Water Surface Profile Calculations with HEC-RAS

Alex Sanchez

USACE, Institute for Water Resources, Hydrologic Engineering Center

US Army Corps of Engineers®

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Overview

- Basic Open Channel Flow Concepts
- Energy Principles
- Cross Section Subdivision for Conveyance
- Computational Procedure
- Critical Depth Determination
- Momentum Equation
- Cross Section Spacing
- 1D Model Limitations

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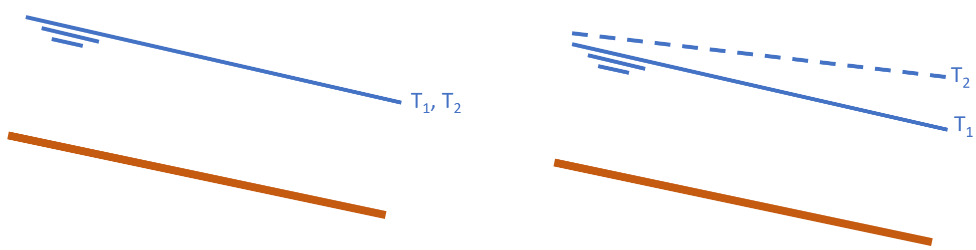
Classifications of Open Channel Flow

Steady **Unsteady**



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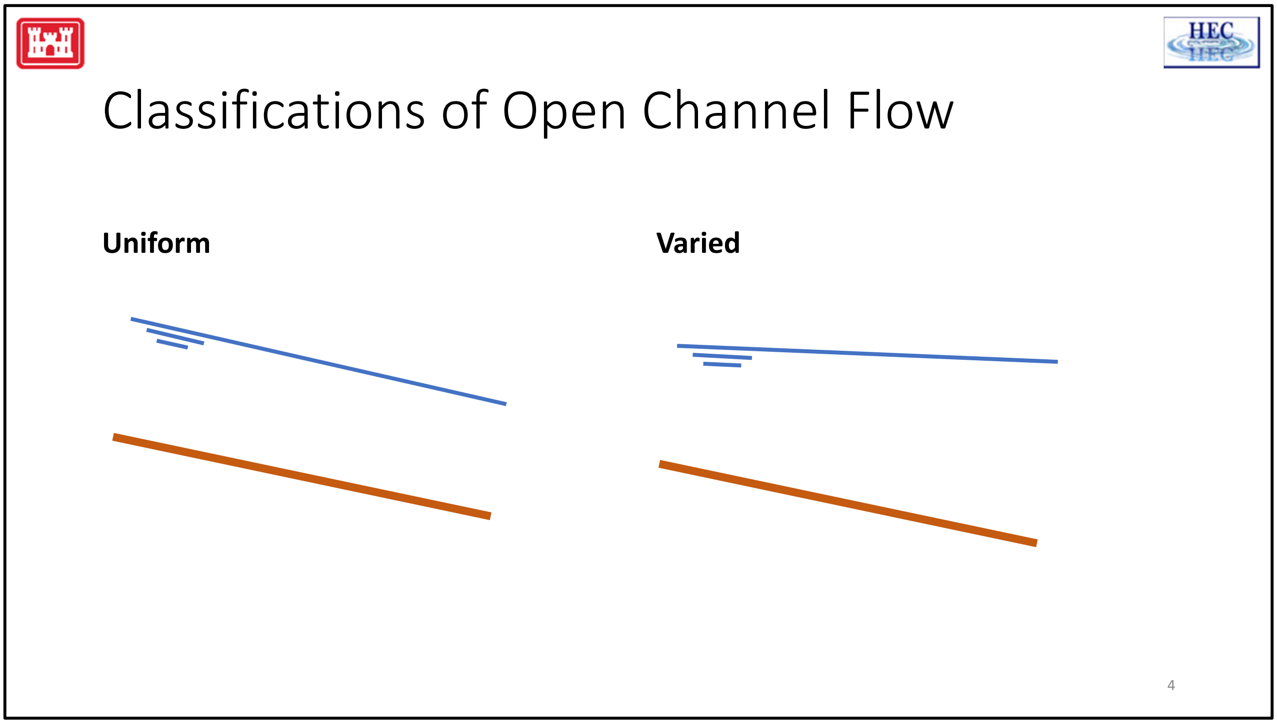
Steady Flow: Depth and velocity at a given location do not vary with time.

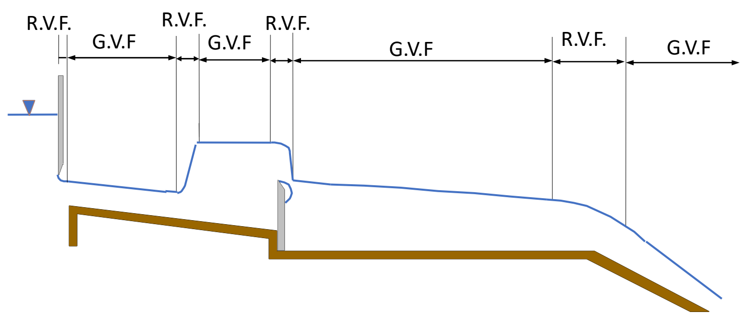
Unsteady Flow: Depth and velocity at a given location vary with time.



Uniform Flow: Depth and velocity are constant with distance along the channel.

Varied Flow: Depth and velocity vary with distance along the channel.

Gradually and Rapidly Varied Flow



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Gradually Varied Flow: Depth changes gradually over a long distance.

Rapidly Varied Flow: Depth changes abruptly over a short distance.

Subcritical vs. Supercritical

$$F_r = \frac{V}{\sqrt{gD}}$$

$F_r < 1$ *The flow is Subcritical*
$F_r = 1$ *The flow is at Critical depth*
$F_r > 1$ *The flow is Supercritical*

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The effect of gravity upon the state of flow is represented by a ratio of inertial forces to gravity forces (Chow, 1959). This ratio is called the Froude Number. The above equation for Froude number only applies where a uniform velocity distribution can be assumed (i.e. main channel only, not including overbank flow).

F_r	= Froude Number
V	= Mean velocity (Q/A)
g	= Gravitational acceleration
D	= Hydraulic depth (A/T)

Velocity Distribution

- Factors influencing velocity distribution:
  - Shape of the cross section
  - Roughness of the boundaries
  - Presence of bends
  - Contractions and expansions
  - Flow obstructions, such as bridge piers, etc…

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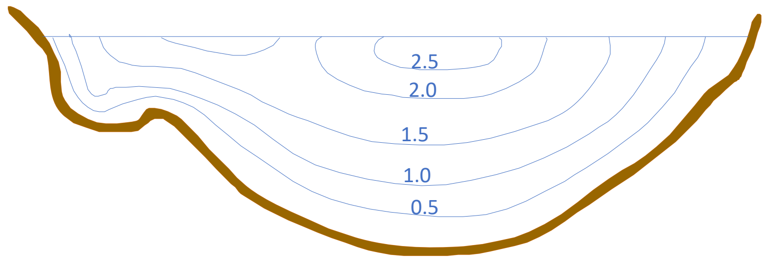
Example 2D Velocity Distribution



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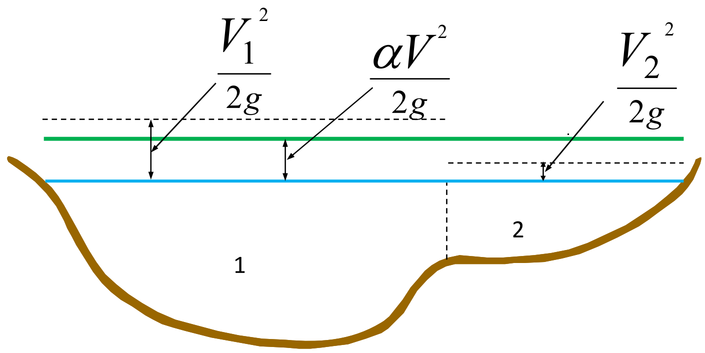
Evaluation of Kinetic Energy Head for a Cross Section (assume constant WS)



V_1= mean velocity for subarea 1
V_2= mean velocity for subarea 2

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Mean Kinetic Energy Head

$$\alpha \frac{\overline{V}^2}{2g} = \frac{Q_1 \frac{V_1^2}{2g} + Q_2 \frac{V_2^2}{2g}}{Q_1 + Q_2} \qquad \alpha = \frac{2g\left[Q_1 \frac{V_1^2}{2g} + Q_2 \frac{V_2^2}{2g}\right]}{\overline{V}^2 (Q_1 + Q_2)}$$

$$\alpha = \frac{Q_1 V_1^2 + Q_2 V_2^2}{\overline{V}^2 (Q_1 + Q_2)}$$

In General:

$$\alpha = \frac{\left[Q_1 V_1^2 + Q_2 V_2^2 + \ldots + Q_N V_N^2\right]}{\overline{V}^2 Q_T}$$

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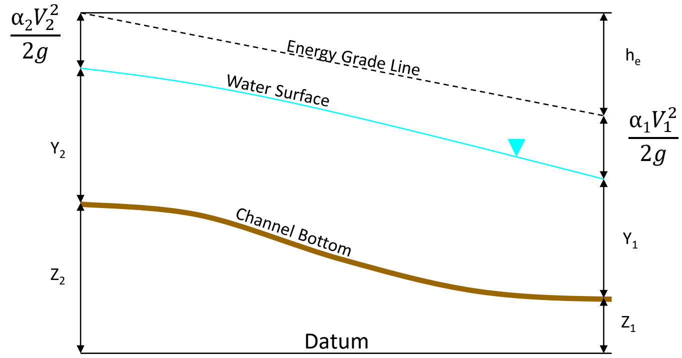
Energy Principles



$$Z_2 + Y_2 + \frac{\alpha_2 {V_2}^2}{2g} = Z_1 + Y_1 + \frac{\alpha_1 {V_1}^2}{2g} + h_e$$

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Water surface profiles are computed from one cross section to the next by solving the Energy equation with an iterative procedure called the standard step method. The Energy equation is written above.

where: Y_1 Y_2 = depth of water at cross sections

Z_1, Z_2 = elevation of the main channel inverts

V_1, V_2 = average velocities (total discharge/ total flow area)

α_1, α_2 = velocity weighting coefficients

g = gravitational acceleration

h_e = energy head loss

Energy Losses

- Energy Losses:

$$h_e = L\overline{S}_f + C\left|\frac{\alpha_2 V_2^2}{2g} - \frac{\alpha_1 V_1^2}{2g}\right|$$

Friction losses + Contraction and Expansion

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The energy head loss (h_e) between two cross sections is comprised of friction losses and contraction or expansion losses. The equation for the energy head loss is shown above.

Friction losses are based on multiplying a flow weighted reach length by a representative friction slope for the reach.

Contraction and expansion losses are based on a user entered contraction or expansion loss coefficient times the absolute value of the change in velocity head. The program assumes that a contraction is occuring whenever the velocity head downstream is greater that upstream. Likewise, when the velocity head upstream is greater than the velocity head downstream, the program assumes an expansion is occurring.

Flow Weighted Reach Length

$$L = \frac{L_{lob}Q_{lob} + L_{ch}Q_{ch} + L_{lob}Q_{rob}}{Q_{lob} + Q_{ch} + Q_{rob}}$$

Where: L_{lob}, L_{ch}, L_{rob} = cross section reach lengths specified for flow in the left overbank, main channel, and right overbank, respectively.

Q_{lob}, Q_{ch}, Q_{rob} = arithmetic average of the flows between sections for the left overbank, main channel, and right overbank, respectively.

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Cross Section Subdivision for Conveyance Calculations

- What is Conveyance?
  - $Q = (1.486/n)\ A\ R^{2/3}\ Sf^{1/2}$ (Manning's Equation)
  - $Q = K\ S_f^{1/2}$
  - $K = (1.486/n)\ A\ R^{2/3}$ (Conveyance)
  - $R = A/P$

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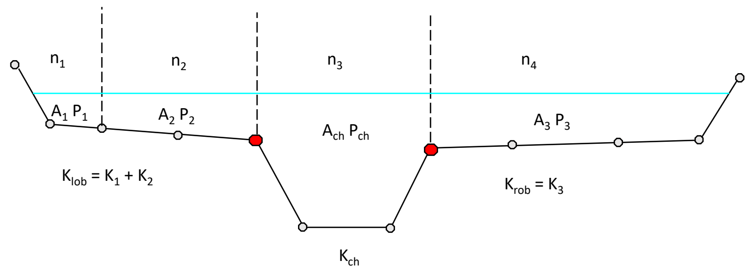
The determination of total conveyance for a cross section requires that flow be subdivided into units for which the velocity is uniformly distributed. Conveyance is based on the Manning equation as shown above. Conveyance is used to distribute flow within a cross section.

Q = Flow rate (cfs)
K = Conveyance (cfs)
n = Manning's roughness coefficient
A = Flow Area (ft^2)
R = Hydraulic Radius
P = Wetted Perimeter

Conveyance Calculations
HEC-RAS Default Method



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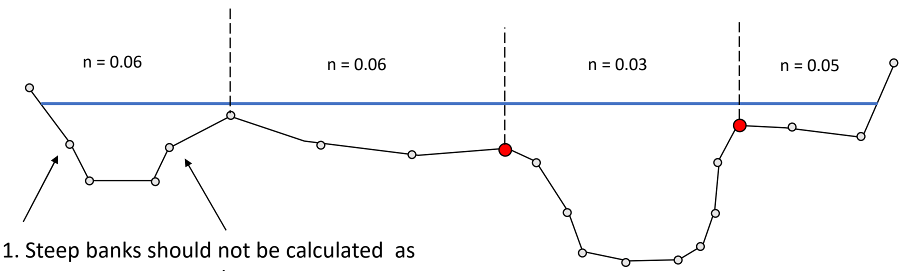
The default approach used in HEC-RAS for calculating conveyance is to subdivide flow in the overbank areas using the input cross section n-value break points (locations where n-values change) as the basis for subdivision. Conveyance is calculated within each subdivision from the Manning equation.

The program sums up all the incremental conveyances in the overbanks to obtain a conveyance for the left overbank and the right overbank. The main channel conveyance is normally computed as a single conveyance element. The total conveyance for the cross section is obtained by summing the three subdivision conveyances (left, channel, and right).

Conveyance Calculations Based on n-value Distribution



1. Steep banks should not be calculated as separate conveyance elements

2. Break up the overbanks into natural conveyance areas by using breaks in Manning's n values, even if the n values are the same.

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The two methods for computing conveyance will produce different answers whenever portions of the overbanks have ground sections with significant vertical slopes. In general, the HEC-RAS default approach will provide a lower total conveyance for the same water surface elevation.

Friction Loss Evaluation

- Friction loss is evaluated in HEC-RAS with the following equation:

$$h_f = L S_f$$

- The energy slope at each cross section is computed from Manning's equation:

$$S_f = \left(\frac{Q}{K}\right)^2$$

- Average Friction slope is computed as:

$$S_f = \left(\frac{Q_1 + Q_2}{K_1 + K_2}\right)^2$$

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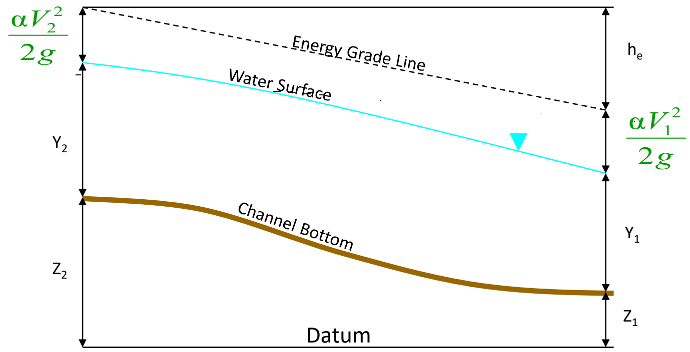
Computational Procedure



$$Z_2 + Y_2 + \frac{\alpha V_2^2}{2g} = Z_1 + Y_1 + \frac{\alpha V_1^2}{2g} + h_e$$

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The following is the computational procedure for computing water surface profiles with the energy equation:

1. Assume a water surface elevation at the upstream cross section.

2. Determine corresponding conveyance and velocity head.

3. Compute the friction slope for the reach and then solve for the energy losses (friction & contraction\expansion).

4. Solve the energy equation for the upstream water surface.

5. Compare the computed water surface with the assumed. If the difference is less than 0.01 feet, it is balanced. If the difference is greater, make an improved guess and repeat steps 2-5.

Computational Procedure
Water Surface Guess

- 1. The first guess of the water surface: downstream depth projected onto upstream cross section.
- 2. Second guess:
  - WSnew = WSassumed + 0.7 (WScomputed - WSassumed)
- 3. Subsequent Trials: Secant projection method
- 4. If secant method fails:
  - WSnew = (WSassumed + WScomputed) / 2

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The criterion used to assume water surface elevations in the iterative procedure varies from trial to trial.

1. The first trial water surface is based on projecting the previous cross section's water depth onto the current cross section.

2. The second trial water surface elevation is set to the assumed water surface elevation plus 70% of the error from the first trial (computed W.S. - assumed W.S.). In other words, W.S. new = W.S. assumed + 0.70 * (W.S. computed - W.S. assumed).

3. The third and subsequent trials are generally based on a "Secant" method of projecting the rate of change of the difference between computed and assumed elevations for the previous two trials.

4. If the secant method fails, the program then switches to a bruit force method of halving the assumed and computed guesses.

Computational Procedure
Tolerances and trials

- Default tolerances and trial settings:
  - Water surface calculation tolerance = 0.01 ft (0.0001 - 0.1)
  - Maximum difference tolerance = 0.3 ft (0.1 - 1.0)
  - Maximum number of iterations = 20 trials (3 - 40)
- What happens if it does not converge after 20 trials?
  - If 0.01 < minimum error < 0.3 then use water surface with minimum error.
  - If minimum error >= 0.3 then default to critical depth.
- Note: minimum error is the error associated with the best guess of the water surface during the 20 trials.

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The program is constrained by a *maximum number of iterations* (the default is 20) for balancing the water surface. While the program is iterating, it keeps track of the water surface that produces the minimum amount of error between the assumed and computed values. This water surface is called the *minimum error water surface*. If the maximum number of iterations is reached before a balanced water surface is achieved, the program will then calculate critical depth (if this has not already been done). The program then checks to see if the error associated with the *minimum error water surface* is within a predefined tolerance (the default is 0.3 ft or 0.1 m). If the minimum error water surface has an associated error less than the predefined tolerance, and this water surface is on the correct side of critical depth, then the program will use this water surface as the final answer and set a warning message that it has done so. If the minimum error water surface has an associated error that is greater than the predefined tolerance, or it is on the wrong side of critical depth, the program will use critical depth as the final answer for the cross section and set a warning message that it has done so. The rationale for using the minimum error water surface is that it is probably a better answer than critical depth.

Critical Depth Determination

- Critical Depth will be computed for a cross section if any of the following conditions are satisfied:
  - Program could not balance the energy equation within specified tolerance and number of trials.
  - Calculated water surface is close to critical depth.
  - Supercritical flow regime is being calculated.
  - Cross section is an external boundary section.
  - User requested critical depth as an output option.

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Critical Depth Determination

- Critical depth is found by computing minimum specific energy.
- HEC-RAS has two methods for computing critical depth.
  - Parabolic Method - built for speed
  - Slicing Method - built for accuracy

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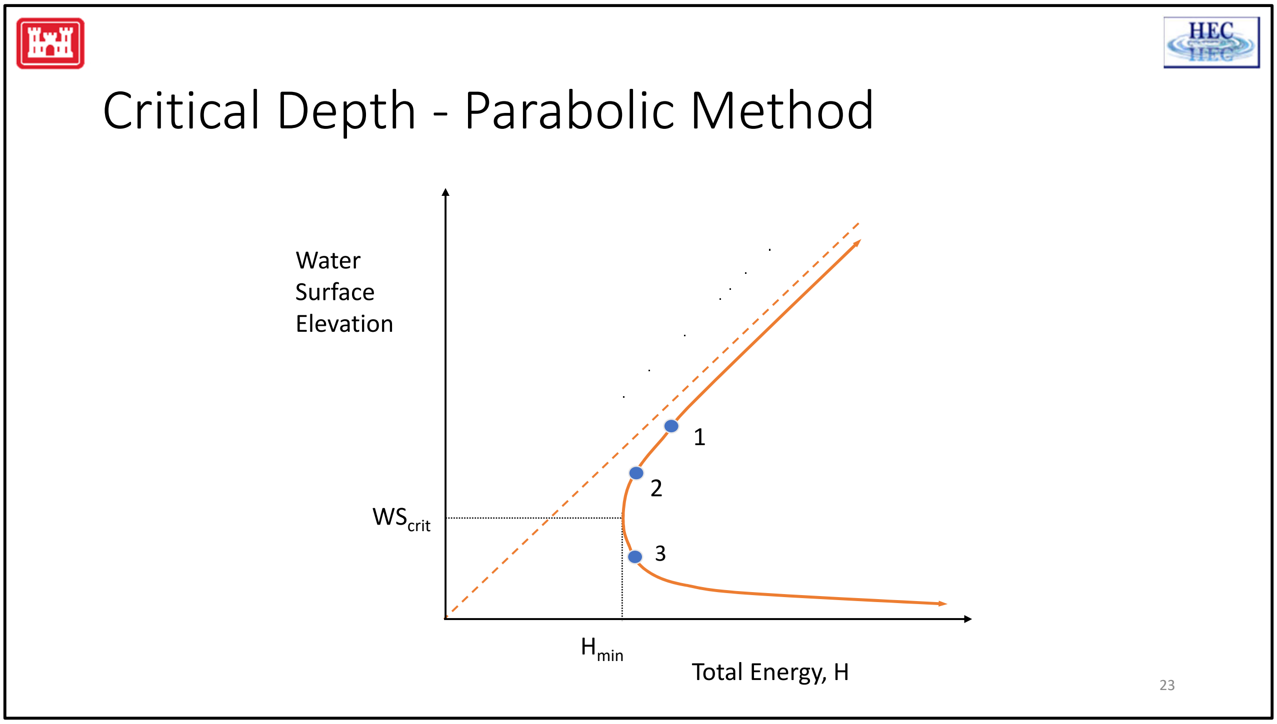
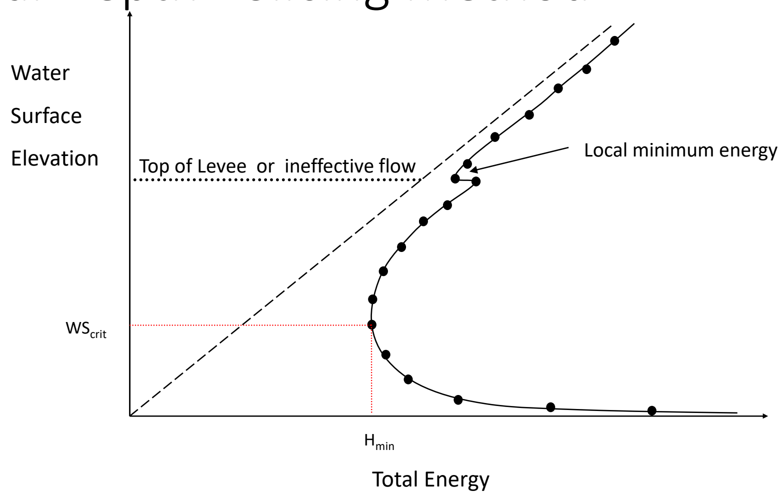
The critical water surface elevation is the elevation for which the total energy head is a minimum (i.e., minimum specific energy for that cross section for the given flow). The critical elevation is determined with an iterative procedure whereby values of water surface are assumed and corresponding values of energy are determined until a minimum value for energy is reached.



The "parabolic" method involves determining values of total energy for three different water surfaces that are spaced at equal intervals. The water surface corresponding to the minimum value for H, defined by a parabola passing through the three points on the H versus WS plane, is used as the basis for the next assumption of a value for WS. It is presumed that critical depth has been obtained when there is less than a 0.01 ft. (0.003 m) change in water depth from one iteration to the next and provided the energy head has not either decreased or increased by more than .01 feet (0.003 m).

Critical Depth - Slicing Method

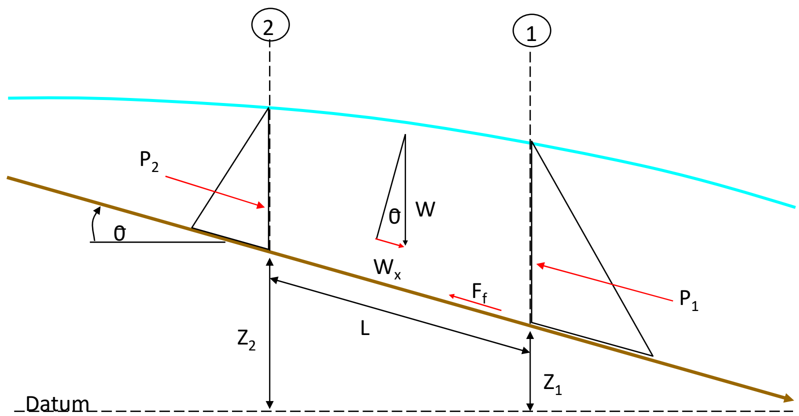


The “secant” method first creates a table of water surface versus energy by slicing the cross section into 30 intervals. The program then searches this table for the location of local minimums. When a point in the table is encountered such that the energy for the water surface immediately above and immediately below are greater than the energy for the given water surface, then the general location of a local minimum has been found. The program will then search for the local minimum by using the secant slope projection method. The program will iterate for the local minimum either thirty times or until the critical depth has been bounded by the critical error tolerance. After the local minimum has been determined more precisely, the program will continue searching the table to see if there are any other local minimums.

The program can locate up to three local minimums in the energy curve. If more than one local minimum is found, the program sets critical depth equal to the one with the minimum energy. If this local minimum is due to a break in the energy curve caused by overtopping a levee or an ineffective flow area, then the program will select the next lowest minimum on the energy curve. If all of the local minimums are occurring at breaks in the energy curve (caused by levees and ineffective flow areas), then the program will set critical depth to the one with the lowest energy. If no local minimums are found, then the program will use the water surface elevation with the least energy. If the critical depth that is found is at the top of the cross section, then this is probably not a real critical depth. Therefore, the program will double the height of the cross section and try again.

Momentum Equation

$$\sum F_x = m\,a$$



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The energy equation is derived for gradually varied flow situations. There are several instances when the transition from subcritical to supercritical and supercritical to subcritical flow can occur. These include significant changes in channel slope, bridge constrictions, drop structures and weirs, and stream junctions. In some of these instances empirical equations can be used (such as at drop structures and weirs), while at others it is necessary to apply the momentum equation in order to obtain a better answer.

The momentum equation is derived from Newton's second law of motion.

Momentum Equation

$$P_2 - P_1 + W_x - F_f = Q \, p \, \Delta V_x$$

Where: P = Hydrostatic Pressure

W_x = Force due to weight of water in X direction

F_f = Force due to external friction from 2 to 1

Q = Discharge

p = Density of water

ΔV_x = Change in velocity from 2 to 1 in X direction

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Applying Newton's second law of motion to a body of water enclosed by two cross sections at locations 1 and 2, the expression for the change in momentum over a unit time can be written as shown above.

Momentum Equation - Forces

Pressure: $P = \gamma A \overline{Y}$

Weight: $W_x = W \sin\theta$

$$W_x = \gamma\left(\frac{A_1 + A_2}{2}\right) L\ S_0$$

Friction: $F_f = \tau\,\overline{p}\,L$ Where: $\tau = \gamma\,\overline{R}\,\overline{S_f}$

$$F_f = \gamma\left(\frac{A_1 + A_2}{2}\right)\overline{S_f}\ L$$

Mass x acceleration: $ma = \frac{Q\gamma}{g}(\beta_1 V_1 - \beta_2 V_2)$

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The assumption of a hydrostatic pressure distribution is only valid for slopes less than 1:10. The cos θ for a slope of 1:10 (approximately 6 degrees) is equal to 0.995. Because the slope of ordinary channels is far less than 1:10, the cos θ correction for depth can be set equal to 1.0 (Chow, 1959).

The weight force is calculated as the unit weight of water times the volume. To get the force along the channel bed it is multiplied by the sin θ , which is the bed slope S_0. Estimating the average bed slope for natural channels is very difficult. A bad estimate of this slope can produce wild estimates of the weight force. HEC-RAS computes a hydraulic depth at each cross section and then subtracts that from the water surface to get a mean bed elevation. The mean bed elevations are then used in estimating the average bed slope between two sections.

Friction losses in the momentum equation are based on boundary friction only. Friction losses are computed as shear stress times the average wetted perimeter, times the flow weighted length between the cross sections. The shear equation is for local shear, it is similar to using Manning's equation for local friction is used in the energy equation.

The mass times the acceleration is computed as the discharge times the density of water, times the change in velocity.

Momentum Equation

- The momentum equation and principles are used in HEC-RAS for the following:
  - Mixed flow regime calculations - locating hydraulic jumps
  - Low flow bridge hydraulics
  - Stream junction analysis - optional method

Cross Section Spacing

- Cross sections should be placed at representative locations to describe the changes in geometry.
- Additional cross sections should be added at locations where changes occur in discharge, slope, velocity, and roughness.
- Cross sections must also be added at levees, bridges, culverts, and other structures.

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Cross sections are required at representative locations throughout a stream reach and at locations where changes occur in discharge, slope, shape, or roughness, at locations where levees begin or end and at bridges or control structures such as weirs. Where abrupt changes occur, several cross sections should be used to describe the change regardless of the distance. Cross section spacing is also a function of stream size, slope, and the uniformity of cross section shape. In general, large uniform rivers of flat slope normally require the fewest number of cross sections per mile. The purpose of the study also affects spacing of cross sections. For instance, navigation studies on large relatively flat streams may require closely spaced (e.g., 200 feet) cross sections to analyze the effect of local conditions on low flow depths, whereas cross sections for sedimentation studies, to determine deposition in reservoirs, may be spaced at intervals on the order of miles.

Cross Section Spacing - Slope

- Bed slope plays an important role in cross section spacing.
  - Steeper slopes require more cross sections
  - Streams flowing under supercritical flow may require cross sections on the order of 50 feet or less.
  - Larger uniform rivers with flat slopes may only require cross sections on the order of 1000 ft or less.

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Estimation of Cross Section Spacing

- Samuel's Equation:

$$\Delta x \leq \frac{0.15D}{S_0}$$

Where: Δx is the cross section spacing distance (ft)
D is the bankfull depth (ft)
S_0 is the bed slope

- Simple rule: Cross sections should gradually transition in size and shape!

Cross Section Spacing - How do you know if you have enough XS:

- Use the HEC-RAS cross section interpolation.
- Make a new plan and run the model.
- Compare the before and after.

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1. Use the HEC-RAS cross section interpolation option to interpolate cross sections. Make sure you set the maximum distance for interpolated cross sections to an interval that will give you at least a few cross sections between all of your existing cross sections.

2. Save the geometry to a new file. Make a new plan and run the model.

3. Compare the results for the before and after interpolation models. If there is no significant difference in the results, then your original model probably had enough cross sections. If there is a significant difference, then in the areas where there are differences you should either get more surveyed sections or leave the interpolation turned on.

4. Be careful using interpolated sections in your final model. Make sure that the interpolated sections are reasonable with what is actually out there. Remember, you can modify the interpolated sections just like any other cross section. Look at a topographical map and make adjustments where necessary.

One Dimensional Model Limitations

- Limitations of the 1D energy Equation:
  - Flow is steady with respect to time.
  - Flow is gradually varied with respect to distance.
  - Dominant forces are in the X-direction.
  - Channel slopes should be less than 1:10.
- Steady vs. Unsteady
- 1 dimensional flow vs. 2 and 3 dimensional

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1. Flow is assumed to be steady because time-dependent terms are not included in the energy equation

2. Flow is assumed to be gradually varied because the energy equation is based on the premise that a hydrostatic pressure distribution exists at each cross section. At locations where the flow is rapidly varied, the program switches to the momentum equation or other empirical equations.

3. Flow is assumed to be one-dimensional because the energy equation is based on the premise that the total energy head is the same for all points in a cross section.

4. Small channel slopes are assumed because the pressure head is represented by the water depth measured vertically, instead of perpendicular to the channel bed. In other words, the cos of theta is assumed to be 1.0, which is a good approximation as long as the slope is less than 1:10. Most natural streams are less than a 1:10 slope, so this should not be to much of a problem.

5. The HEC-RAS software should only be applied to stream flows that are predominantly one dimensional.

Questions?

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