris flow</td>
<td>Coussot et al. (1998)</td>
</tr>
<tr>
<td>Acquabona</td>
<td>611/850</td>
<td>60/1700</td>
<td>1</td>
<td>2000 (63%)</td>
<td>Debris flow</td>
<td>Genevois et al. (2000)</td>
</tr>
</tbody>
</table>

\( \tau_y \) tend to be larger in the prototype scale... more like Julian’s Range.

Whipple and Dunn (1992)
The Influence of Debris-Flow Rheology on Fan Morphology, Owens Valley, California
Geological Society of America Bulletin
104(7):887-900
Image analysis for debris flow properties estimation

Velocity 2-5 m/s
6-15 ft/s

https://www.researchgate.net/publication/248403705_Image_analysis_for_debris_flow_properties_estimation
<table>
<thead>
<tr>
<th>Project</th>
<th>Eqn</th>
<th>$C_v$</th>
<th>$\tau_y$ (Pa)</th>
<th>$\mu$ (Pa·s)</th>
<th>Manning’s $n$</th>
<th>District</th>
</tr>
</thead>
<tbody>
<tr>
<td>Santa Barbara, CA</td>
<td>Bingham</td>
<td>45</td>
<td>0.45 Pa*</td>
<td>122 Pa·s*</td>
<td>0.08</td>
<td>HEC/SPL</td>
</tr>
<tr>
<td>Santa Barbara, CA with parameters from Bessette-Kirton et al. (2019)</td>
<td>Bingham</td>
<td>45</td>
<td>1,000 Pa</td>
<td>100 Pa·s</td>
<td>0.08</td>
<td>HEC</td>
</tr>
<tr>
<td>Brumadinho Dam Failure, Brazil</td>
<td>Bingham</td>
<td>23</td>
<td>800 Pa</td>
<td>100 Pa·s</td>
<td>0.167</td>
<td>HEC/SAM</td>
</tr>
<tr>
<td>Corbett Creek Hydraulic Analysis, Ouray County, Colorado</td>
<td>O'Brien Equation (Quadratic)</td>
<td>48-59</td>
<td>3,000 Pa</td>
<td>4.2-5.9*</td>
<td>0.25</td>
<td>SPK</td>
</tr>
<tr>
<td>Ether Hollow Fire, Mapleton and Springville, Utah</td>
<td>O'Brien Equation (Quadratic)</td>
<td>80</td>
<td>700-2500</td>
<td>11*</td>
<td>0.04-0.12</td>
<td>SPK</td>
</tr>
<tr>
<td>Tule River Post-Wildfire Debris Flow Assessment</td>
<td>Bingham</td>
<td>20-60</td>
<td>0.02-0.45*</td>
<td>5-122*</td>
<td>0.035-0.2</td>
<td>SPK</td>
</tr>
<tr>
<td>Santa Cruz County Post-Fire Debris Flow Flood Mapping</td>
<td>Bingham</td>
<td>24</td>
<td>600</td>
<td>100</td>
<td>0.5</td>
<td>SPD/SPK</td>
</tr>
<tr>
<td>USGS Debris Flow</td>
<td>O'Brien Equation (Quadratic)</td>
<td>61.2</td>
<td>50</td>
<td>Maron &amp; Pierce</td>
<td></td>
<td>HEC</td>
</tr>
<tr>
<td>Eagle Creek, Oregon</td>
<td>O'Brien Equation (Quadratic)</td>
<td>40-60</td>
<td>0.4-3.67*</td>
<td>24-121*</td>
<td>0.035-0.082</td>
<td>NWP</td>
</tr>
</tbody>
</table>

* Computed from Julian’s “typical soil” coefficients for the exponential equation (using the $a10^b$ in the $ae^b$ form.)
### \( \tau_y \) Summary

<table>
<thead>
<tr>
<th>Type</th>
<th>( \tau_y ) Range (Pa)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Julian (1995)</td>
<td>375-5,000</td>
<td>For high ( C_v ) (60-80%)</td>
</tr>
<tr>
<td>Experimental Data</td>
<td>25-400</td>
<td>Lower than other sources</td>
</tr>
<tr>
<td>Arattano and Franzi (2007)</td>
<td>200-2,000</td>
<td>Calibrated Values - Dropped lowest and highest</td>
</tr>
<tr>
<td>Phillips and Davies (1991)</td>
<td>5-2,500 ( \times 2,000-3,000 )</td>
<td>Reported ( \tau_y ) ( \times ) Recommended</td>
</tr>
<tr>
<td>Whipple &amp; Zimbelma (1991)</td>
<td>100-6,000</td>
<td>Survey of several recommended values and Owen’s Valley work</td>
</tr>
<tr>
<td>Genovis et al (2001)</td>
<td>500-1,100</td>
<td>Front of the observed debris flow</td>
</tr>
<tr>
<td>USACE Experience</td>
<td>600-3,000</td>
<td>Excluding estimates from ( 10^{aC_v} )</td>
</tr>
</tbody>
</table>

**\( \tau_y \) summary:** High hundreds to low thousands.
Rheological Parameter Estimation

1. Yield Strength ($\tau_y$)
2. Viscosity ($\mu$)
3. Coarse/Fine Impacts
4. Sensitivity
5. HB Power ($n$)
Exponential Method

\[ \mu = 0.001 e^{B \cdot C_v} \]

Like \( \tau_y \) many investigators have reported an exponential relationship between \( \mu \) and \( C_v \).
HEC-RAS Includes the Exponential Method

\[
\tau_y = ae^{b \cdot C_v}
\]

Julian (1995) reports coefficient for:

\[
\tau_y = a10^{b \cdot C_v}
\]

To Convert Julien’s (1995) Parameters from:

\[
\tau_y = a10^{b \cdot C_v}
\]

leave “a” the same and multiply “b” by 2.303

<table>
<thead>
<tr>
<th>Type</th>
<th>Liquid Limit (Cv)</th>
<th>B</th>
<th>Range (Pa-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Soil</td>
<td>65-80%</td>
<td>18.4</td>
<td>160-2,500</td>
</tr>
<tr>
<td>Kaolinite</td>
<td>40-50%</td>
<td>18.4</td>
<td>1.6-10</td>
</tr>
<tr>
<td>Sensitive Clays</td>
<td>35-60%</td>
<td>11.5</td>
<td>0.1-1</td>
</tr>
<tr>
<td>Bentonite</td>
<td>5-20%</td>
<td>230.3</td>
<td>100-1E+17</td>
</tr>
</tbody>
</table>

Non-Newtonian Methods and Parameters

Concentration and Bulk:

Volumetric Concentration (Cv) (%)

Select Bulk Method:

Shear Components

Yield Strength: Exponential

Mixture Dynamic Viscosity: Exponential

Convert Viscosity Units
Jong et al. (2010) Meta Analysis of Experimental Data

Viscous porosity, $\mu$ (Pa-s) vs. Yield stress, $\tau_c$ (Pa)

Possible boundary of rheological transition

- Sand rich
- Silt rich
- Clay rich
- Iron tailings

Locat (1997)

- Phillips and Davies (1991)
- Major and Pierson (1992)
- Whipple and Dunne (1992)
- Coussot and Piau (1995)
- Coussot et al. (1998)
- Parsons et al. (2001)
- Schatzman et al. (2003)
- Ilistad et al. (2004)
- Jeong (2006)
<table>
<thead>
<tr>
<th>Torrent</th>
<th>( \tau_y ) (Pa)</th>
<th>( \mu ) (Pa·s(^n))</th>
<th>( \eta )</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( CV )%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anhui Dam</td>
<td>38</td>
<td>2.1</td>
<td>1</td>
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<td>3200</td>
<td>1</td>
<td>2000</td>
<td>63%</td>
</tr>
<tr>
<td>Faucon stream</td>
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<td>5/100</td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>Yosemite valley</td>
<td>150/300</td>
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<td>1</td>
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<td></td>
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<td>Moscardo</td>
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<td>0.3</td>
<td>–</td>
<td></td>
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<td>611/850</td>
<td>60/1700</td>
<td>1</td>
<td>2000</td>
<td>63%</td>
</tr>
</tbody>
</table>

Arattano and Franzi (2007):
\[ \mu \sim 2\text{-}1,700 \text{ Pa-s} \]

Philips and Davies (1991):
\[ \mu \sim 20\text{-}320 \text{ Pa-s} \]
Whipple and Dunn (1992)
Owen’s Valley

Genovis et al. (2001)
Image Analysis

Bingham viscosity ($\mu_B$ (Pa s))

<table>
<thead>
<tr>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>430 ± 50</td>
<td></td>
</tr>
<tr>
<td>380 ± 70</td>
<td></td>
</tr>
<tr>
<td>180 ± 45</td>
<td></td>
</tr>
<tr>
<td>30 ± 15</td>
<td></td>
</tr>
<tr>
<td>225 ± 75</td>
<td></td>
</tr>
<tr>
<td>18 ± 3</td>
<td></td>
</tr>
</tbody>
</table>

$\mu \sim 20-430$ Pa-s

Shear strength $K$

Newtonian viscosity $\mu_N$

Bingham viscosity $\mu_B$

$\mu \sim 20-70$ Pa-s
## \( \mu \) Summary

<table>
<thead>
<tr>
<th>Type</th>
<th>( \tau_y ) Range (Pa-s)</th>
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</tr>
</thead>
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<tr>
<td>Experimental Data</td>
<td>0.2-50</td>
<td>Lower than other sources</td>
</tr>
<tr>
<td>Arattano and Franzi (2007)</td>
<td>2-1,700</td>
<td>Calibrated Values - Dropped lowest and highest</td>
</tr>
<tr>
<td>Phillips and Davies (1991)</td>
<td>20-320</td>
<td>Reported</td>
</tr>
<tr>
<td>Whipple &amp; Zimbelma (1991)</td>
<td>50-1,000</td>
<td>Survey of several recommended values and Owen’s Valley work</td>
</tr>
<tr>
<td>Genovis et al (2001)</td>
<td>20-70</td>
<td>Front of the observed debris flow</td>
</tr>
<tr>
<td>USACE Experience</td>
<td>~100</td>
<td>Excluding estimates from ( 10^{ACv} )</td>
</tr>
</tbody>
</table>

### \( \mu \) Summary: high tens to low hundreds
Viscosities of various materials

(Typical examples, approximate values. At room temperature around 20–24 °C)

Low viscosity
- Water
- Worcester sauce
- Salad oil
- Petrol engine oil
- Silicone adhesive
- Water-based printing ink
- Laundry starch
- Egg yolk
- Yogurt
- Gum syrup
- Strawberry jam
- Mayonnaise
- Honey

Medium viscosity

High viscosity
- Toothpaste
- Pomade
- Rice Jelly/Tar
- Mustard paste
- Mustard
- Shortening/Lard

Ultra-high viscosity

## Credibility Testing Viscosity

<table>
<thead>
<tr>
<th>Material</th>
<th>$\mu$ (Pa-s)</th>
<th>$X_{H_20}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peanut Butter</td>
<td>250</td>
<td>250,000</td>
</tr>
<tr>
<td>Ketchup</td>
<td>100</td>
<td>100,000</td>
</tr>
<tr>
<td>Chocolate Syrup</td>
<td>20</td>
<td>20,000</td>
</tr>
<tr>
<td>Molasses</td>
<td>10</td>
<td>10,000</td>
</tr>
<tr>
<td>Corn Syrup</td>
<td>1</td>
<td>1,000</td>
</tr>
<tr>
<td>Motor Oil</td>
<td>0.3</td>
<td>300</td>
</tr>
<tr>
<td>Olive Oil</td>
<td>0.1</td>
<td>100</td>
</tr>
<tr>
<td>Water</td>
<td>0.001</td>
<td>1</td>
</tr>
</tbody>
</table>
Rheological Parameter Estimation

1. Yield Strength ($\tau_y$)

2. Viscosity ($\mu$)

3. Coarse/Fine Impacts

4. Sensitivity

5. HB Power ($n$)
Sand and fine concentrations are “not purely additive”
- Sand $\downarrow \tau_y$ relative to the fine-only trend.
- Viscosity is more complicated.
- Low sand concentrations $\uparrow \mu$ relative to fine-only, but higher concentrations $\downarrow \mu$
Sand “concentrates” solids into low surface-area clasts. “Fluid” phase has more water and fewer fine-particle interactions. Sand contacts do not yet compensate.

“Fluid” phase has all solids. Maximum viscous resistance from fine particle interactions.

More of the solid phase is in sand. “Fluid” is more dilute, further reducing vasospastic effects. But the increase in sand collision and/or friction more than compensates.
USGS DEBRIS-FLOW FLUME
Debris flows with 20% loam vs. 2% loam on rough rigid bed (each flow is 10 m³ water-saturated sand, gravel and loam).
Rheological Parameter Estimation

1. Yield Strength ($\tau_y$)
2. Viscosity ($\mu$)
3. Coarse/Fine Impacts
4. Sensitivity
5. HB Power ($n$)
\[ \tau_y = 700 \text{ Pa (All)} \]

**Viscosity Sensitivity**

- \( \mu = 10,000 \text{ Pa-s} \)
- \( \mu = 5,000 \text{ Pa-s} \)
- \( \mu = 1,000 \text{ Pa-s} \)
- \( \mu = 500 \text{ Pa-s} \)
- \( \mu = 100 \text{ Pa-s} \)
- \( \mu = 0 \text{ Pa-s} \)
Young's Modulus

Yield Stress

Sensitivity

\( \tau_y = 5,000 \text{ Pa} \)

\( \tau_y = 2,500 \text{ Pa} \)

\( \tau_y = 1,000 \text{ Pa} \)

\( \tau_y = 500 \text{ Pa} \)

\( \tau_y = 100 \text{ Pa} \)

\( \tau_y = 50 \text{ Pa} \)

\( \mu = 500 \text{ Pa-s (All)} \)

Yield Stress Sensitivity
# Sensitivity Summary

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sensitivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_y$</td>
<td>Affects runout and final, inundation. $\uparrow \tau_y$ can $\downarrow$ run times because fewer wet nodes</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Affects:</td>
</tr>
<tr>
<td></td>
<td>- depth     ($\uparrow \mu \rightarrow \uparrow$ depth)</td>
</tr>
<tr>
<td></td>
<td>- arrival time ($\uparrow \mu \rightarrow \downarrow$ velocity $\rightarrow \downarrow$ arrival time)</td>
</tr>
<tr>
<td></td>
<td>- stability ($\uparrow \mu \rightarrow \downarrow$ stable)</td>
</tr>
</tbody>
</table>
Rheological Parameter Estimation

1. Yield Strength ($\tau_y$)
2. Viscosity ($\mu$)
3. Coarse/Fine Impacts
4. Sensitivity
5. HB Power ($n$)
\[
\tau = \tau_y + K \left( \frac{d\nu_x}{dz} \right)^{n>1}
\]

\[
\tau = \tau_y + K \left( \frac{d\nu_x}{dz} \right)^{n=1}
\]

\[
\tau = \tau_y + K \left( \frac{d\nu_x}{dz} \right)^{n<1}
\]
In the lab, cohesive-only mixtures tend to be strongly shear thinning, while most observations of shear thickening included sand.