Bridge Modeling Approach

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Objective

- Describe the Hydraulic computations through bridges
  - Low Flow Hydraulic Methods
  - High Flow Hydraulic Methods
- Learn how to selecting the Appropriate Modeling Approach

References

- FHWA, 1978. Hydraulics of Bridge Waterways, HDS No. 1, U.S. Department of Transportation, Second Edition , March 1978.
- HEC, 2016. HEC-RAS Hydraulic Reference Manual, Chapter 5, Hydrologic Engineering Center, February 2016.

Hydraulic Computations Through the Bridge

- The bridge routines in HEC-RAS have the ability to model:
  - Low Flow
    - Class A – Subcritical flow
    - Class B – Mixed flow regime
    - Class C – Supercritical flow
  - Low flow and weir flow
  - Pressure flow (orifice or sluice gate types)
  - Pressure and weir flow
  - Highly submerged flows (energy equation)

Low-Flow computations

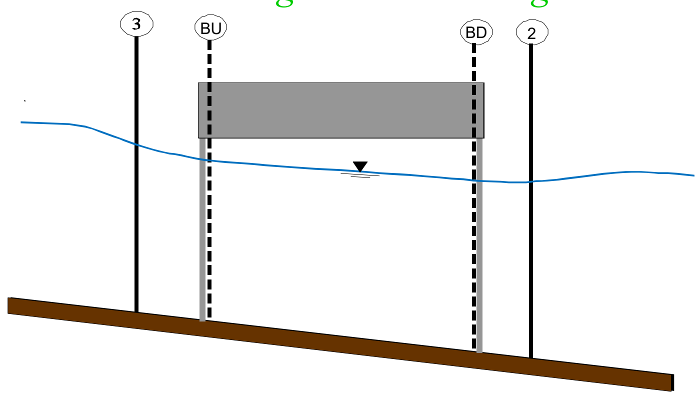
- Momentum is used to determine Class A, B or C
- Class A - If the momentum downstream is greater than the critical depth momentum inside the bridge, the flow is considered subcritical.
- Class B - If the momentum downstream is less than the momentum at critical depth inside the bridge, then it is assumed that the flow will pass through critical depth and a hydraulic jump will occur downstream.
- Class C - The profile is considered completely supercritical through the bridge.

Class A Low-Flow Methods

- All methods use Standard Step calculations for the transition sections (1 to 2 and 3 to 4).
- Four methods are available to compute bridge hydraulic losses (Between Sections 2 to 3):
  - Energy Equation (standard step method)
  - Momentum Balance
  - Yarnell's Equation
  - WSPRO low-flow model

Energy Method (standard step)

$$Y_3 + Z_3 + \frac{\alpha V_3^2}{2g} = Y_2 + Z_2 + \frac{\alpha V_2^2}{2g} + h_e$$



The energy based method treats a bridge in the same manner as a natural river cross section, except the area of the bridge below the water surface is subtracted from the total area, and the wetted perimeter is increased where the water is in contact with the bridge structure. The program performs an energy balance by stepping from cross section 2 to cross section BD. Then an energy balance is performed through the bridge, and finally out of the bridge to section 3.

Energy Method (standard step)

- Friction losses are computed as a length times an average friction slope. Wetted perimeter and blocked areas from piers, abutments and deck decrease conveyance in the bridge.
- Energy losses associated with the flow contracting and expanding are computed with an empirical coefficient times a change in velocity head
- Does not account for pier and abutment shape.

Momentum Balance Method

From 2 to BD:

$$A_{BD}\,\overline{Y}_{BD} + \frac{\beta_{BD}\,Q_{BD}^2}{g\,A_{BD}} = A_2\,\overline{Y}_2 + \frac{\beta_2\,Q_2^2}{g\,A_2} - A_{P_{BD}}\,\overline{Y}_{P_{BD}} + F_f - W_x$$

From BD to BU:

$$A_{BU}\,\overline{Y}_{BU} + \frac{\beta_{BU}\,Q_{BU}^2}{g\,A_{BU}} = A_{BD}\,\overline{Y}_{BD} + \frac{\beta_{BD}\,Q_{BD}^2}{g\,A_{BD}} + F_f - W_x$$

From BU to 3:

$$A_3\,\overline{Y}_3 + \frac{\beta_3\,Q_3^2}{g\,A_3} = A_{BU}\,\overline{Y}_{BU} + \frac{\beta_{BU}\,Q_{BU}^2}{g\,A_{BU}} + A_{P_{BU}}\,\overline{Y}_{P_{BU}} + \frac{1}{2}C_D\,\frac{A_{P_{BU}}\,Q_3^2}{g\,A_3^2} + F_f - W_x$$

Where: A_2, A_{BD} = Active flow area at section 2 and BD.

A_{pBD} = Obstructed area of the pier on the downstream side

Y_2, Y_{BD} = Vertical distance from water surface to center of gravity of flow area A_2 and A_{BD}

Y_{pBD} = Vertical distance from water surface to center of gravity of wettec pier area

β_2, β_{BD} = Velocity weighting coefficients for momentum

Q_2, Q_{BD} = Discharge

g = Gravitational acceleration

F_f = Force due to friction

W_x = Force due to weight of water in the direction of flow

Momentum Drag Coefficients

Typical drag coefficients for various pier shapes

Pier Shape	**Drag Coefficient C_D**
Circular pier	1.20
Elongated piers with semi-circular ends	1.33
Elliptical piers with 2:1 length to width	0.60
Elliptical piers with 4:1 length to width	0.32
Elliptical piers with 8:1 length to width	0.29
Square nose piers	2.00
Triangular nose with 30 degree angle	1.00
Triangular nose with 60 degree angle	1.39
Triangular nose with 90 degree angle	1.60
Triangular nose with 120 degree angle	1.72

Momentum Method

- Friction losses are external skin friction, computed as avg. wetted perimeter times the length between cross sections times the avg. shear stress.
- Weight force is computed as the avg. area times the length between cross sections times the average slope. Computing the average bed slope is difficult!
- Pier shape is considered based on drag force computed with an empirical drag coefficient.

Yarnell's Equation

$$H_{3-2} = 2K(K + 10\omega - 0.6)(\alpha + 15\alpha^4)\frac{V^2}{2g}$$

Where: H_{3-2} = The drop in water surface from section 3 to 2

K = Yarnell's pier shape coefficient

ω = Ratio of velocity head to depth at section 2

α = Obstructed area of the piers divided by the total unobstructed area

V_2 = Velocity at cross section 2

The Yarnell equation is an empirical equation that computes the change in water surface from just downstream of the bridge (section 2 of Figure 3) to just upstream of the bridge (section 3). The Yarnell equation only provides hydraulic information at cross-sections 2 and 3. No information is provided inside the bridge (sections BU and BD).

Yarnell's Pier Coefficient, K

Pier Shape	Yarnell K Coefficient
Semi-circular nose and tail	0.90
Twin-cylinder piers with connecting diaphragm	0.95
Twin-cylinder piers without diaphragm	1.05
90 degree triangular nose and tail	1.05
Square nose and tail	1.25
Ten pile trestle bent	2.50

Yarnell's Equation

- Based on 2600 lab experiments with varied pier shape, width, length, angle and flow rate.
- Experiments were run with rectangular and trapezoidal channels only.
- Method is sensitive to the pier size and shape, and the velocity of the flow.
- Not sensitive to the shape of the bridge opening, abutment types, or changes to cross-sections.
- Should be limited to uniform sections through the bridge and where piers are the primary obstruction.

FHWA WSPRO Method

Energy balance from section 1 to 4:

$$Z_4 + Y_4 + \frac{a_4 V_4^2}{2g} = Z_1 + Y_1 + \frac{a_1 V_1^2}{2g} + h_f + h_e$$

Where: Z = Bed invert elevations

Y = Water surface elevations

V = Average velocity

h_f = Friction losses

h_e = Expansion losses from section 1 to 2

The low-flow hydraulic computations of the Federal Highway Administration's (FHWA) WSPRO computer program has been adapted as an option for low flow hydraulics in HEC-RAS. The WSPRO methodology had to be modified slightly in order to fit into the HEC-RAS concept of cross-section locations.

The WSPRO method computes the water surface profile through a bridge by solving the energy equation. The method is an iterative solution performed from the exit cross section (1) to the approach cross section (4). The energy balance is performed in steps from the exit section (1) to the cross section just downstream of the bridge (2); from just downstream of the bridge (2) to inside of the bridge at the downstream end (BD); from inside of the bridge at the downstream end (BD) to inside of the bridge at the upstream end (BU); From inside of the bridge at the upstream end (BU) to just upstream of the bridge (3); and from just upstream of the bridge (3) to the approach section (4). A general energy balance equation from the exit section to the approach section is written in the slide above.

WSPRO - Continued

From Section 1 to 2:

$$h_{f_{1-2}} = \frac{BQ^2}{K_2 K_1}$$

$$h_e = \frac{Q^2}{2gA_1^2}\left[2\beta_1 - a_1 - 2\beta_2\left(\frac{A_1}{A_2}\right) + a_2\left(\frac{A_1}{A_2}\right)^2\right]$$

Losses from section 1 to section 2 are based on friction losses and an expansion loss. Friction losses are calculated using the geometric mean friction slope times the flow weighted distance between sections 1 and 2. B is the flow weighted distance between sections 1 and 2, and K_1 and K_2 are the total conveyance at sections 1 and 2 respectively.

The expansion loss from section 2 to section 1 is computed by and idealized expansion loss equation. Where α_1 and β_1 are energy and momentum correction factors for nonuniform flow. α_2 and β_2 are related to the bridge geometry and are defined as $\alpha_2 = 1/C^2$ and $\beta_2 = 1/C$. Where C is an empirical discharge coefficient for the bridge, which was originally developed as part of the Contracted Opening method by Kindswater, Carter, and Tracy (USGS, 1953), and subsequently modified by Matthai (USGS, 1968). The computation of the discharge coefficient, C, is explained in detail in appendix D of the HEC-RAS Hydraulic Reference Manual.

WSPRO - Continued

From section 2 to 3:

1. Losses are based on friction only.
2. Energy balance is done in 3 steps: from section 2 to BD, BD to BU, and BU to 3.
3. Friction losses are calculated using the geometric mean friction slope time the flow weighted length.

$$h_{f(BU-BD)} = \frac{L_B Q^2}{K_{BU} K_{BD}}$$

WSPRO - Continued

From section 3 to 4:

1. Energy losses are based on friction only.
2. The effective flow length (L_{av}) is computed as the average length of 20 equal conveyance stream tubes.

$$h_{f(3-4\_)} = \frac{L_{av} Q^2}{K_3 K_4}$$

The computation of the effective flow length by the stream tube method is explained in appendix D of the HEC-RAS Hydraulic Reference Manual.

Class B and C Low-flow Methods

- Class B Flow
  - Momentum - the default method. (With irregular cross-section data and rapidly changing water surface elevation, the estimate of bed slope can be erratic. Therefore, the weight component is automatically turned off for Class B flow.)
  - Energy – only if momentum fails (During Class B flow, a dramatic change in depth can occur with resulting large changes in velocity head. Contraction and Expansion energy losses may be overestimated with "traditional" contraction and expansion coefficients.)
- Class C Flow – both methods can be used

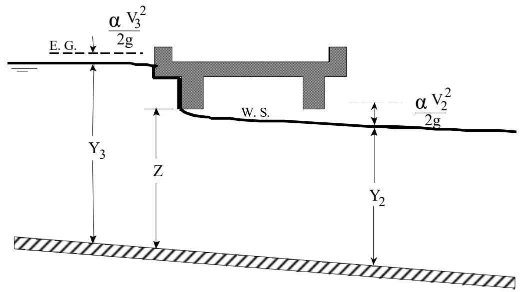
High Flow Bridge Methods

- Energy Method - The area of the deck is subtracted and additional wetted perimeter is added. The water surface elevation represents the hydraulic grade line.
  - This method does not account for the shape of the entrance or piers.
  - Conveyance is calculated treating the bridge as a cross section, including flow over the roadway.

High Flow Methods, continued

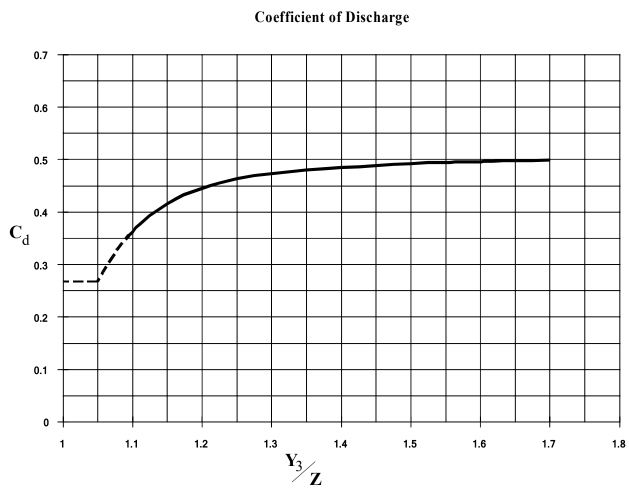
- Pressure and Weir Method - Treats the flow as two separate components.
- Flow through the opening is pressure flow.
  - Full submerged pressure when tailwater is above low chord. Input coefficient is for total loss.
  - Gate equation when tailwater is unsubmerged. Coefficient curve is programmed; however, it is not well defined from point of contact to 1.1 times depth.
- Flow over the roadway is weir flow, with tailwater submergence. (Energy method for high tailwater)

Sluice Gate Pressure Flow



$$Q = C_d A_{BU} \sqrt{2g}\left[Y_3 - \frac{Z}{2} + \frac{a_3 V_3^2}{2g}\right]^{1/2}$$

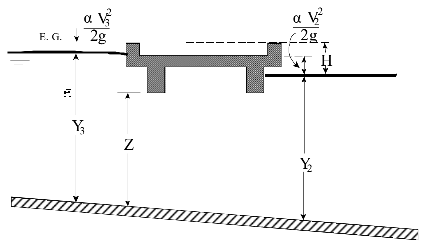
Cd For Sluice Gate Pressure Flow



The discharge coefficient C_d, can vary depending upon the depth of water upstream, ranging in values from 0.35 to 0.5 with a value of 0.5 commonly used, The user can enter a fixed value for this coefficient or the program will compute one based on the amount that the inlet is submerged, see the Figure above relating C_d to Y_3/Z .

As shown in the Figure, the limiting value of Y_3/Z is 1.1. There is a transition zone somewhere between Y_3/Z = 1.0 and 1.1 where free surface flow changes to orifice flow. The type of flow in this range is unpredictable, and the sluice gate pressure flow equation may not be applicable.

Orifice Pressure Flow

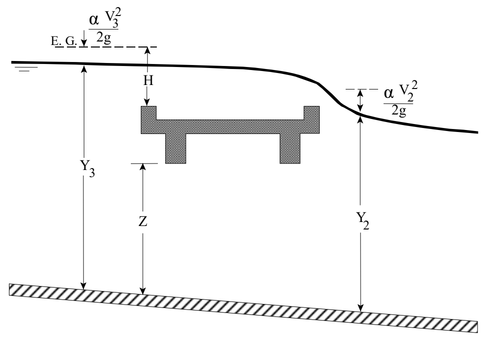


$$Q = CA\sqrt{2gH}$$

Typical values for the discharge coefficient C range from 0.7 to 0.9, with a value of 0.8 commonly used for most bridges. The user must enter a value for C whenever the pressure flow method is selected.

The program will begin checking for the possibility of pressure flow when the computed low flow energy grade line is above the maximum low chord elevation at the upstream side of the bridge. Once pressure flow is computed, the pressure flow answer is compared to the low flow answer, the higher of the two is used. The user has the option to tell the program to use the water surface, instead of energy, to trigger the pressure flow calculation.

Weir Flow Computations



$$Q = CLH^{3/2}$$

Flow over the bridge, and roadway approaching the bridge, can be calculated as weir flow. The approach velocity is included by using the energy grade line elevation in lieu of the upstream water surface elevation for computing the head, H.

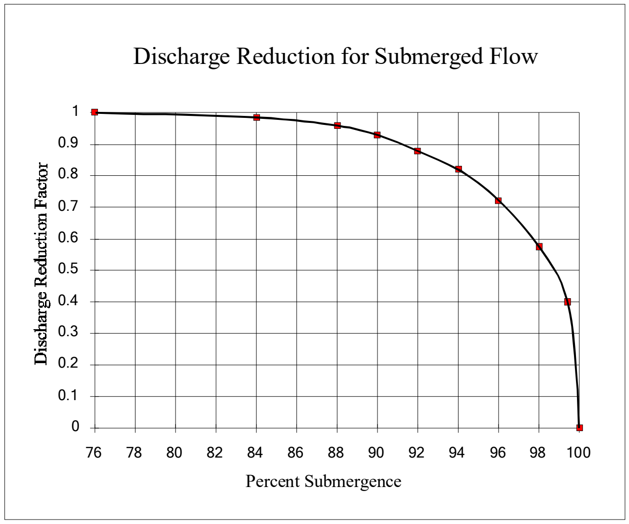
Under free-flow conditions (discharge independent of tailwater) the coefficient of discharge C, ranges from 2.5 to 3.1 (1.38 - 1.71 metric) for broad-crested weirs depending primarily upon the gross head on the crest (C increases with head). Increased resistance to flow caused by obstructions such as trash on bridge railings, curbs, etc. would decrease the value of C.

From King's Handbook (King, 1963), weir coefficients C are given for broad-crested weirs in with the value of C varying with measured head H and breadth of weir. Rectangular weirs with a breadth of 15 feet and a H of 1 foot or more, the given value is 2.63 (1.45 for metric). Trapezoidal shaped weirs generally have a higher coefficient, with values ranging from 2.7 to 3.08 (1.49 to 1.70 for metric).

Hydraulics of Bridge Waterways [Bradley, 1978] provides a curve of C versus the head on the roadway. The roadway section is shown as a trapezoid and the coefficient rapidly changes from 2.9 for a very small H to 3.03 for H = 0.6 feet, and the curve levels off at 3.05 (1.69 for metric).

With very little prototype data available, it seems the assumption of a rectangular weir for flow over the bridge deck (assuming the bridge can withstand the forces) and a coefficient of 2.6 (1.44 for metric) would be reasonable. If the weir flow is over the roadway approaches to the bridge, a value of 3.0 (1.66 for metric) would be consistent with available data. If weir flow occurs as a combination of bridge and roadway, an average coefficient (weighted by weir length) could be used.

Weir Flow Submergence



The program will automatically reduce the weir coefficient to account for submergence on the weir. Submergence corrections are based on a trapezoidal weir shape (default), or optionally an ogee spillway shape. The submergence correction for a trapezoidal weir shape, shown in the Figure above, is from "Hydraulics of Bridge Waterways" [Bradley, 1978].

When the weir becomes highly submerged the program will automatically switch to the energy equation (standard step backwater) to calculate the upstream water surface, instead of using the pressure and weir flow equations. The criterion can be set by the user, with a default maximum submergence of 0.98 (98 percent).

Combination Flow

- Low Flow and Weir Flow
  - Iterative solution for upstream energy
  - Can be done with any of the low flow methods except the momentum method

Selecting a Low-Flow Modeling Approach

- Where the bridge piers are a small obstruction to the flow, and friction losses are the predominate consideration, the energy-based methods should give good answers.

Selecting a Low-Flow Modeling Approach - Continued

- Where pier losses and friction losses are both predominant, the Momentum should be the most applicable. But the Energy and WSPRO methods can be used..

Selecting a Low-Flow Modeling Approach - Continued

- For bridges in which the piers are the dominant contributor to energy losses and the change in water surface, either the momentum method or the Yarnell equation would be most applicable.

For bridges in which the piers are the dominant contributor to energy losses and the change in water surface, either the momentum method or the Yarnell equation would be most applicable. However, the Yarnell equation is only applicable to Class A low flow.

Selecting a High Flow Bridge Method

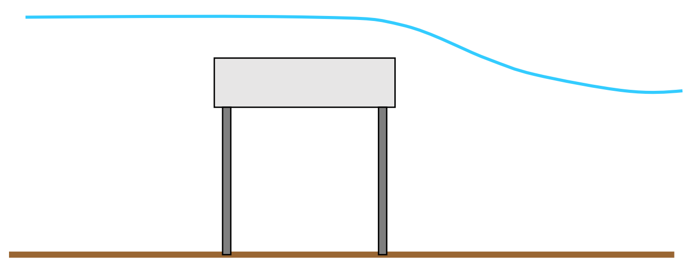
- When the bridge and the roadway approaches are a small obstruction to the flow, and the bridge opening is not acting like a pressurized orifice, the energy-based method should be used.

Selecting a High Flow Bridge Method - Continued

- When the bridge and road embankment are a large obstruction to the flow, and a backwater is created due to the constriction of the flow, the pressure and weir method should be used.

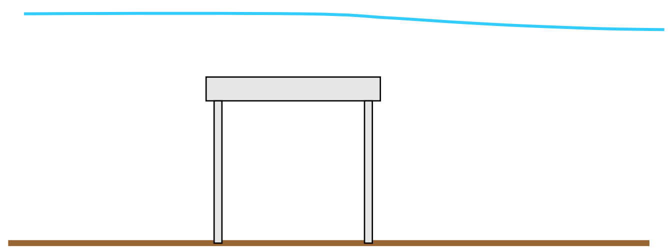
Selecting a High Flow Bridge Method - Continued

- When the bridge and\or road embankment is overtopped, and the water going over top of the bridge is not highly submerged by the downstream tailwater, the pressure and weir method should be used.



Selecting a High Flow Bridge Method - Continued

- When the bridge is highly submerged, and flow over the road is not acting like weir flow, the energy based method should be used.



Questions?

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