# Equation Selection: Diffusion Wave vs Shallow Water Equations

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# Objectives

- Overview of the Diffusion Wave and Shallow Water Equations
- Learn the positive and negative attributes of
  - Diffusion Wave Equations
  - Shallow Water Equations
- Understand the impacts through examples

# Hydraulic Equations

- **Shallow Water Equations**
  - **Mass Conservation** (Continuity)
  - **Momentum Equation**
    - Friction
    - Pressure gradient
    - Accelerations (local and advective)
    - Diffusion (optional)
    - Coriolis term (optional)
    - Wind Forces (optional)

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\boldsymbol{V}) = q$$

$$\frac{\partial \boldsymbol{V}}{\partial t} + \left(\boldsymbol{V} \cdot \nabla\right)\boldsymbol{V} + f_c \boldsymbol{k} \times \boldsymbol{V} = -g\nabla z_s + \frac{1}{h}\nabla \cdot \left(\boldsymbol{v}_t h \nabla \boldsymbol{V}\right) - \frac{\boldsymbol{\tau}_b}{\rho R} + \frac{\boldsymbol{\tau}_s}{\rho h}$$

- **Diffusion Wave Equation**
  - **Mass Conservation** (Continuity)
  - **Momentum Equation**
    - Friction
    - Pressure gradient

$$\frac{\partial h}{\partial t} = \nabla \cdot \left(\beta \nabla z_s\right) + q$$

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# Diffusion Wave Positive Attributes

- Flow is mainly driven by **gravity** and **friction**
  - Good for steep to moderate sloping streams (S > 2 ft/mi)
  - Hydrographs that rise and fall slowly
- **Very Stable Computationally**
  - Can handle larger time step Courant C > 2 (C = 5 max)
- Good for computing **rough global estimates**, such as flood extent
- Good for assessing **rough effects of dam breaks**
- Good for assessing **interior areas due to levee breeches**
- Good for **quick estimations before a SWE run**
  - Often used to get model up and running stable before use SWE

# Diffusion Wave Negative Attributes

- **Not as good for fast rising and falling flood waves** due to lack of acceleration terms (Dam break or flash floods)
- **Not good for sharp contractions and expansions**
  - Will generally under compute water surface upstream due to no contraction force
  - Will not accurately predict expansion zones and recirculation patterns
- **Can't handle tidal boundary conditions accurately**
  - No wave propagation up stream (This requires acceleration terms)
- **Not good for sharp bends** – can't predict any super elevation
- **Note good for predicting detailed velocity distributions** in channels or around objects.
- **Does not work well for mixed flow regimes and hydraulic jumps**

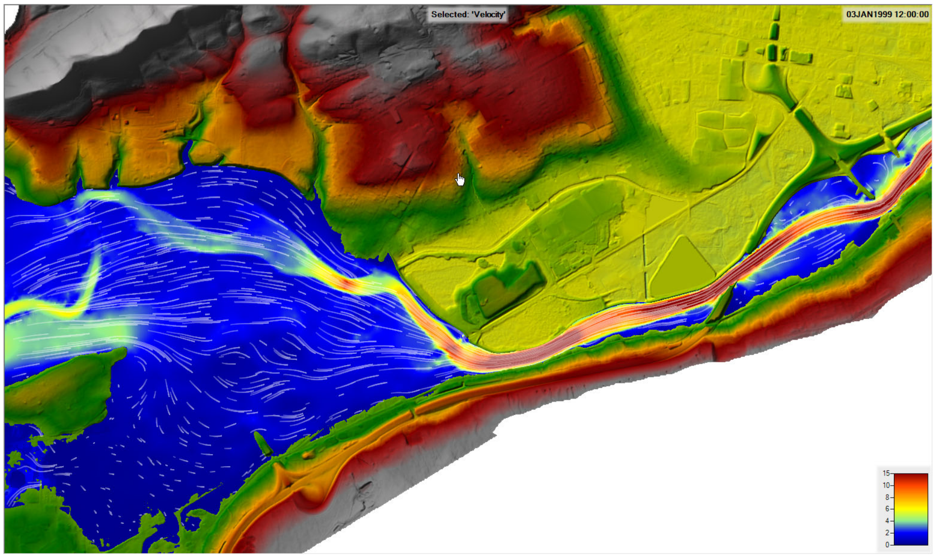
# Shallow Water Equations Applications

- **Highly Dynamic Flood Waves -** Rapidly rising and falling flood waves (dam break, flash floods, etc..)
- **Abrupt Contractions and Expansions -** flow with high velocities, as well as flow approaching structures on an angle.
- **Flat Sloping River Systems:** Slopes less than 2 ft/mile
- **Detailed Velocities and Water Surface Elevations:** (natural channels and around structurers)
- **Mixed Flow Regime**: sub to supercritical flow transitions, and hydraulic jumps (super to subcritical)
- **Tidal boundary conditions** (wave propagation upstream)
- **Super elevation around bends**
- **General Wave Propagation**: If the user needs to model wave propagation due to rapidly opening or closing of gated structures, or wave run-up on a wall or around an object
- **Simulations influenced by turbulence, wind, or Coriolis effects**
- **River Morphodynamics**

# Testing if Diffusion Wave is Appropriate?

1. Create two Plans: Diffusion Wave and Shallow Water
2. Run both
3. Compare the Water surface, velocities, and flow rates
4. Where differences are significant, means you should be using the SWE

# Sharp Contraction – Bald Eagle Creek, PA

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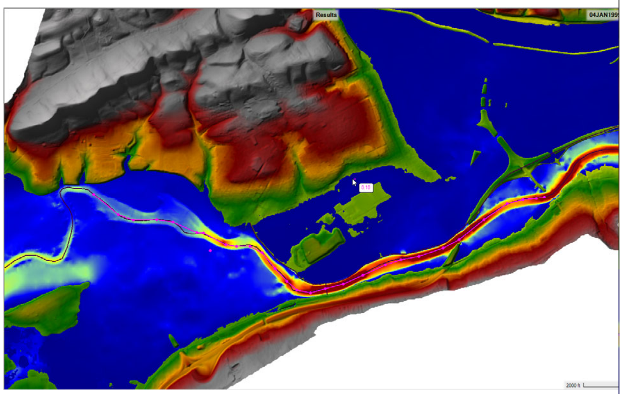
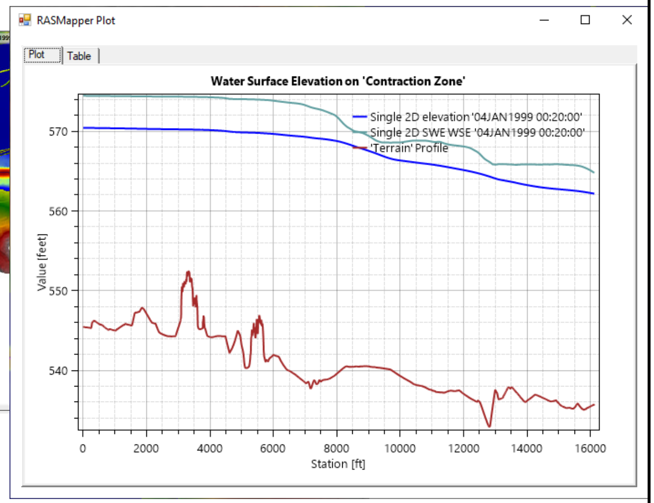
# Sharp Contraction – WS Profiles



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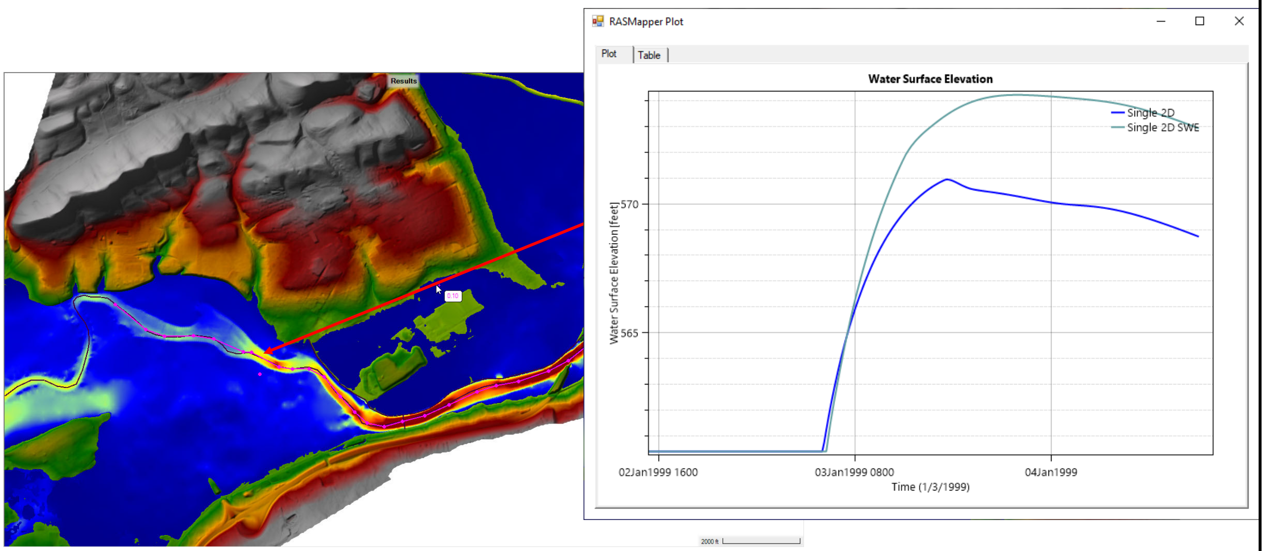
# Sharp Contraction – WS Time Series

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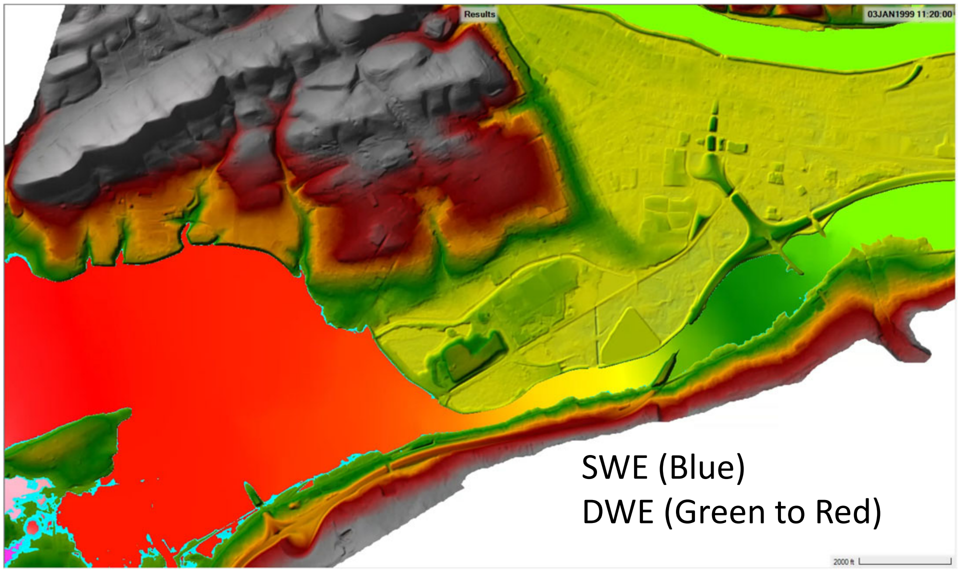
# Inundation Map

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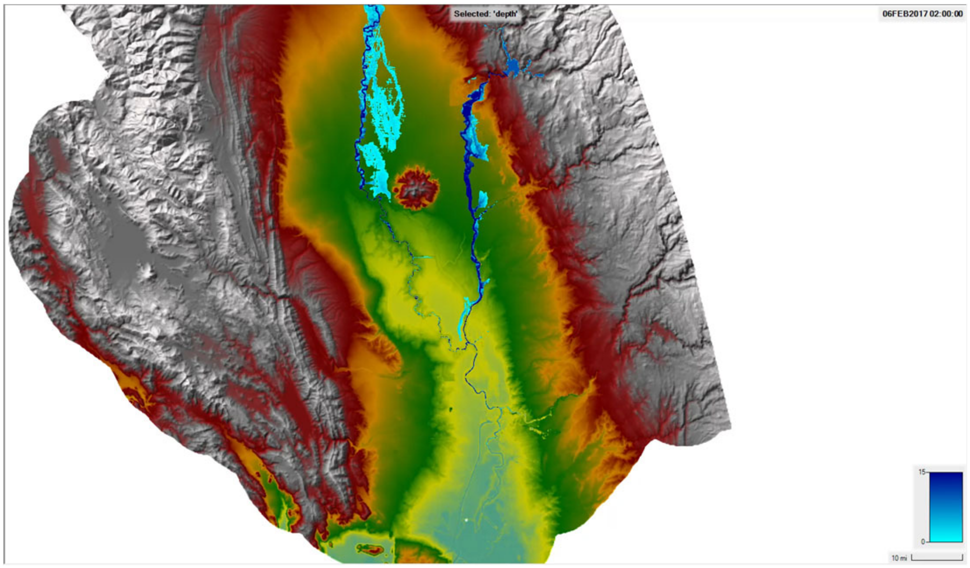
# Dam Break – Oroville Dam, Sacramento Valley

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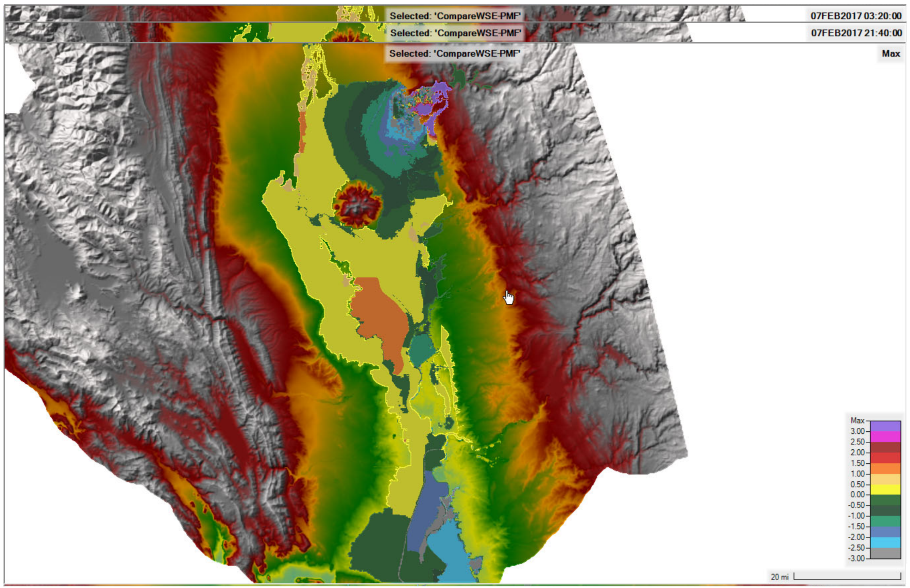
- Animation is from SWE

# Dam Break – Oroville Dam – WS Comparison
## Difference = SWE WS – DWE WS

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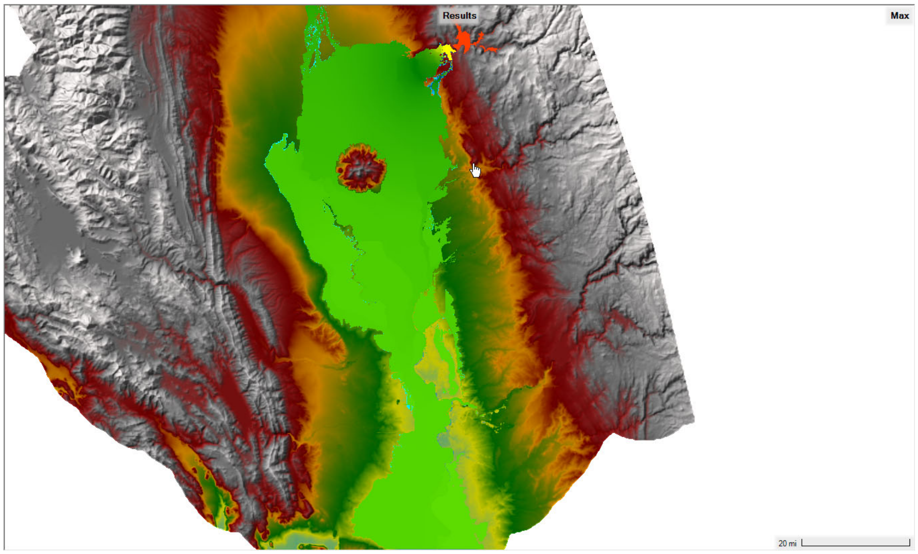
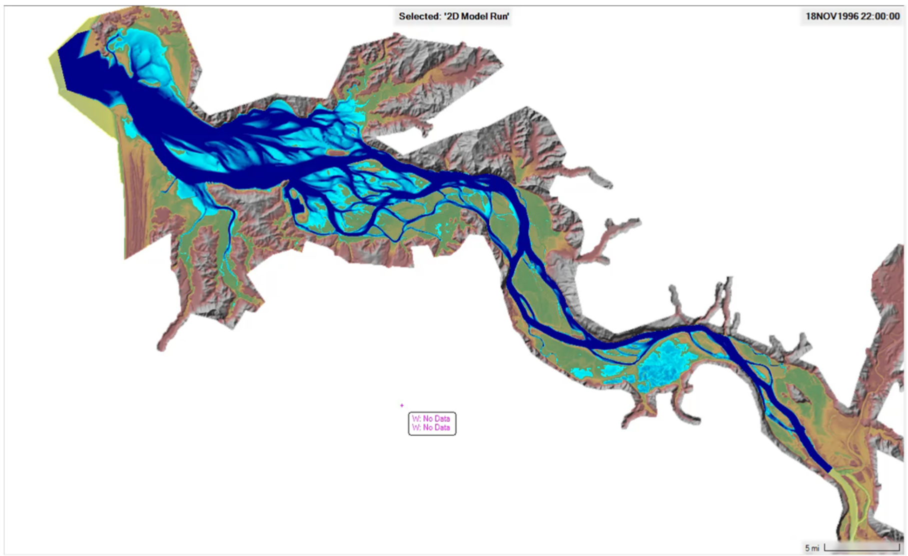
# Dam Break – Oroville Dam – WS Max



# Tidal Boundary Condition – Lower Columbia

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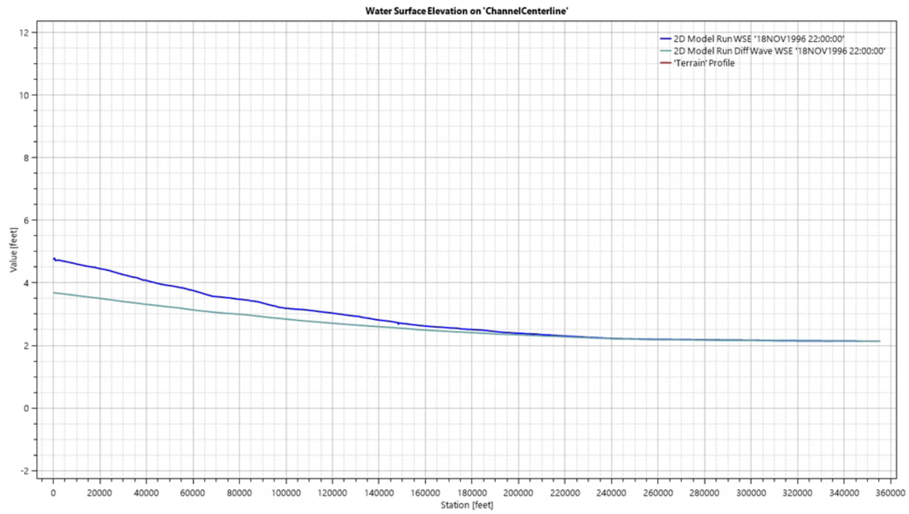
# Tidal BC – Lower Columbia – WS Profiles

SWE (Dark Blue) and DWE (light Blue)

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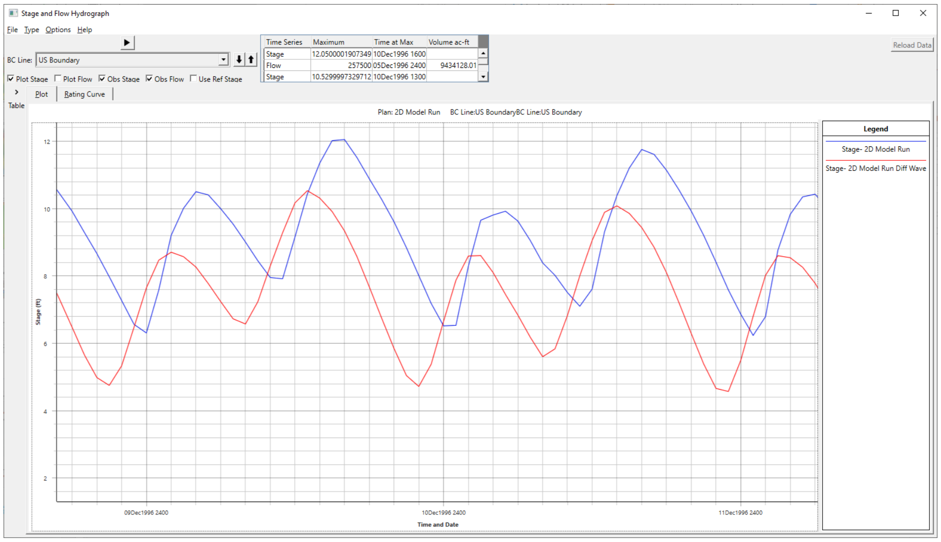
# Tidal BC – Lower Columbia – US Hydrograph

## SWE (Blue) and DWE (Red)

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# Local Inertia Approximation to Shallow Water Equations

- Shallow Water Equations
  - Mass Conservation (Continuity)
  - Momentum Equation
    - Friction
    - Pressure gradient
    - Local acceleration
    - Coriolis term (optional)
    - Wind Forces (optional)
- Ignoring advection and turbulence
  - Simplifies model
  - Reduces computational costs
  - Allows for larger time steps
  - Faster run times

$$\frac{\partial h}{\partial t} + \nabla \cdot (h\boldsymbol{V}) = q$$

$$\frac{\partial \boldsymbol{V}}{\partial t} + \cancel{(\boldsymbol{V} \cdot \nabla)\boldsymbol{V}} + f_c \boldsymbol{k} \times \boldsymbol{V} = -g\nabla z_s + \cancel{\frac{1}{h}\nabla \cdot (\boldsymbol{\nu}_t h \nabla \boldsymbol{V})} - \frac{\boldsymbol{\tau}_b}{\rho R} + \frac{\boldsymbol{\tau}_s}{\rho h}$$

Coming soon for V6.3

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# Questions?

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