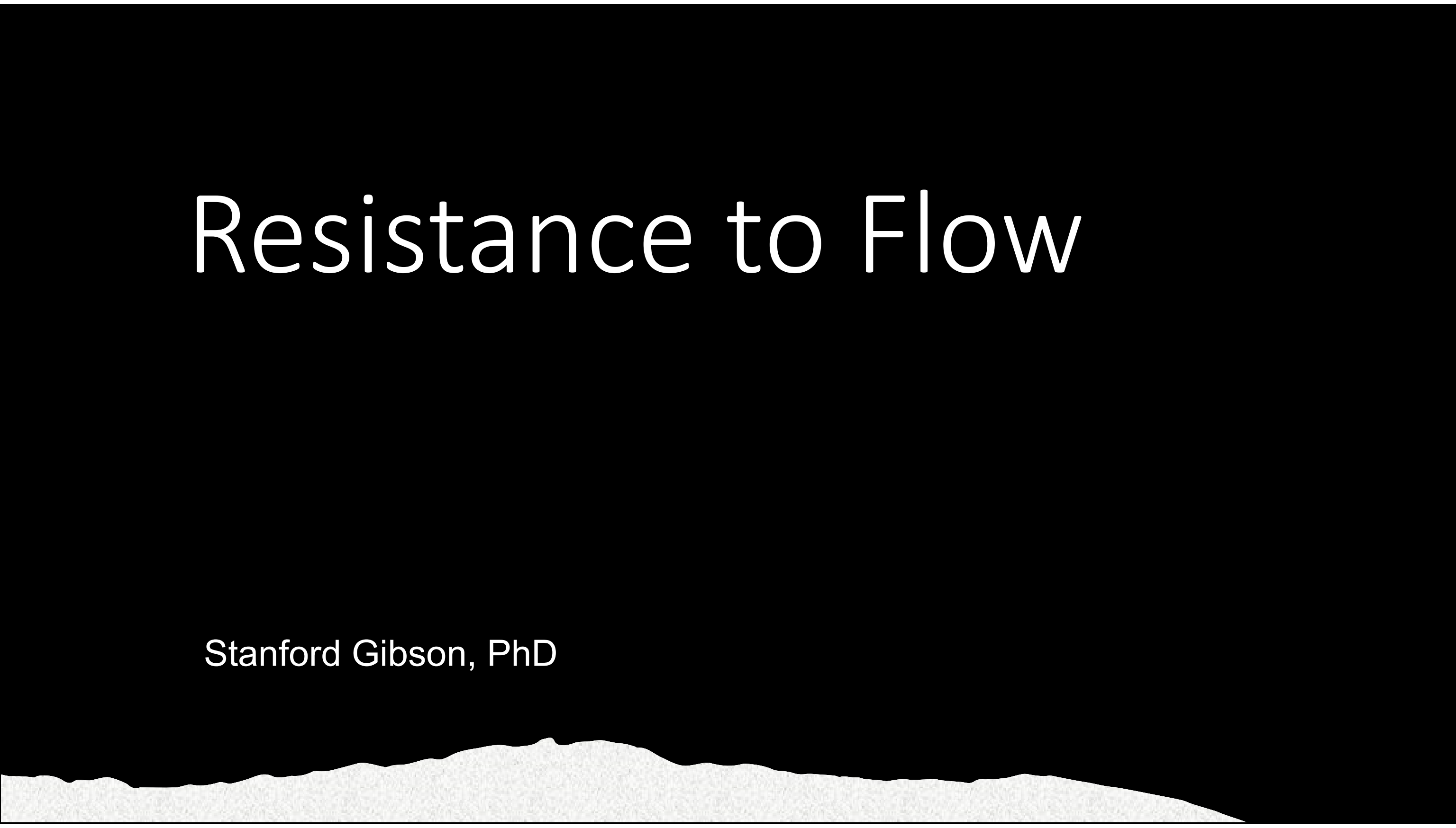

Resistance to Flow

Stanford Gibson, PhD

The two most commonly used equations for the computation of steady flow in natural channels are the Chezy and Manning equations. Both express velocity as a function of a roughness coefficient, the hydraulic radius and friction slope.

The obvious difference in these two equations aside from the use of “C” or “n” as an empirical roughness coefficient is the extra R to the 1/6 power in the Manning equation. Chow (1959) discourses at length (in a 3 page foot note) on the implications of the non-dimensionality of the Manning equation and its effect on the Manning roughness coefficient n.

Manning’s equation is used more often for open channel flow.

Road Map

1. Flow Resistance Equations
2. Factors Affecting Roughness Coefficients
3. Determination of Manning's n
    i. Charts
    ii. Pictures
    iii. Equations
4. Roughness Options in HEC-RAS
    i. Equivalent Roughness – k Values
    ii. Horizontal/Vertical Variation of n
    iii. Flow Verses Roughness

Road Map

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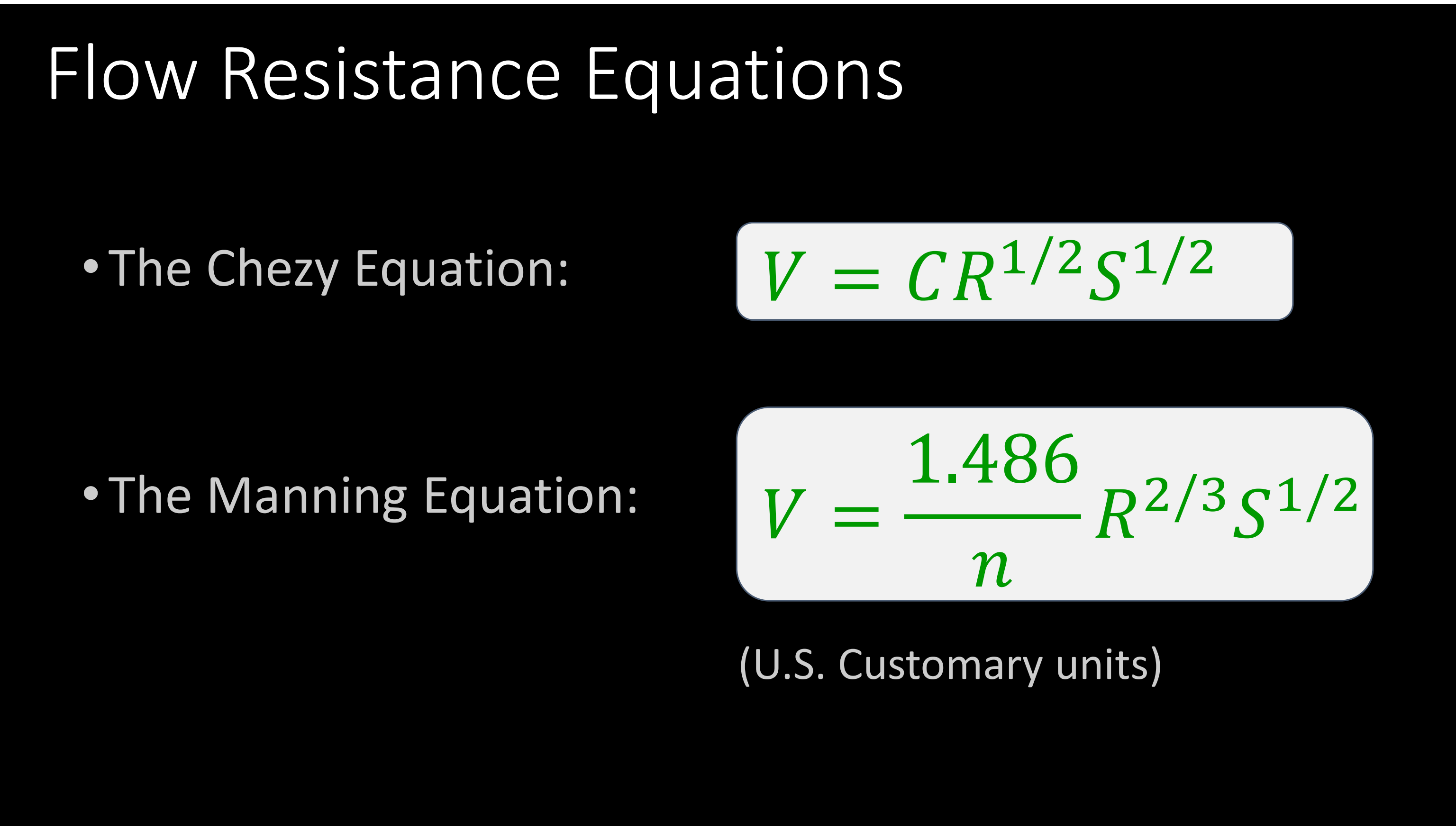

Flow Resistance Equations

- The Chezy Equation: $V = CR^{1/2}S^{1/2}$
- The Manning Equation: $V = \frac{1.486}{n}R^{2/3}S^{1/2}$

(U.S. Customary units)

The two most commonly used equations for the computation of steady flow in natural channels are the Chezy and Manning equations. Both express velocity as a function of a roughness coefficient, the hydraulic radius and friction slope.

The obvious difference in these two equations aside from the use of “C” or “n” as an empirical roughness coefficient is the extra R to the 1/6 power in the Manning equation. Chow (1959) discourses at length (in a 3 page foot note) on the implications of the non-dimensionality of the Manning equation and its effect on the Manning roughness coefficient n.

Manning’s equation is used more often for open channel flow.

Friction Loss Calculations in HEC-RAS

- Friction loss is evaluated by the following equation in HEC-RAS:

$$h_f = \overline{S}_f L$$

- The energy slope is from Manning’s equation:

$$S_f = \left(\frac{Q}{K}\right)^2$$

where:

$$K = \frac{1.486}{n} A R^{2/3}$$

Road Map

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Factors Affecting Roughness Coefficients

- Surface roughness (material and bed form)
- Vegetation (possible seasonal effects)
- Channel irregularities
- Channel alignment (sinuosity)
- Scour and deposition
- Obstructions (debris)
- Stage and discharge
- Temperature
- Suspended material and bedload

As pointed out in Chow's discourse on the effects of the "extra" R to 1/6 power in the Manning equation, the n value varies with depth of flow. For n values greater than 0.022 the effect of depth is to decrease the apparent roughness, for n values less than 0.022 the effect of depth is to increase the apparent roughness. This effect may be seen graphically in Plate B-4 of EM 1110-2-1601. It is particularly important to consider this effect when analyzing concrete lined flood control channels.

'The height…of the bedforms was much smaller when the water was cold than whne it was warm. This accounts for the lower friction factor of the cold water."
Vanoni Manual 54 p110

Resistance to Flow

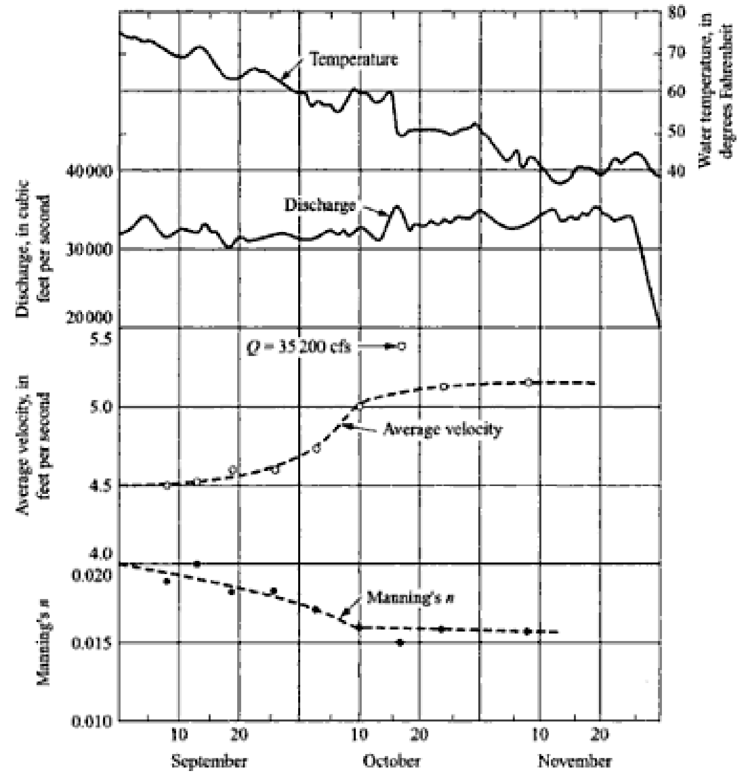

Figure 2.91 Variation of water temperature, discharge, average velocity, and Manning's *n* for Missouri River at Omaha, Neb., during fall of 1966 (United States Army Corps of Engineers, 1969).

Road Map

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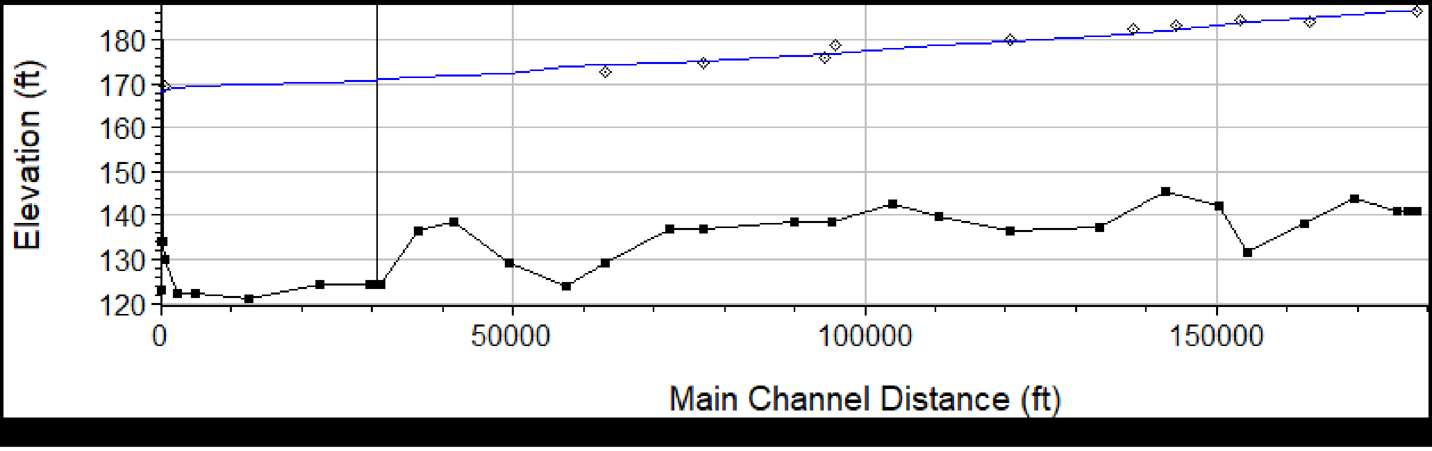
Selecting Manning’s n Values

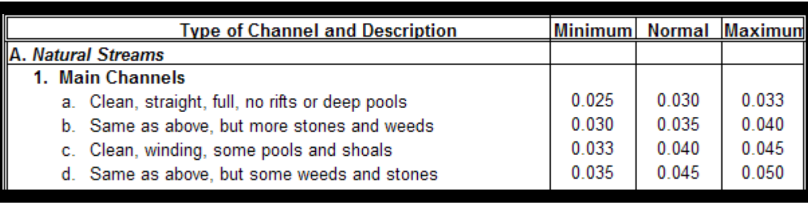
- Channel roughness is one of the primary sources of uncertainty in water surface profile computations.
- Therefore, Manning’s n values should be calibrated to observed data.



Selecting Manning’s n Values

- There are three tools that help select n values:

Lookup Tables



Type of Channel and Description	Minimum	Normal	Maximum
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1. **Main Channels**			
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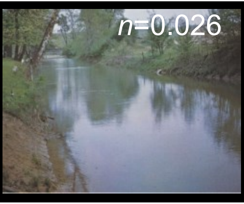
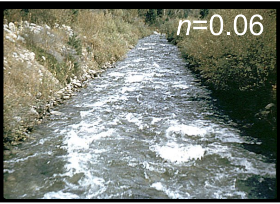
b. Same as above, but more stones and weeds	0.030	0.035	0.040
c. Clean, winding, some pools and shoals	0.033	0.040	0.045
d. Same as above, but some weeds and stones	0.035	0.045	0.050

Photographic Comparison


n=0.026


n=0.043


n=0.06

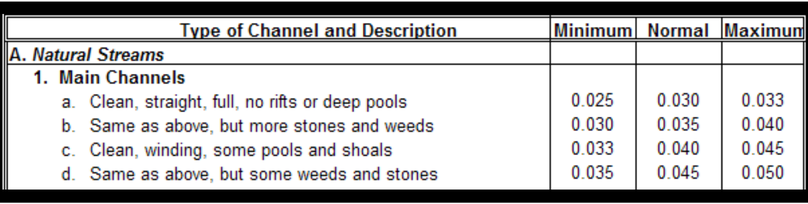
Analytical Equations

$$n = 0.39S^{0.38}R^{-0.16}$$

Selecting Manning’s n Values

- There are three tools that help select n values:

Lookup Tables



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Photographic Comparison

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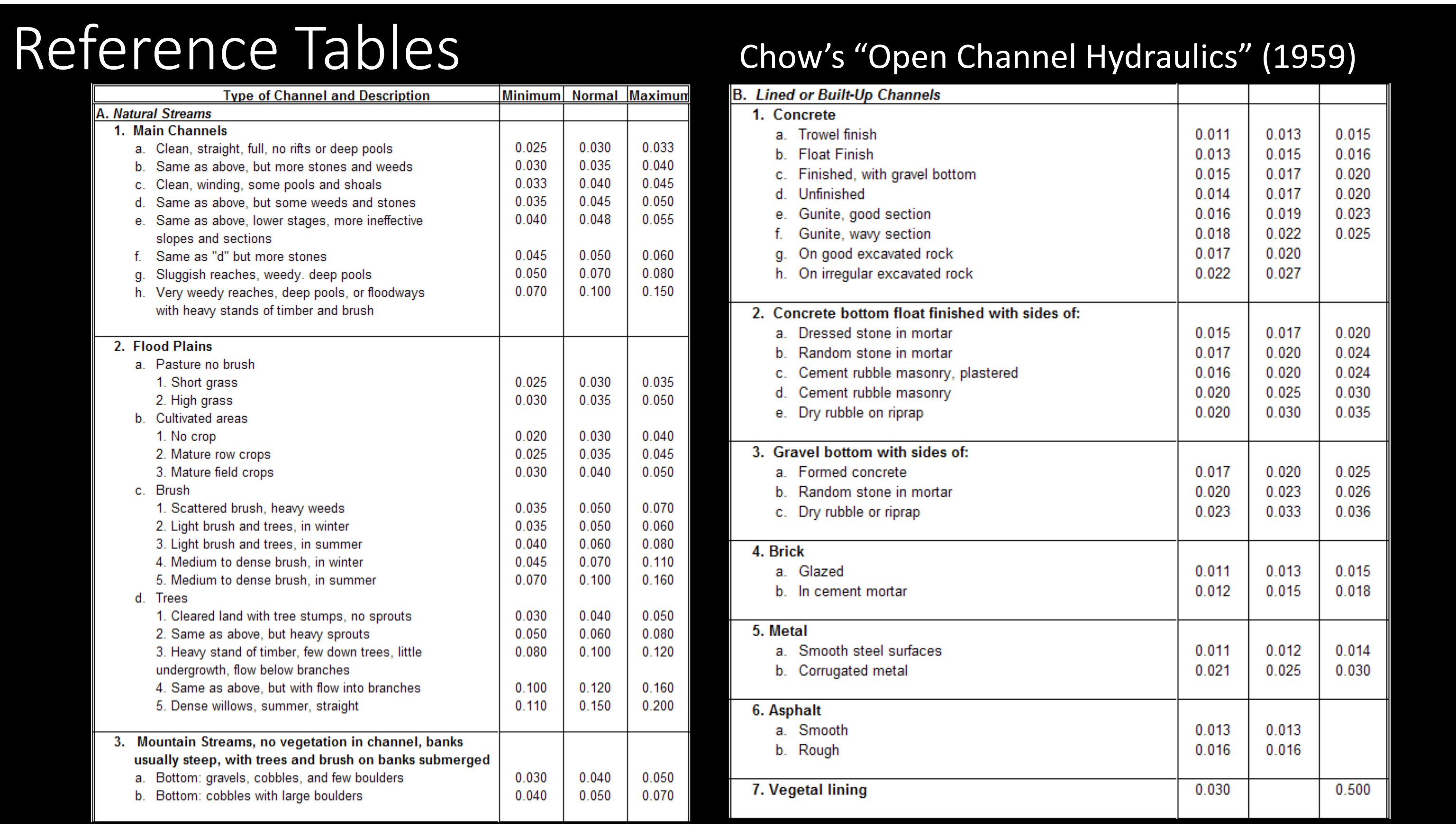

Reference Tables

Chow's "Open Channel Hydraulics" (1959)

Type of Channel and Description	Minimum	Normal	Maximum
A. Natural Streams			
1. Main Channels			
a. Clean, straight, full, no rifts or deep pools	0.025	0.030	0.033
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c. Clean, winding, some pools and shoals	0.033	0.040	0.045
d. Same as above, but some weeds and stones	0.035	0.045	0.050
e. Same as above, lower stages, more ineffective slopes and sections	0.040	0.048	0.055
f. Same as "d" but more stones	0.045	0.050	0.060
g. Sluggish reaches, weedy. deep pools	0.050	0.070	0.080
h. Very weedy reaches, deep pools, or floodways with heavy stands of timber and brush	0.070	0.100	0.150
2. Flood Plains			
a. Pasture no brush			
1. Short grass	0.025	0.030	0.035
2. High grass	0.030	0.035	0.050
b. Cultivated areas			
1. No crop	0.020	0.030	0.040
2. Mature row crops	0.025	0.035	0.045
3. Mature field crops	0.030	0.040	0.050
c. Brush			
1. Scattered brush, heavy weeds	0.035	0.050	0.070
2. Light brush and trees, in winter	0.035	0.050	0.060
3. Light brush and trees, in summer	0.040	0.060	0.080
4. Medium to dense brush, in winter	0.045	0.070	0.110
5. Medium to dense brush, in summer	0.070	0.100	0.160
d. Trees			
1. Cleared land with tree stumps, no sprouts	0.030	0.040	0.050
2. Same as above, but heavy sprouts	0.050	0.060	0.080
3. Heavy stand of timber, few down trees, little undergrowth, flow below branches	0.080	0.100	0.120
4. Same as above, but with flow into branches	0.100	0.120	0.160
5. Dense willows, summer, straight	0.110	0.150	0.200
3. Mountain Streams, no vegetation in channel, banks usually steep, with trees and brush on banks submerged			
a. Bottom: gravels, cobbles, and few boulders	0.030	0.040	0.050
b. Bottom: cobbles with large boulders	0.040	0.050	0.070

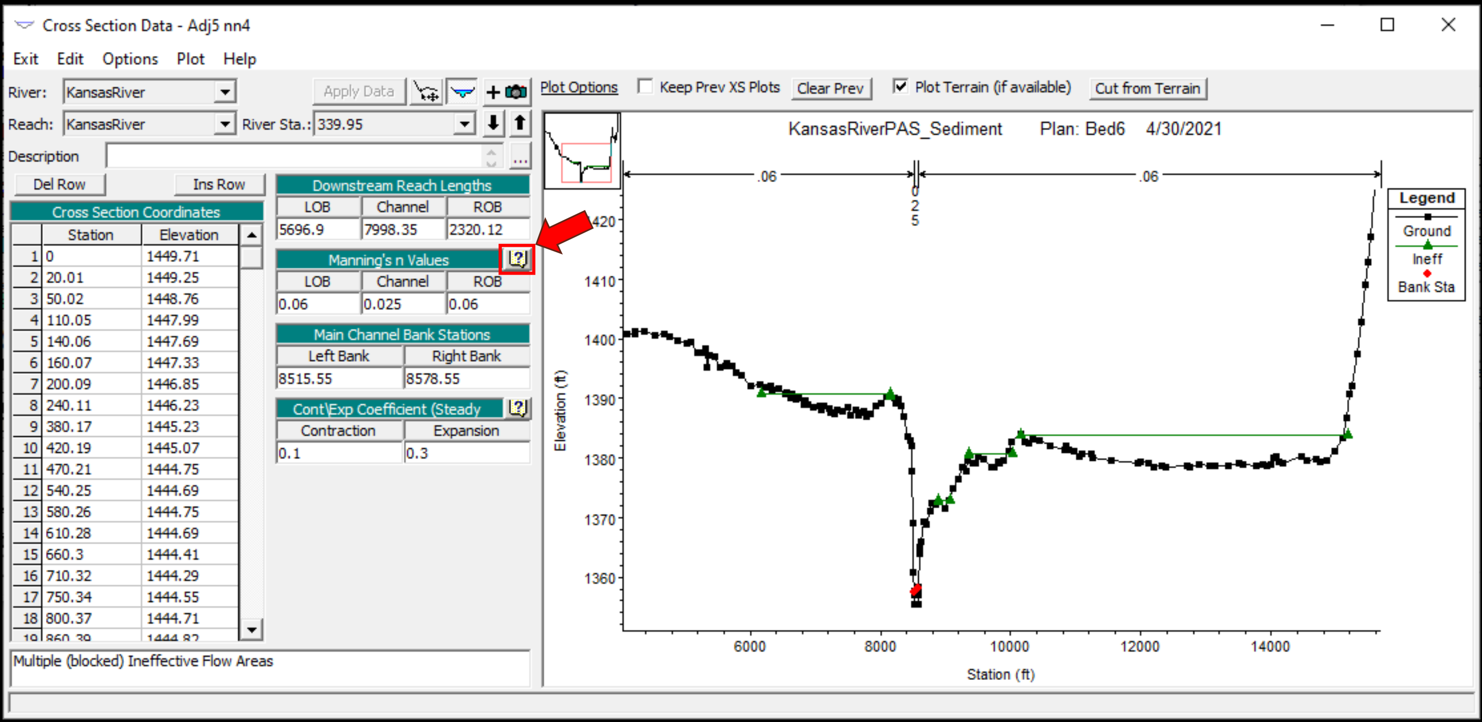
B. Lined or Built-Up Channels			
1. Concrete			
a. Trowel finish	0.011	0.013	0.015
b. Float Finish	0.013	0.015	0.016
c. Finished, with gravel bottom	0.015	0.017	0.020
d. Unfinished	0.014	0.017	0.020
e. Gunite, good section	0.016	0.019	0.023
f. Gunite, wavy section	0.018	0.022	0.025
g. On good excavated rock	0.017	0.020	
h. On irregular excavated rock	0.022	0.027	
2. Concrete bottom float finished with sides of:			
a. Dressed stone in mortar	0.015	0.017	0.020
b. Random stone in mortar	0.017	0.020	0.024
c. Cement rubble masonry, plastered	0.016	0.020	0.024
d. Cement rubble masonry	0.020	0.025	0.030
e. Dry rubble on riprap	0.020	0.030	0.035
3. Gravel bottom with sides of:			
a. Formed concrete	0.017	0.020	0.025
b. Random stone in mortar	0.020	0.023	0.026
c. Dry rubble or riprap	0.023	0.033	0.036
4. Brick			
a. Glazed	0.011	0.013	0.015
b. In cement mortar	0.012	0.015	0.018
5. Metal			
a. Smooth steel surfaces	0.011	0.012	0.014
b. Corrugated metal	0.021	0.025	0.030
6. Asphalt			
a. Smooth	0.013	0.013	
b. Rough	0.016	0.016	
7. Vegetal lining	0.030		0.500

Chow's "Open Channel Hydraulics" (1959)

Lists natural streams, flood plains, and constructed channels.

Tables include maximum, normal, and minimum values for type of channel.

HEC-RAS Option to Display Chow’s n Value Table



River: KansasRiver

Reach: KansasRiver

River Sta.: 339.95

Description

☐ Keep Prev XS Plots

☑ Plot Terrain (if available)

Cross Section Coordinates

	Station	Elevation
1	0	1449.71
2	20.01	1449.25
3	50.02	1448.76
4	110.05	1447.99
5	140.06	1447.69
6	160.07	1447.33
7	200.09	1446.85
8	240.11	1446.23
9	380.17	1445.23
10	420.19	1445.07
11	470.21	1444.75
12	540.25	1444.69
13	580.26	1444.75
14	610.28	1444.69
15	660.3	1444.41
16	710.32	1444.29
17	750.34	1444.55
18	800.37	1444.71
19	860.39	1444.82

Downstream Reach Lengths

LOB	Channel	ROB
5696.9	7998.35	2320.12

Manning's n Values

LOB	Channel	ROB
0.06	0.025	0.06

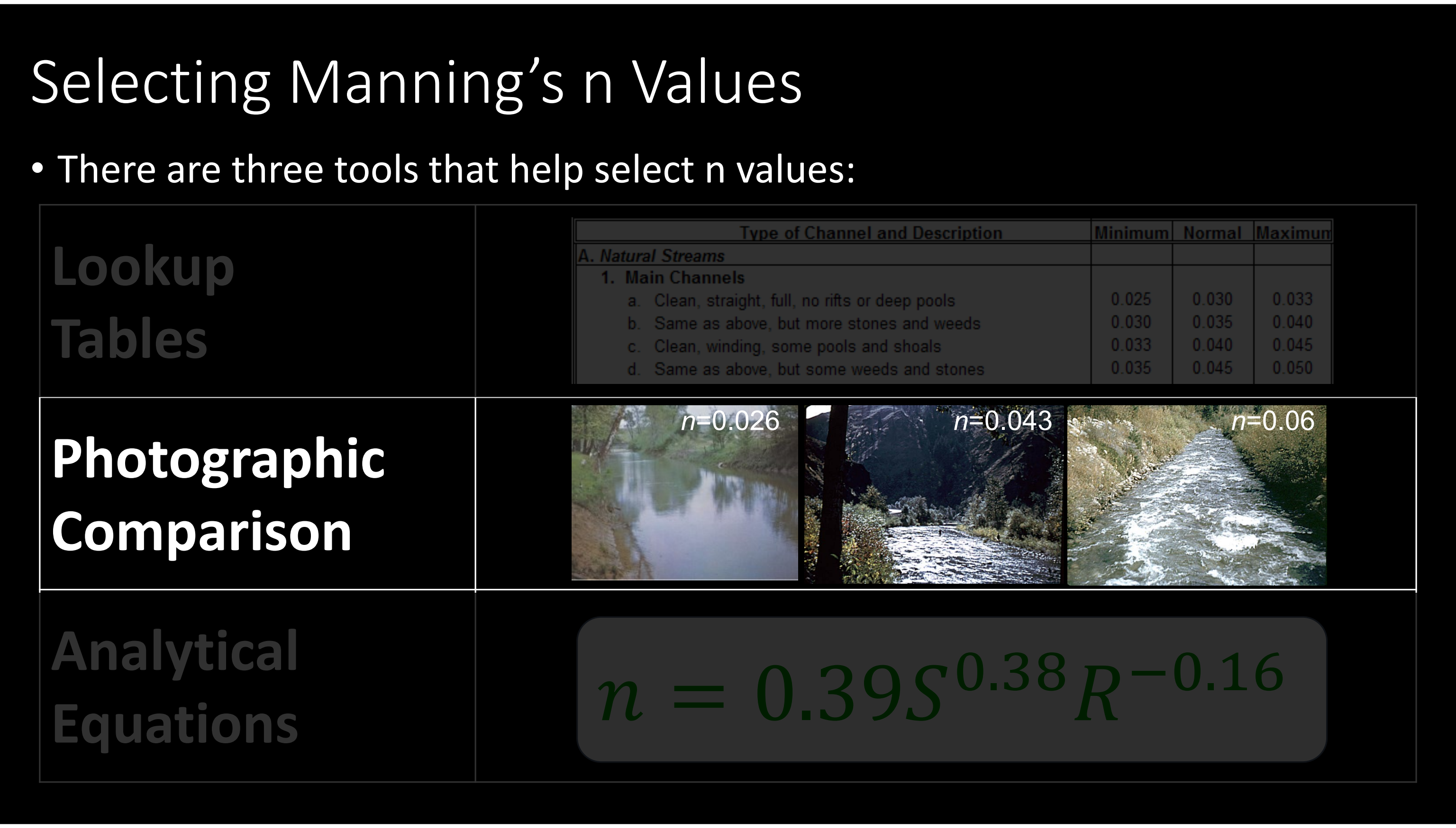
Main Channel Bank Stations

Left Bank	Right Bank
8515.55	8578.55

Cont\Exp Coefficient (Steady

Contraction	Expansion
0.1	0.3

Multiple (blocked) Ineffective Flow Areas



Selecting Manning’s n Values

- There are three tools that help select n values:

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Photographic Comparison

Analytical Equations

$$n = 0.39 S^{0.38} R^{-0.16}$$

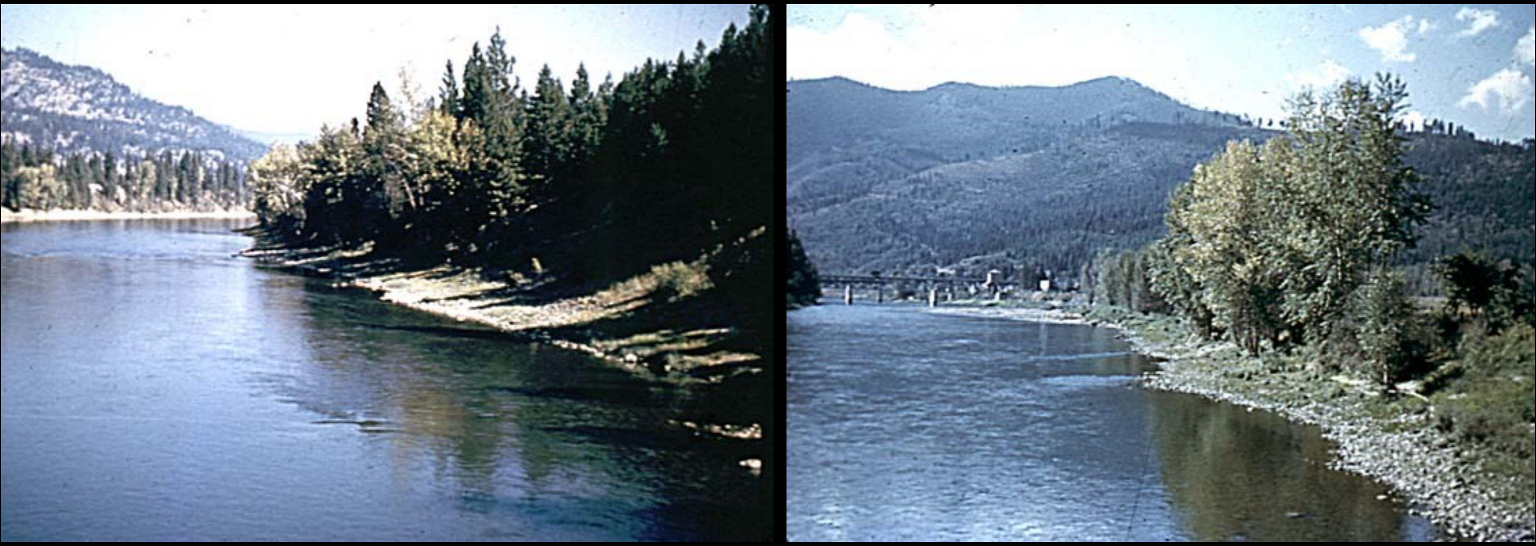
The following publications have photographs of calibrated streams:

Barnes (1967)
Arcement and Schneider (1989)
Chow (1959)
Hicks and Mason (1991)
Fasken (1963)

Indian Fork below Atwood Dam New Cumberland, Ohio

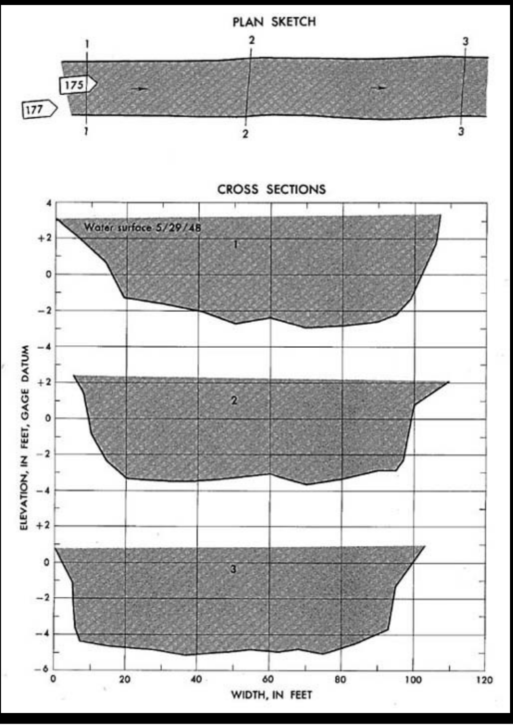
n=0.026

Clark Fork at St. Regis, Montana



$n=0.028$

West Fork Bitterroot River near Conner, Montana



n=0.036

USGS data includes the cross sections and slopes associated with the pictures used to calibrate the stream sections for Manning's n values

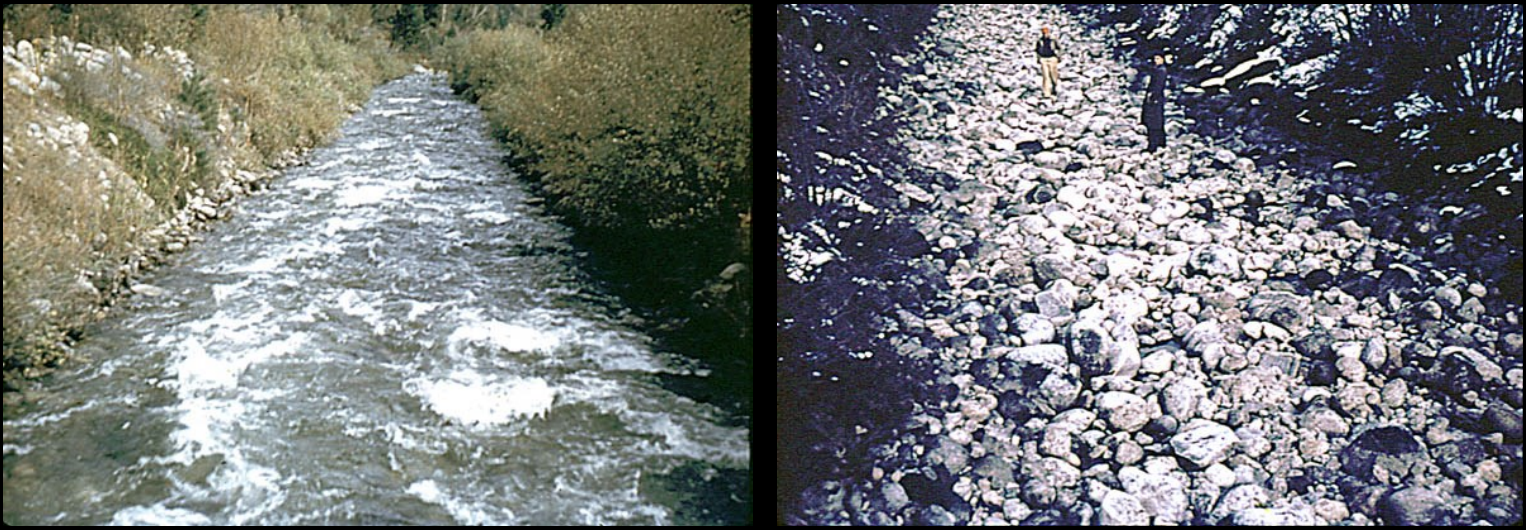
Grande Ronde River at La Gande, Oregon

n=0.043

Mission Creek near Cashmere, Washington

$n=0.057$

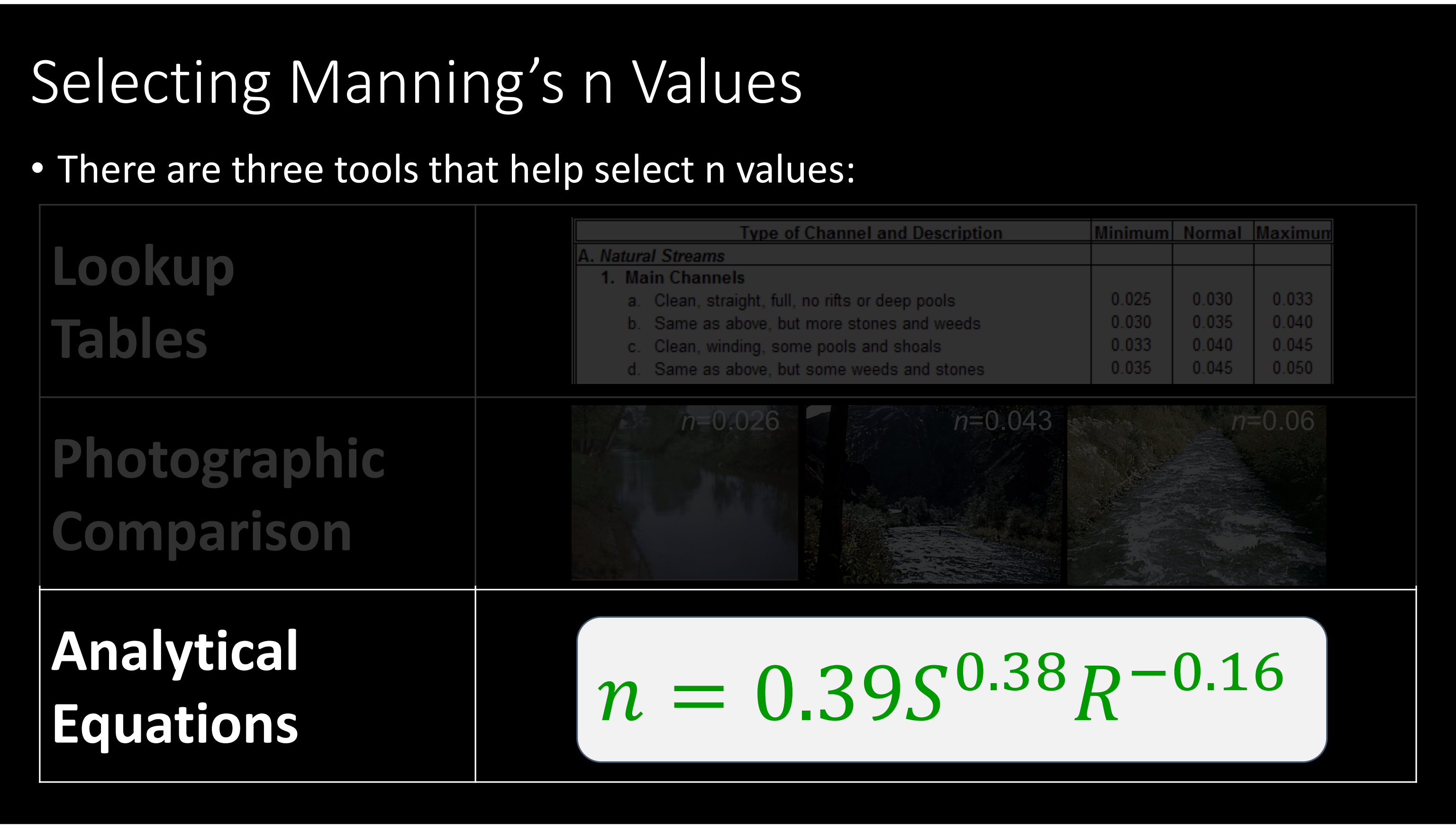
Rock Creek Canal near Darby, Montana



$n=0.06$

Rock Creek near Darby, Montana

n=0.075



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The following publications have photographs of calibrated streams:

Barnes (1967)
Arcement and Schneider (1989)
Chow (1959)
Hicks and Mason (1991)
Fasken (1963)

Cowen’s Method (1956):

$$n = (n_b + n_1 + n_2 + n_3 + n_4) m$$

where:

n_b = Base value of n for a straight uniform, smooth channel in natural materials.

n_1 = Value added to for surface irregularities

n_2 = Value for variations in shape and size

n_3 = Value for obstructions

n_4 = Value for vegetation and flow conditions

m = Correction factor to account for meandering

Although there are many factors that affect the selection of the n value for the channel, some of the most important factors are the type and size of materials that compose the bed and banks of a channel, and the shape of the channel. Cowan (1956) developed a procedure for estimating the effects of these factors to determine the value of Manning’s n of a channel.

A detailed description of Cowan’s method can be found in “Guide for Selecting Manning’s Roughness Coefficients for Natural Channels and Flood Plains” (FHWA, 1984). This report was developed by the U.S. Geological Survey (Arcement, 1989) for the Federal Highway Administration. The report also presents a method similar to Cowan’s for developing Manning's n values for flood plains, as well as some additional methods for densely vegetated flood plains.

Limerinos Equation (1970)

$$n = \frac{0.0926\,R^{1/6}}{1.16 + 2.0\log\left(\frac{R}{d_{84}}\right)}$$

Where : R = hydraulic radius in feet (data range was 1.0 to 6.0 feet)

d_{84} = particle diameter in feet that equals or exceeds 84 percent of the particles (data range was 1.5 mm to 250 mm)

Limerinos (1970) related n values to hydraulic radius and bed particle size based on samples from 11 stream channels having bed materials ranging from small gravel to medium size boulders.

The Limerinos equation fits the data that he used very well, in that the coefficient of correlation ^{2} = 0.88 and the standard error of estimates for values of $n/R^{1/6}$ = 0.0087.

Limerinos selected reaches that had a minimum amount of roughness, other than that caused by the bed material. The Limerinos equation provides a good estimate of the base n value. The base n value should then be increased to account for other factors, as shown above in Cowen's method.

"In mobile boundary channels the bed roughness is composed of grain roughness and form roughness. The grain roughness refers to the effective surface roughness height of the mixture of sediment particles on the streambed. Form roughness refers to bed features described as ripples, dunes, transition, plain bed, standing waves, and antidunes. These bed features, called bed forms, are grouped into the general categories of lower regime, transitional, and upper regime." **EM 1110-2-1601**

Jarrett's (1984) Equation for Steep Streams

$$n = 0.39S^{0.38}R^{-0.16}$$

Where: S = friction slope*

*The slope of the water surface can be used when the friction slope is unknown.

Range of S = 0.002 to 0.04

R = 0.5 to 7.0 feet

Jarrett (1984) developed an equation for high gradient streams (slopes greater than 0.002). Jarrett performed a regression analysis on 75 data sets that were surveyed from 21 different streams. Jarrett (1984) states the following limitations for the use of his equation:

1. The equations are applicable to natural main channels having stable bed and bank materials (gravels, cobbles, and boulders) without backwater.

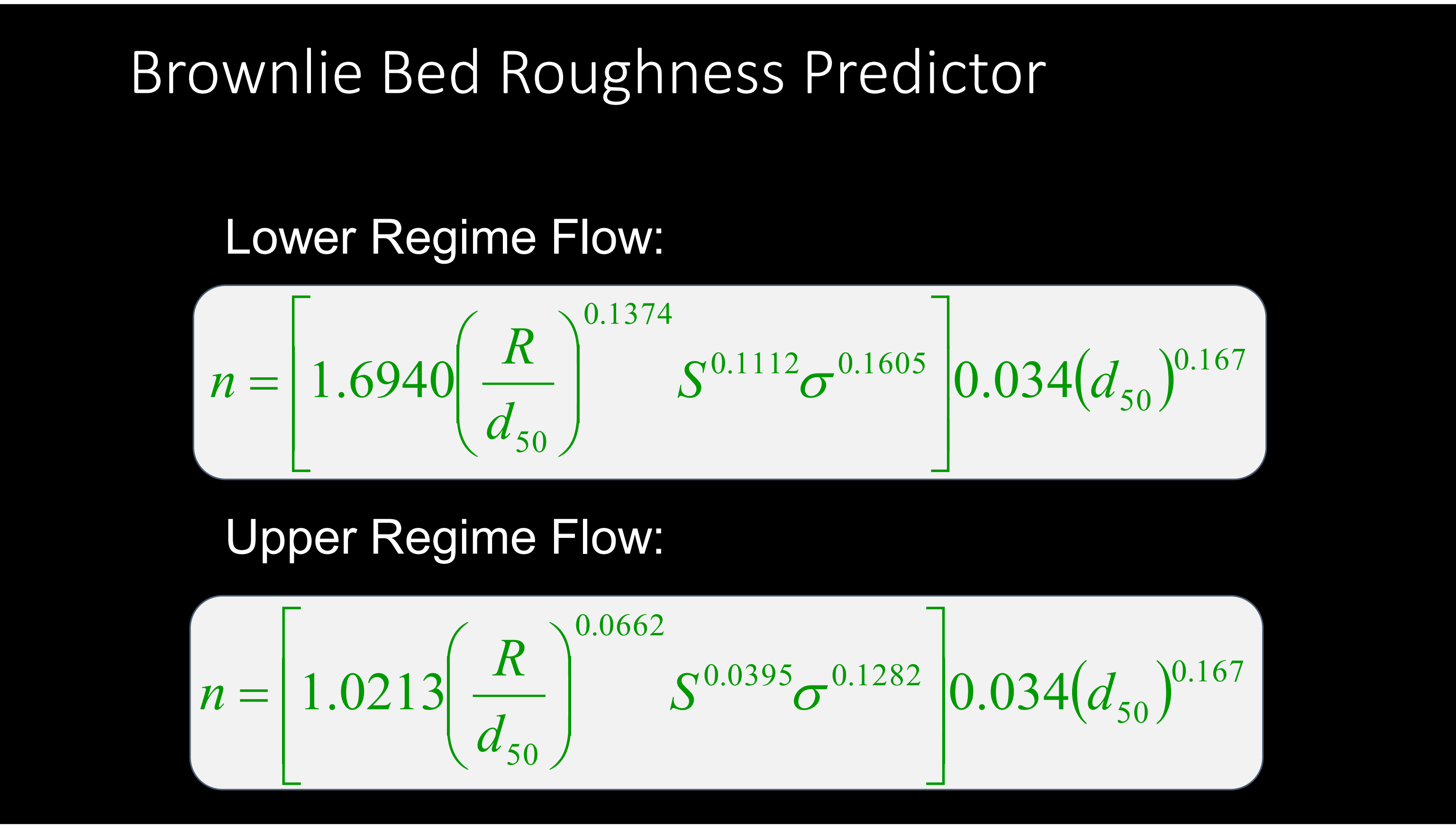
2. The equations can be used for slopes from 0.002 to 0.04 and for hydraulic radii from 0.5 to 7.0 feet (0.15 to 2.1 m). The upper limit on slope is due to a lack of verification data available for the slopes of high-gradient streams. Results of the regression analysis indicate that for hydraulic radius greater than 7.0 feet (2.1 m), n did not vary significantly with depth; thus extrapolating to larger flows should not be too much in error as long as the bed and bank material remain fairly stable.

3. During the analysis of the data, the energy loss coefficients for contraction and

expansion were set to 0.0 and 0.5, respectively.

4. Hydraulic radius does not include the wetted perimeter of bed particles.

5. These equations are applicable to streams having relatively small amounts of suspended sediment.



Brownlie Bed Roughness Predictor

Lower Regime Flow:

$$n = \left[1.6940 \left(\frac{R}{d_{50}} \right)^{0.1374} S^{0.1112} \sigma^{0.1605} \right] 0.034 \left(d_{50} \right)^{0.167}$$

Upper Regime Flow:

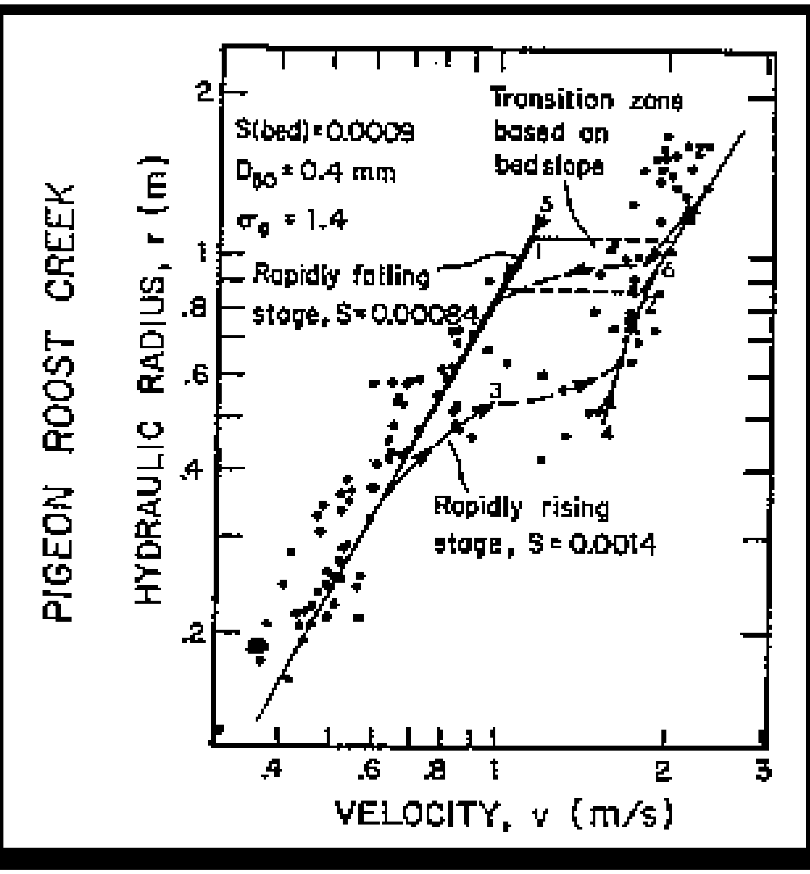
$$n = \left[1.0213 \left(\frac{R}{d_{50}} \right)^{0.0662} S^{0.0395} \sigma^{0.1282} \right] 0.034 \left(d_{50} \right)^{0.167}$$

From EM 1110-2-1601 - HYDRAULIC DESIGN OF FLOOD CONTROL CHANNELS

In sediment transport calculations it is important to link n values to the bed regime. This is particularly true when hydraulic conditions shift between upper regime and lower regime flow. There are several methods in Vanoni (1975) that express n value in terms of sediment parameters, but Brownlie (1983) is the only method that calculates the transition. This method postdates Vanoni (1975).

Brownlie sought to reconstitute the most fundamental process--the discontinuity in the graph of hydraulic radius versus velocity (Figure 5-4). In the process of this research, he collected the known sediment data sets--77 in all, containing 7,027 data points. Of the total, 75 percent were from flume studies and 25 percent from field tests. He used 22 of these data sets and demonstrated a significant agreement with both field and laboratory data.

Resistance to Flow



From EM 1110-2-1601 - HYDRAULIC DESIGN OF FLOOD CONTROL CHANNELS

Road Map

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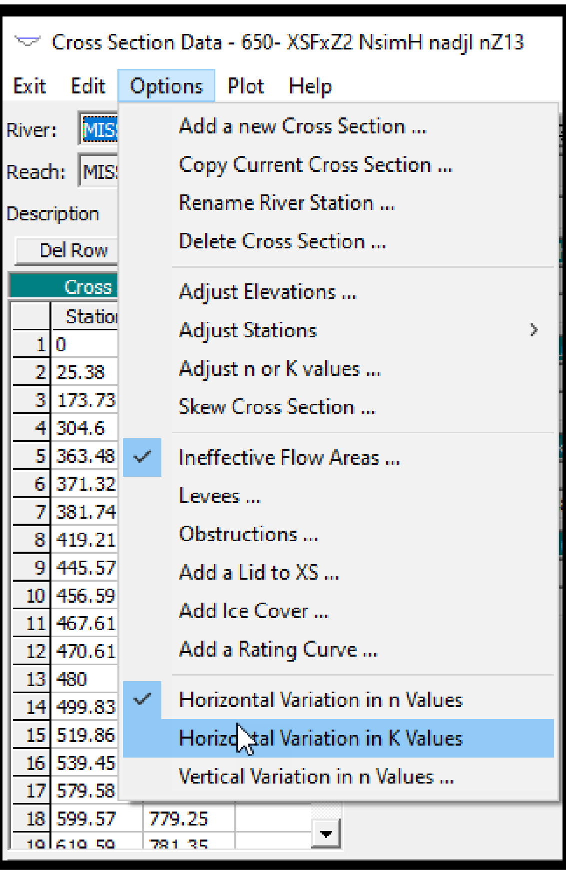
Equivalent Roughness, k Values

- The value *k* represents the average roughness height in the channel that effects flow movement
- Equivalent roughness, k values, are commonly used in hydraulic design of channels. This is an option in HEC-RAS.
- The advantage of using equivalent roughness k instead of Manning's n is that k reflects changes in the friction factor due to stage, whereas Manning's n alone does not.



An equivalent roughness parameter "k", commonly used in the hydraulic design of channels, is provided as an option for describing boundary roughness in HEC-RAS. Equivalent roughness, sometimes called "roughness height," is a measure of the linear dimension of roughness elements, but is not necessarily equal to the actual, or even the average, height of these elements. In fact, two roughness elements with different linear dimensions may have the same "k" value because of differences in shape and orientation [Chow, 1959].

The advantage of using equivalent roughness "k" instead of Manning's n is that "k" reflects changes in the friction factor due to stage, whereas Manning's n alone does not. This influence can be seen in the definition of Chezy's "C" (English units) for a rough channel.

Equivalent Roughness – Conversion Equations

$$C = 32.6 \log_{10}\left(\frac{12.2\,R}{k}\right)$$

$$n = \frac{1.486\,R^{1/6}}{32.6 \log_{10}\left(\frac{12.2\,R}{k}\right)}$$

English Units

$$n = \frac{R^{1/6}}{18.0 \log_{10}\left(\frac{12.2\,R}{k}\right)}$$

Metric Units

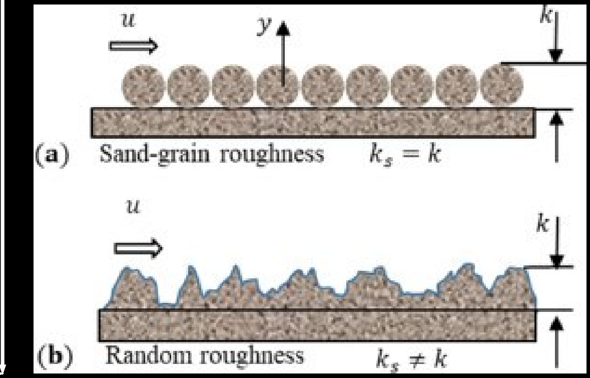
Note that as the hydraulic radius increases (which is equivalent to an increase in stage), the friction factor “C” increases. In HEC-RAS, “k” is converted to a Manning’s n by using the above equation and equating the Chezy and Manning’s equations.

Again, this equation is based on the assumption that all channels (even concrete-lined channels) are “hydraulically rough.” A graphical illustration of this conversion is available [USACE, 1991].

Horizontal variation of “k” values is described in the same manner as horizontal variation of Manning's n values. See chapter 6 of the HEC-RAS user’s manual, to learn how to enter k values into the program. Up to twenty values of “k” can be specified for each cross section.

Equivalent Roughness Values for Various Bed Materials

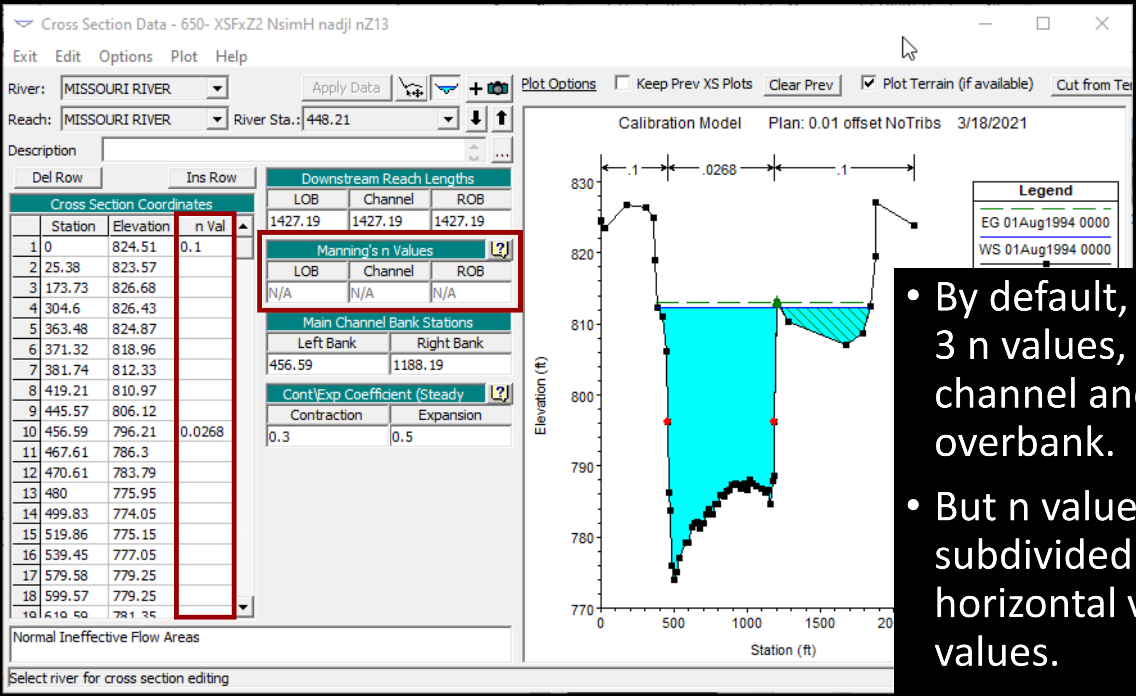
Bed Materials	k values in feet
Brass, Cooper, Lead, Glass	0.0001 - 0.0030
Wrought Iron, Steel	0.0002 - 0.0080
Asphalted Cast Iron	0.0004 - 0.0070
Galvanized Iron	0.0005 - 0.0150
Cast Iron	0.0008 - 0.0180
Wood Stave	0.0006 - 0.0030
Cement	0.0013 - 0.0040
Concrete	0.0015 - 0.0100
Drain Tile	0.0020 - 0.0100
Riveted Steel	0.0030 - 0.0300
Natural River Bed	0.1000 - 3.0000



Tables and charts for determining ‘k’ values for concrete-lined channels are provided in EM 1110-2-1601 [USACE, 1991]. Values for riprap-lined channels may be taken as the theoretical spherical diameter of the median stone size. Approximate ‘k’ values [Chow, 1959] for a variety of bed materials, including those for natural rivers are shown in the Table above.

The values of ‘k’ (0.1 to 3.0 ft.) for natural river channels are normally much larger than the actual diameters of the bed materials to account for boundary irregularities and bed forms.

Horizontal Variation of n Values



River: MISSOURI RIVER

Reach: MISSOURI RIVER

River Sta.: 448.21

Description

Cross Section Coordinates

	Station	Elevation	n Val
1	0	824.51	0.1
2	25.38	823.57	
3	173.73	826.68	
4	304.6	826.43	
5	363.48	824.87	
6	371.32	818.96	
7	381.74	812.33	
8	419.21	810.97	
9	445.57	806.12	
10	456.59	796.21	0.0268
11	467.61	786.3	
12	470.61	783.79	
13	480	775.95	
14	499.83	774.05	
15	519.86	775.15	
16	539.45	777.05	
17	579.58	779.25	
18	599.57	779.25	

Downstream Reach Lengths

LOB	Channel	ROB
1427.19	1427.19	1427.19

Manning's n Values

LOB	Channel	ROB
N/A	N/A	N/A

Main Channel Bank Stations

Left Bank	Right Bank
456.59	1188.19

Cont\Exp Coefficient (Steady

Contraction	Expansion
0.3	0.5

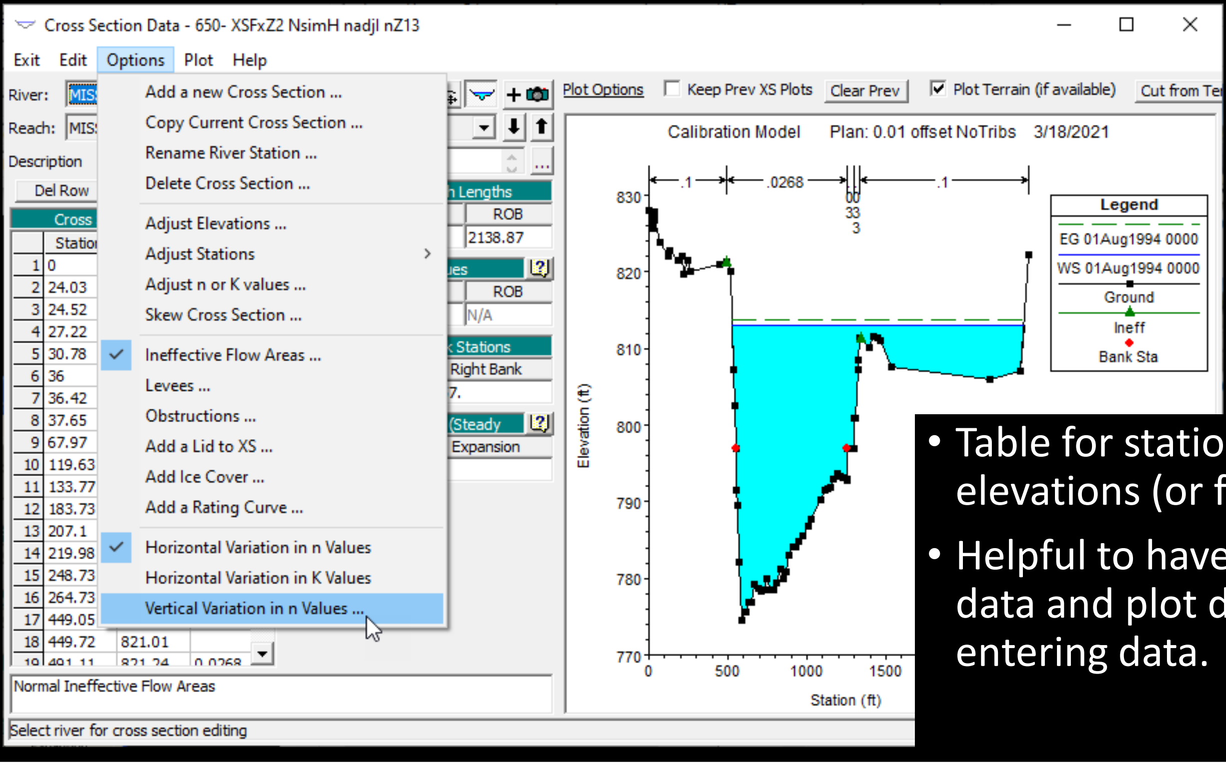
Normal Ineffective Flow Areas

Select river for cross section editing

- By default, HEC-RAS requires 3 n values, one for the main channel and one for each overbank.
- But n values can be further subdivided by selected horizontal variation in n values.

Horizontal Variation in n Values. This option allows the user to enter more than three Manning's n values for the current cross section. When this option is selected, an additional column for n values is added to the cross section coordinates table as shown in the Figure. A Manning's n value must be placed in the first row of the table. This n value is good for all cross section stations until a new n value shows up in the table. The user does not have to enter an n value for every station, only at the locations where the n value is changing.

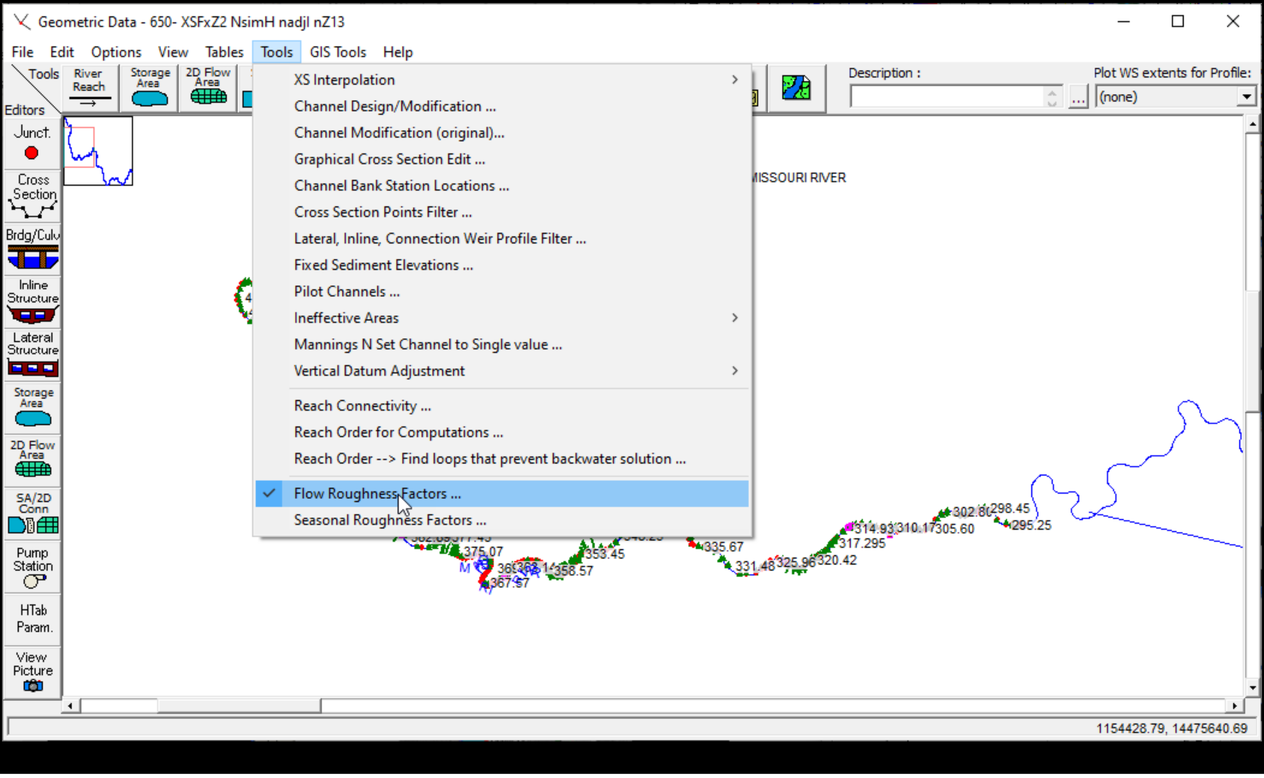
Vertical Variation of Manning’s n



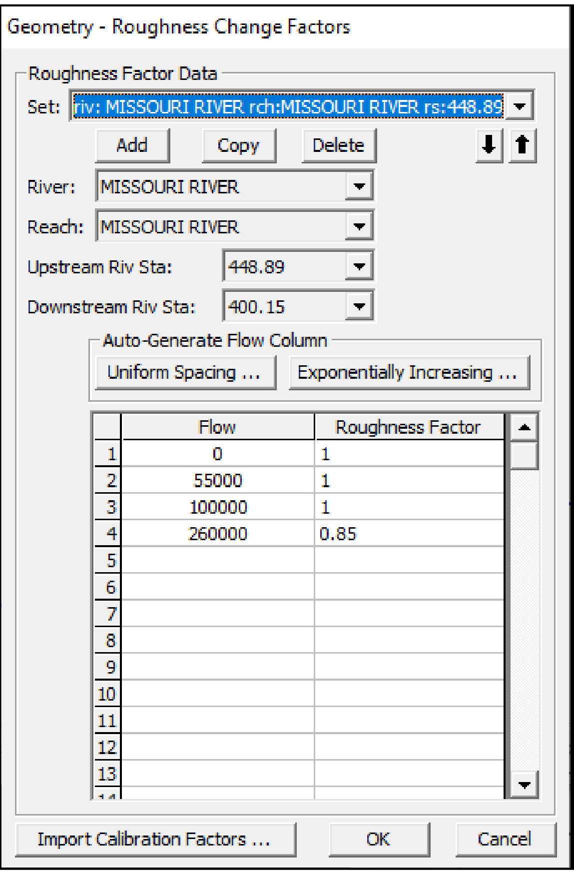
	Statio		
1	0		
2	24.03		
3	24.52		
4	27.22		
5	30.78		
6	36		
7	36.42		
8	37.65		
9	67.97		
10	119.63		
11	133.77		
12	183.73		
13	207.1		
14	219.98		
15	248.73		
16	264.73		
17	449.05		
18	449.72	821.01	
19	491.11	821.24	0.0268

- Table for stations and elevations (or flow)
- Helpful to have Cross section data and plot displayed while entering data.

Flow Roughness Factors



Flow roughness can be a function of flow. Turbulence losses resulting from roughness elements at low flows can result in higher roughness coefficients than in a bank flow flood event. RAS allows you to specify a multiplier that is applied to the calibrate n values for different flow magnitudes.



Geometry - Roughness Change Factors

Roughness Factor Data

Set: riv: MISSOURI RIVER rch:MISSOURI RIVER rs:448.89

River: MISSOURI RIVER

Reach: MISSOURI RIVER

Upstream Riv Sta: 448.89

Downstream Riv Sta: 400.15

	Flow	Roughness Factor
1	0	1
2	55000	1
3	100000	1
4	260000	0.85
5		
6		
7		
8		
9		
10		
11		
12		
13		

Flow Roughness Factors

- Modified in version 4 to include variable flow increments and an exponentially increasing increments.

In this example, the model was calibrated at 25,000 cfs and therefore has a factor of 1 at that flow (i.e. the n values determined in the calibrated model are used). However, for lower flows larger roughness coefficient are used so a multiplier >1 is applied to the Manning's n values. At higher flows the effects bed roughness effects are reduced and a multiplier <1 are applied to the 'n' values.

Selected References

Arcement, J. A., and Schneider, V. R., AGuide for Selecting Manning=s Roughness Coefficients for Natural Channels and Flood Plains@, U.S.G.S. *Water-Supply Paper 2339*, 1989.

Barnes, H. H., Jr., 1967, ARoughness Characteristics Of Natural Channels@; *U.S.G.S. Water-Supply Paper 1849*, 1967.

Chow, V.T., *Open-Channel Hydraulics*, McGraw-Hill, 1959.

Fasken, G.B., *Guide for Selecting Roughness Coefficient An@ Values for Channels*, Soil Conservation Service (currently the Natural Resource Conservation Service), 1963.

French, R.H., *Open-Channel Hydraulics*, McGraw-Hill, 1985.

Hicks, D.M., and Mason, P.D., Roughness *Characteristics of New Zealand River*s, New Zealand Water Resources Survey, 1991.

Hydrologic Engineering Center, *HEC-RAS Hydraulic Reference Manual*, 1998.

Jarrett, R.D., A Hydraulics of High-Gradient Streams@, *Journal of the Hydraulics Division*, ASCE, Vol 110, No. 11, Nov, 1984.

U.S. Army Corps of Engineers, *Hydraulic Design of Flood Control Channels*, EM 1110-2-1601, 1991 (with Change 1, June 1994).