Partial Duration Series and Peaks Over Threshold

Flood Frequency Analysis PROSPECT

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Gregory S. Karlovits, *P.E., PH, CFM*

Statistical Hydrologist

US Army Corps of Engineers

Hydrologic Engineering Center

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Purpose

- Understand a different approach for estimating flood frequency
- Identify when applying peaks over threshold can be beneficial

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Outline

1. Peaks over threshold approach
2. Models for partial duration series
3. Computing AEP using partial duration series
4. Why PDS?

Peaks Over Threshold Approach

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Peaks Over Threshold

- Choose a high flow value as a threshold
- Keep **all independent** flood peaks greater than the threshold
- Varying number of floods per year
  - 0 or more

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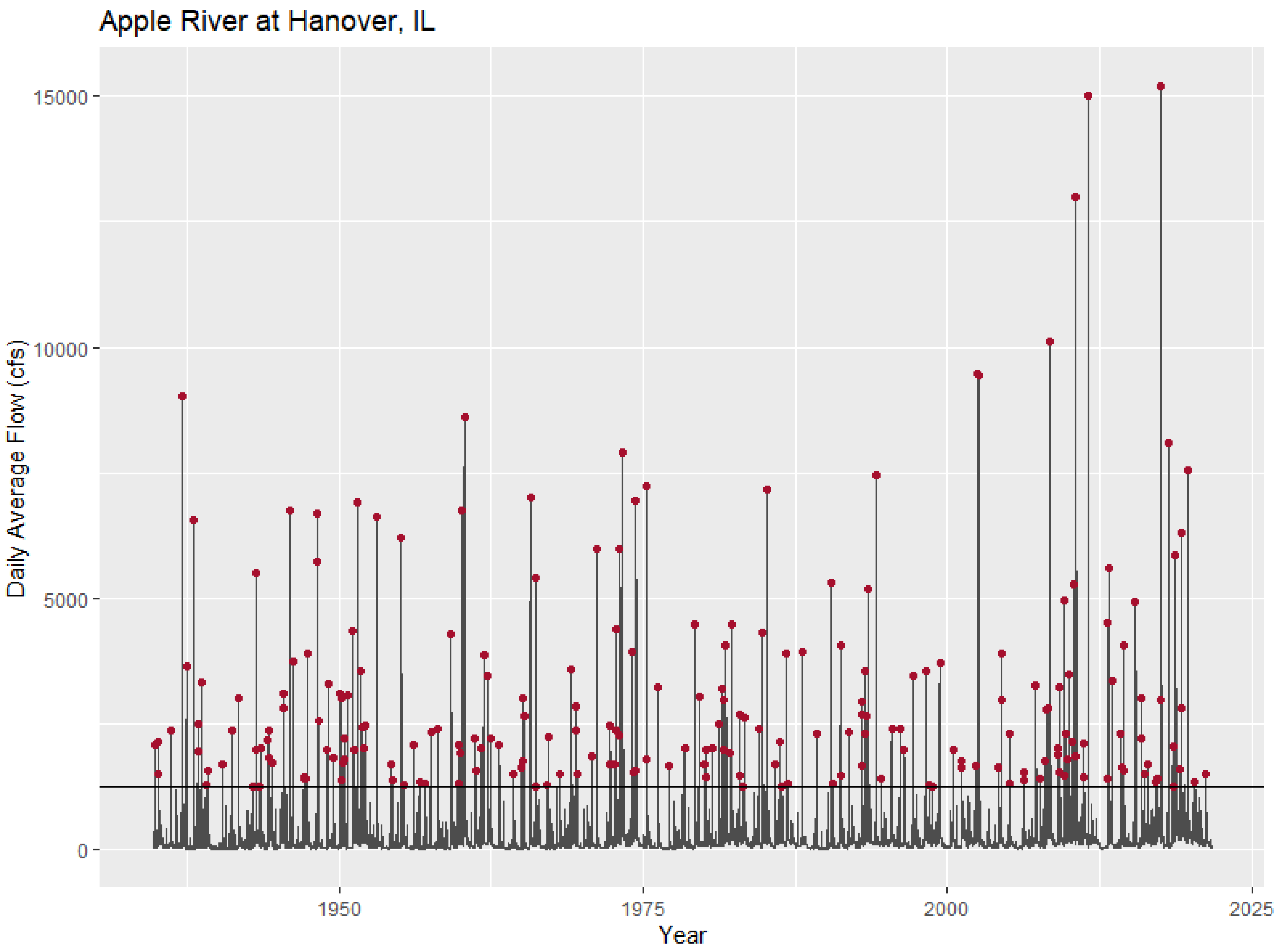
Peaks Over Threshold



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Partial Duration Series

- The collection of independent peaks over a sufficiently high threshold is called the *partial duration series (PDS)*
- Number of events can be different than in the annual maximum series (AMS)

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Inference Assumptions

- Sample is representative of population
- Sample values are independent
- Sample values are identically-distributed

I.I.D.

Representative Sample

- High threshold ensures sample made only of floods
- Too high of a threshold limits sample size

- Violation: model for population doesn’t actually represent it

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Identical Distribution

- High threshold ensures sample made only of floods
- May still be a mixture of flood-generating mechanisms

- Violation: population model misrepresents extremes

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Independence

- Floods you keep should be independent of others
- Enough time passes between floods
- Hydrograph sufficiently recedes between floods

- Violation: we think large floods happen more often than they actually do

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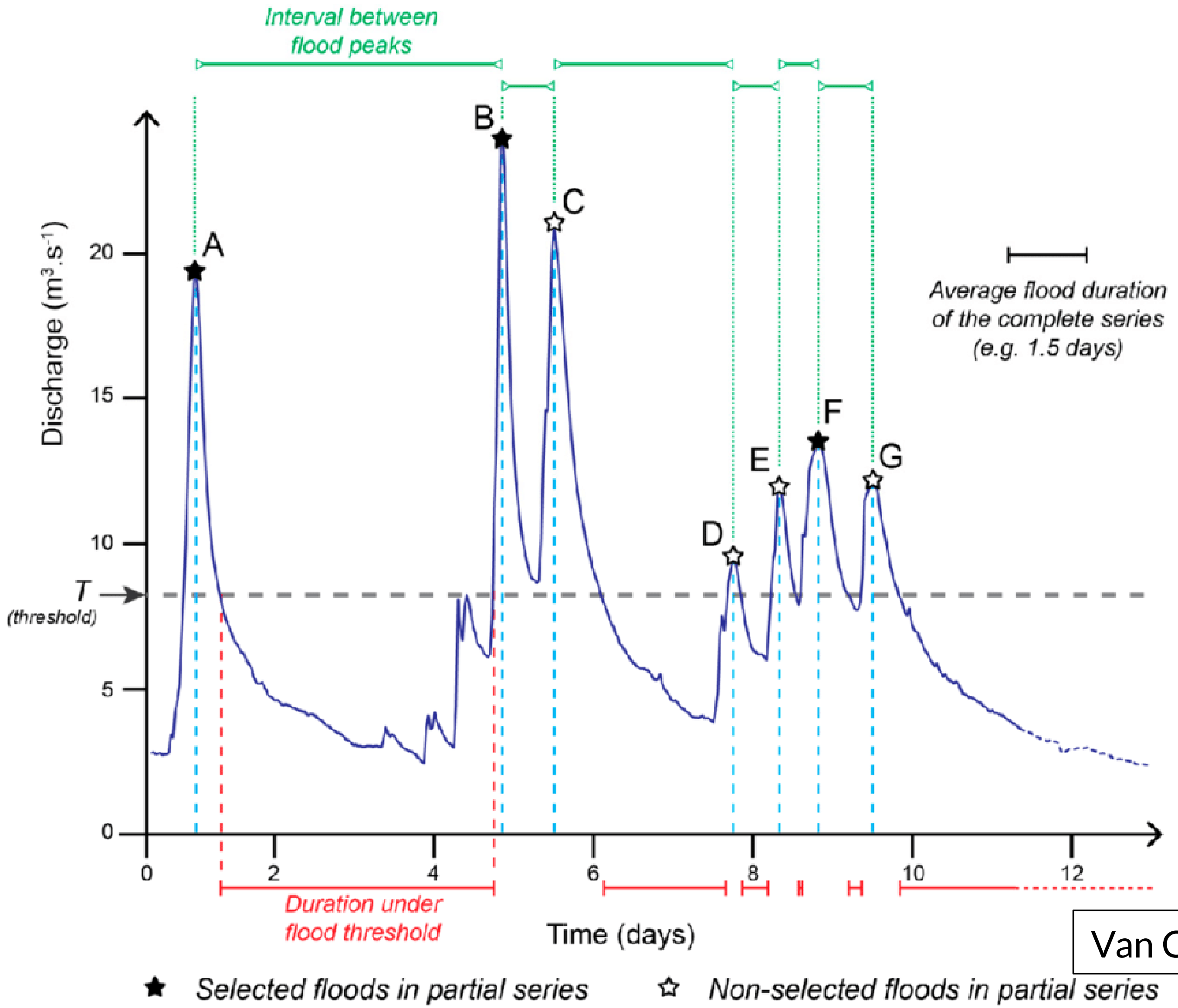
Independence



Van Campenhout, et al. (2020)

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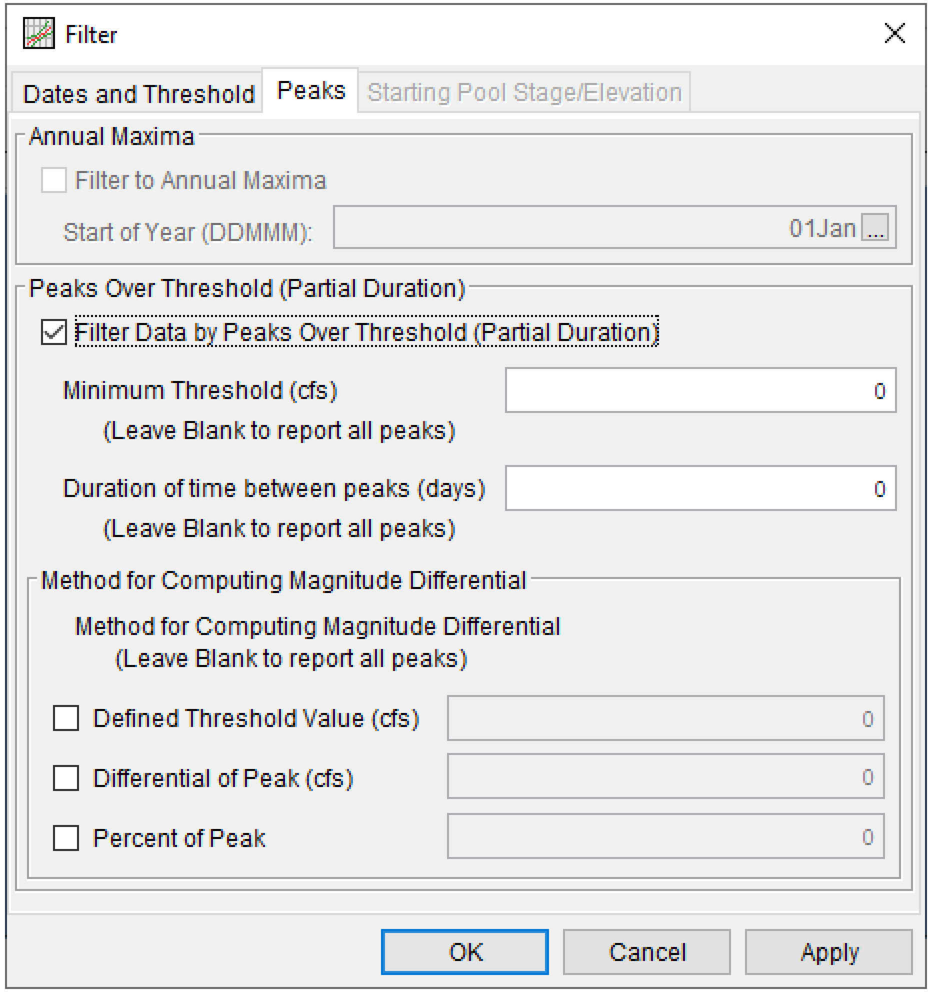
Filtering Peaks in HEC-SSP

- Threshold
- Time between peaks
- Magnitude difference
  - Recede below a threshold
  - Recede by x cfs
  - Recede by y% of peak



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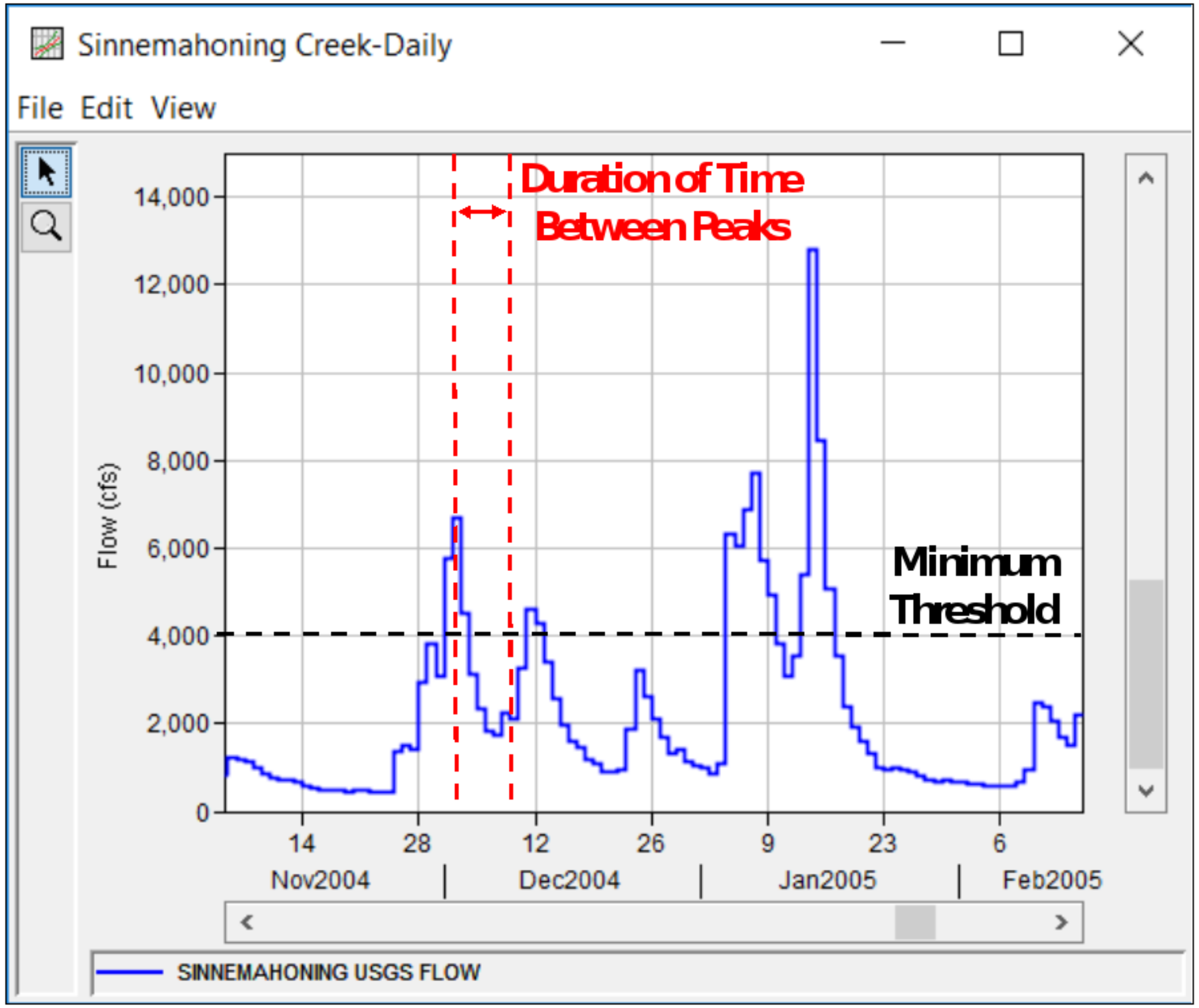
Filtering Peaks



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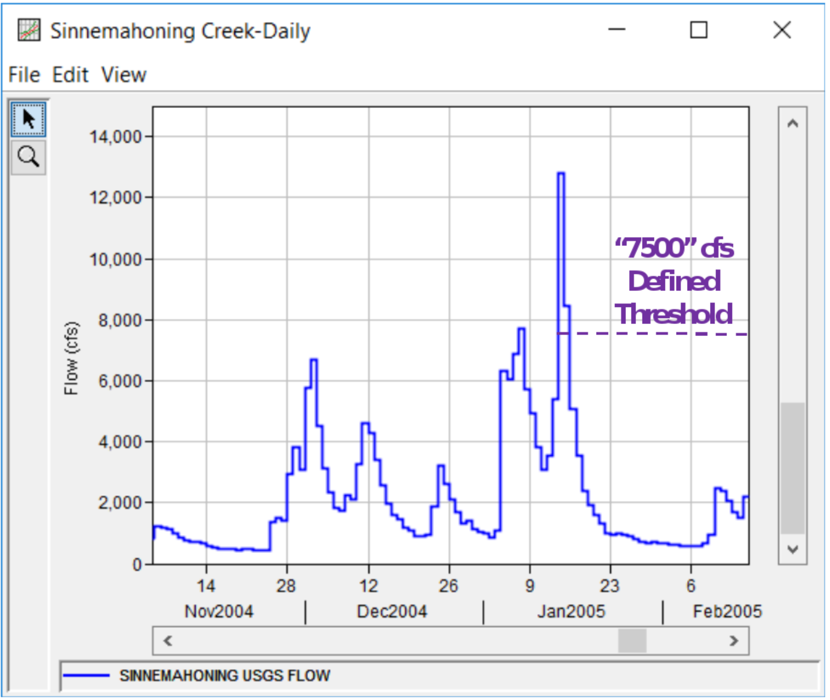
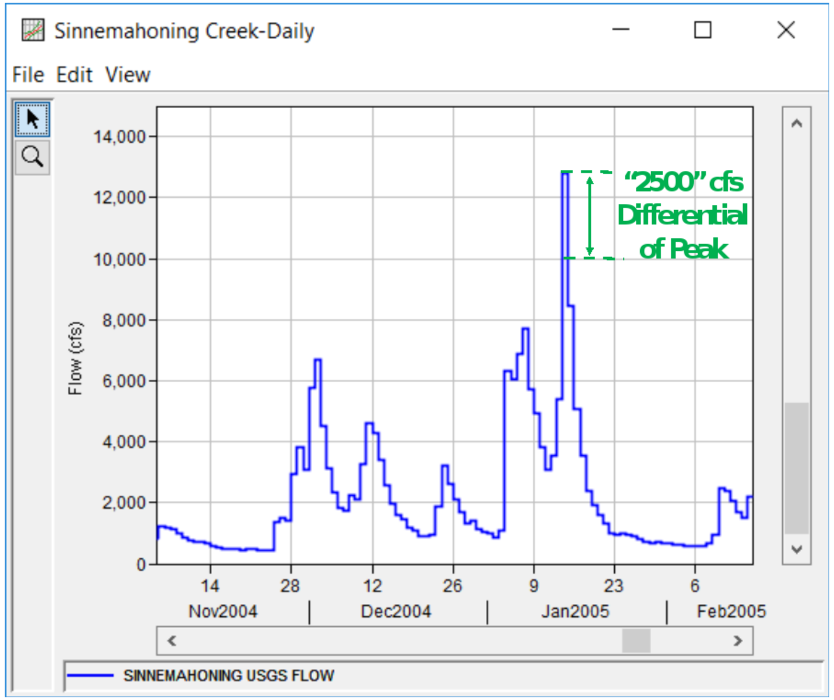
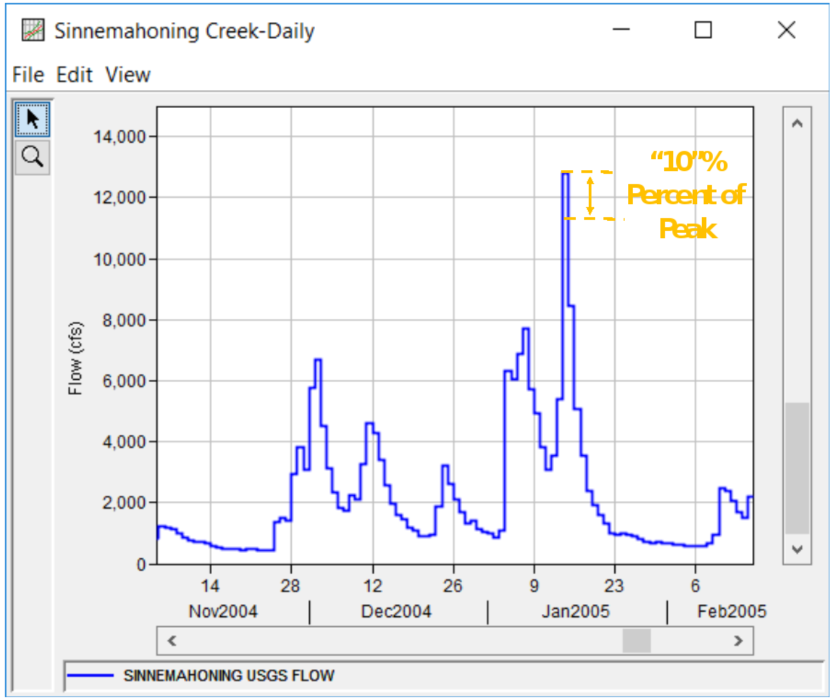
Filtering Peaks







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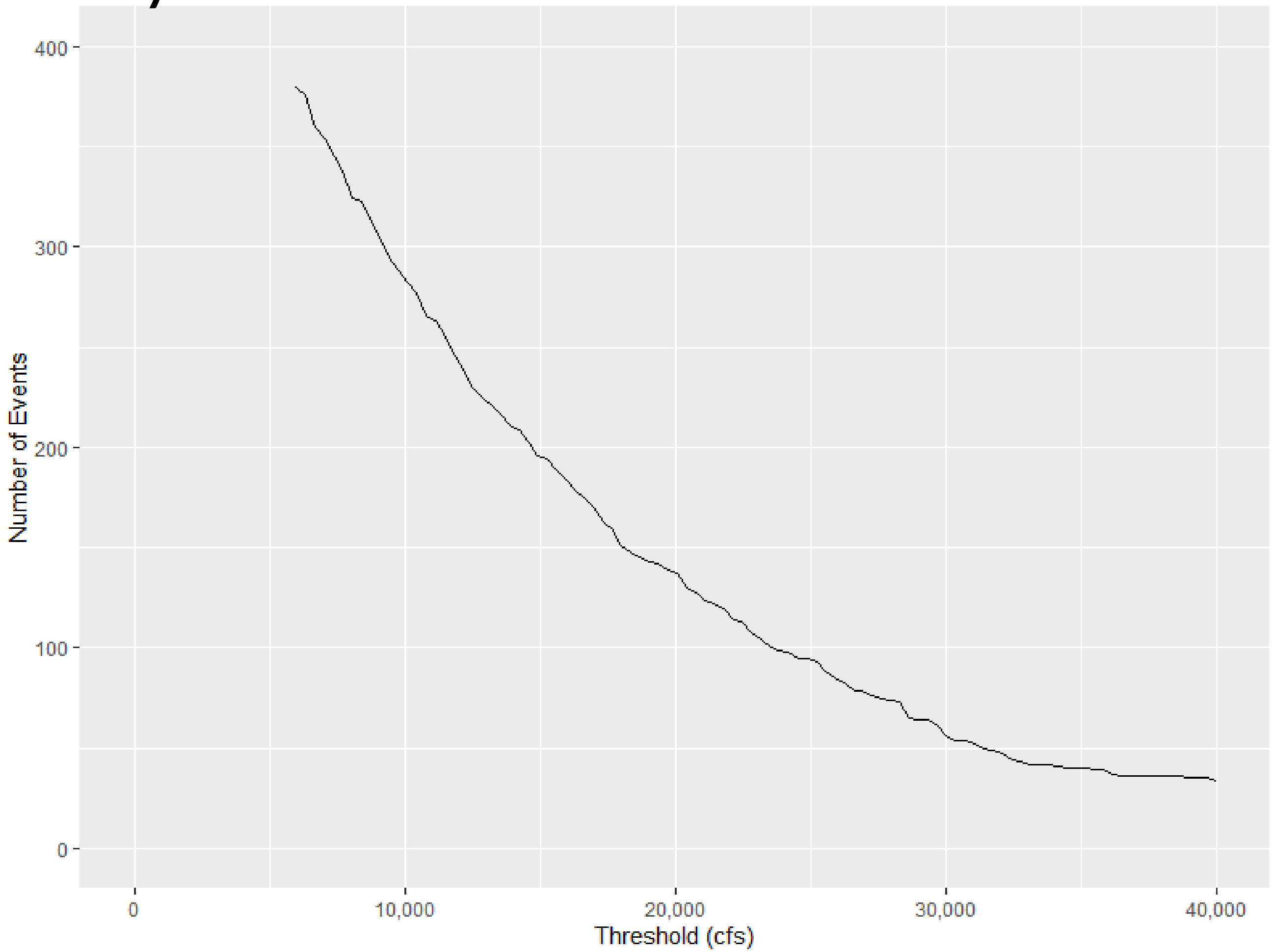
Choosing a Threshold

- Sufficiently high threshold necessary
  - Helps with assumptions
  - Makes our model work
- Too high a threshold results in small sample sizes



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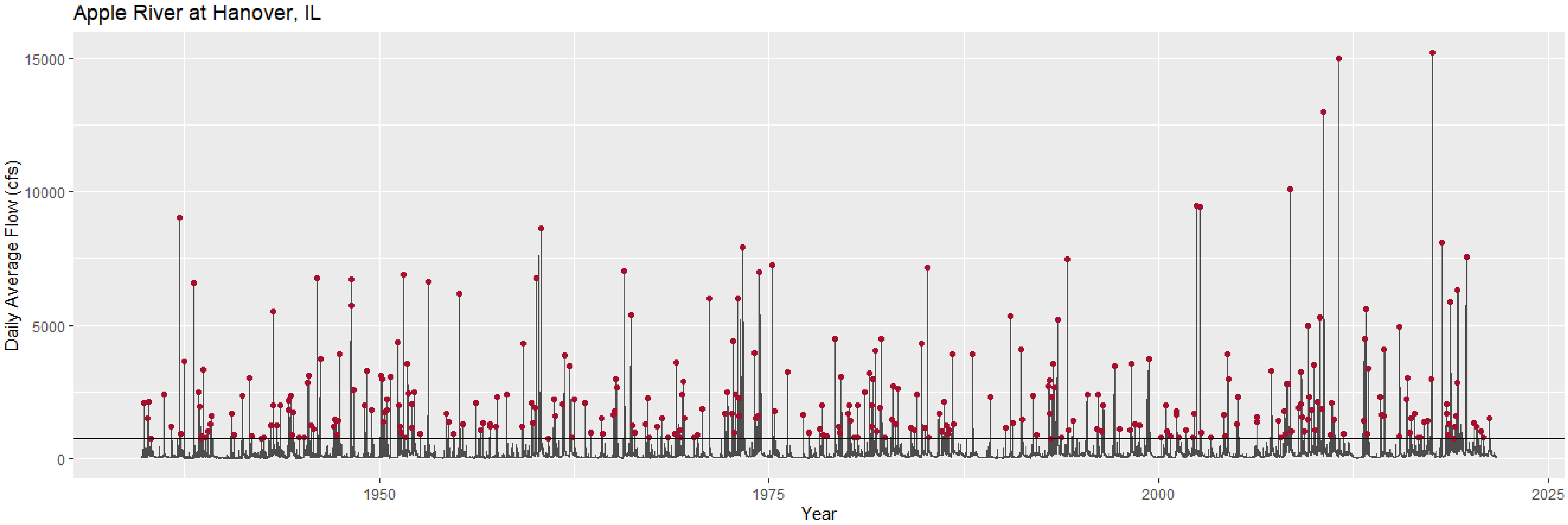

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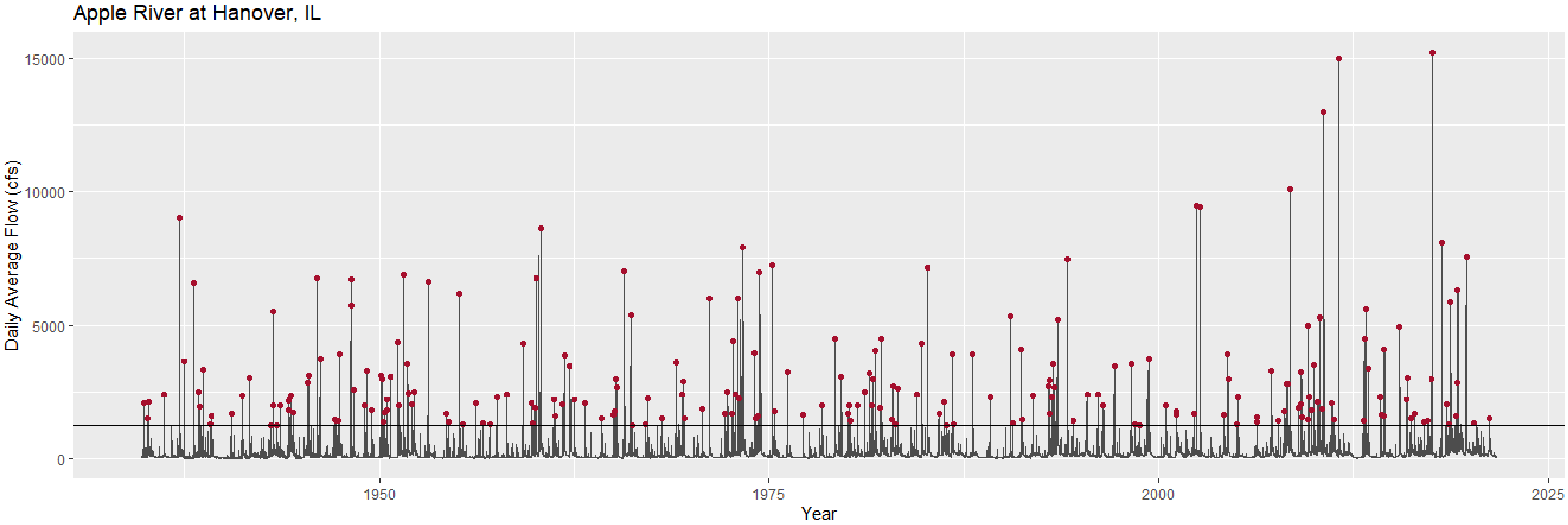
750 cfs
11 days minimum separation
250 cfs minimum recession
318 events



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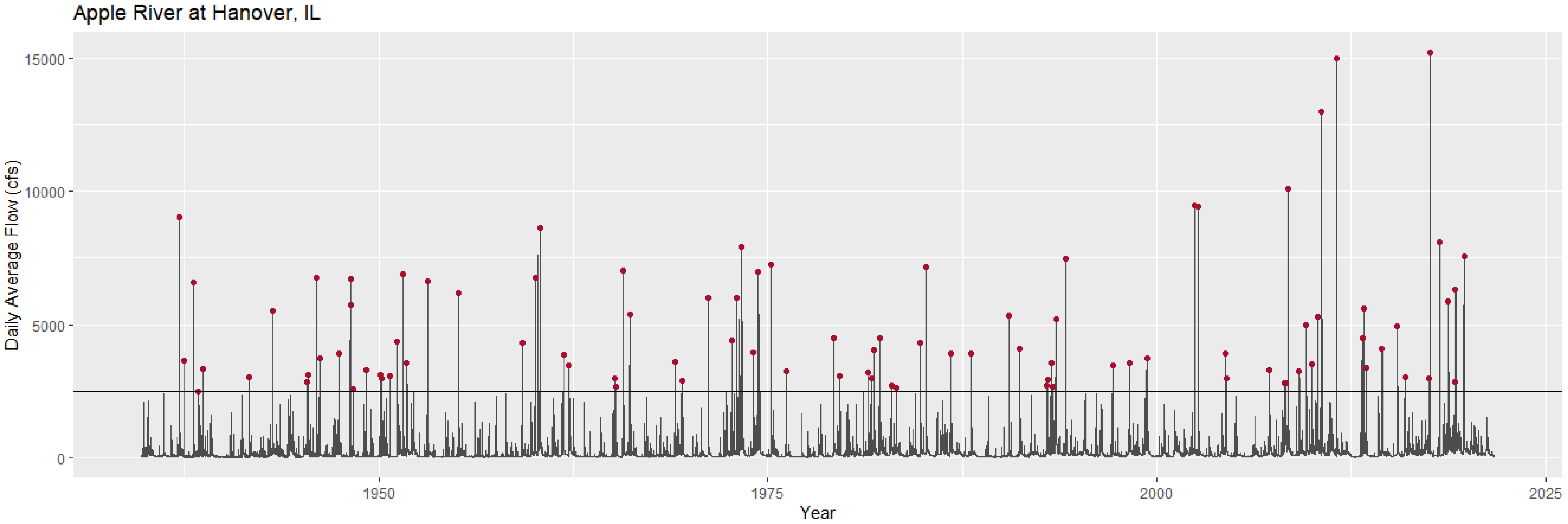
1,250 cfs
11 days minimum separation
250 cfs minimum recession
220 events



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2,500 cfs
11 days minimum separation
250 cfs minimum recession
93 events

Choosing a Threshold: Ideas

- Minimum annual maximum value
- Bankfull discharge
- Mean rate = 1 event / year (# events = # years)
- More "advanced" diagnostics
  - Pareto shape plot
  - Mean excess plot

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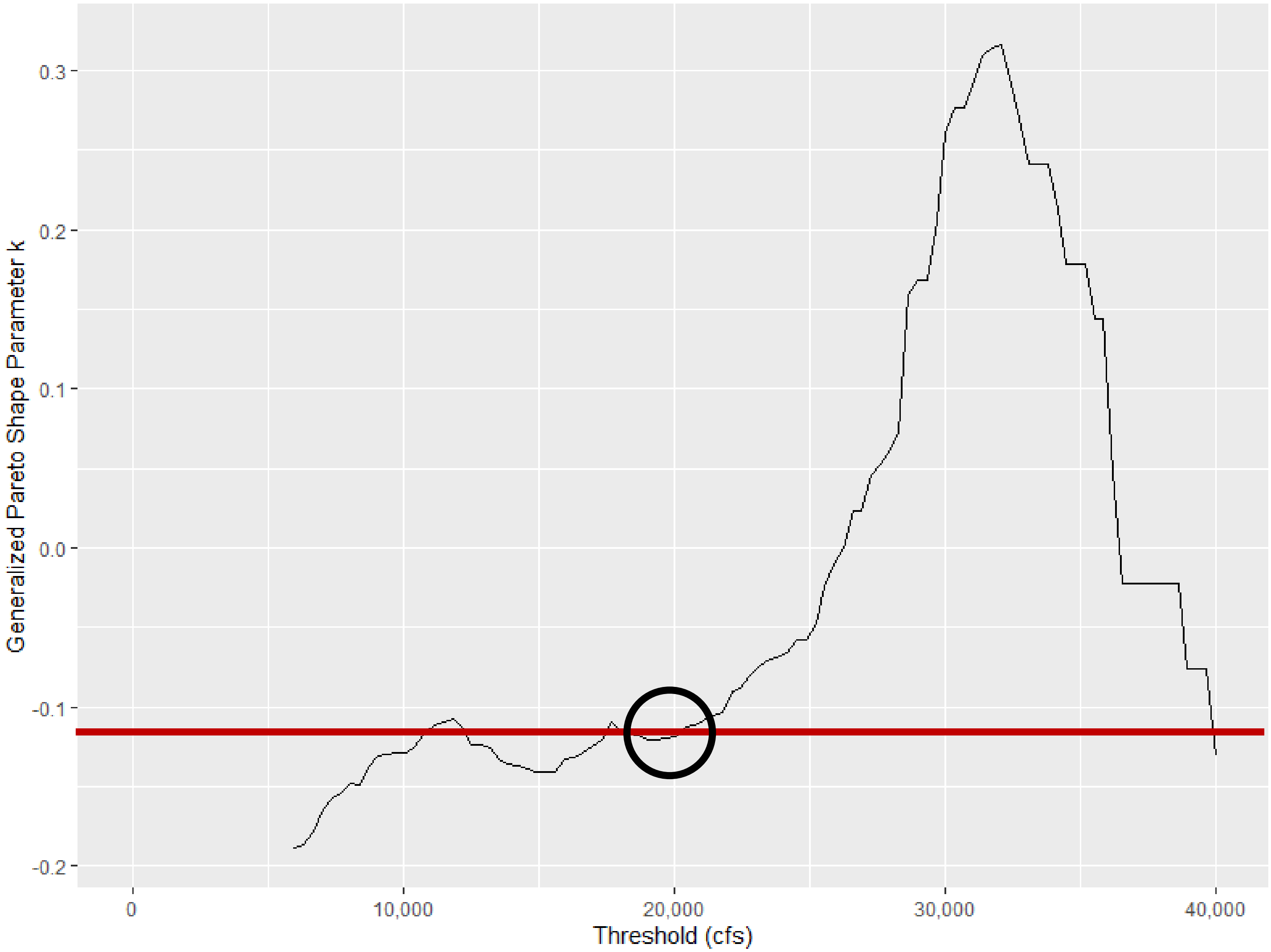
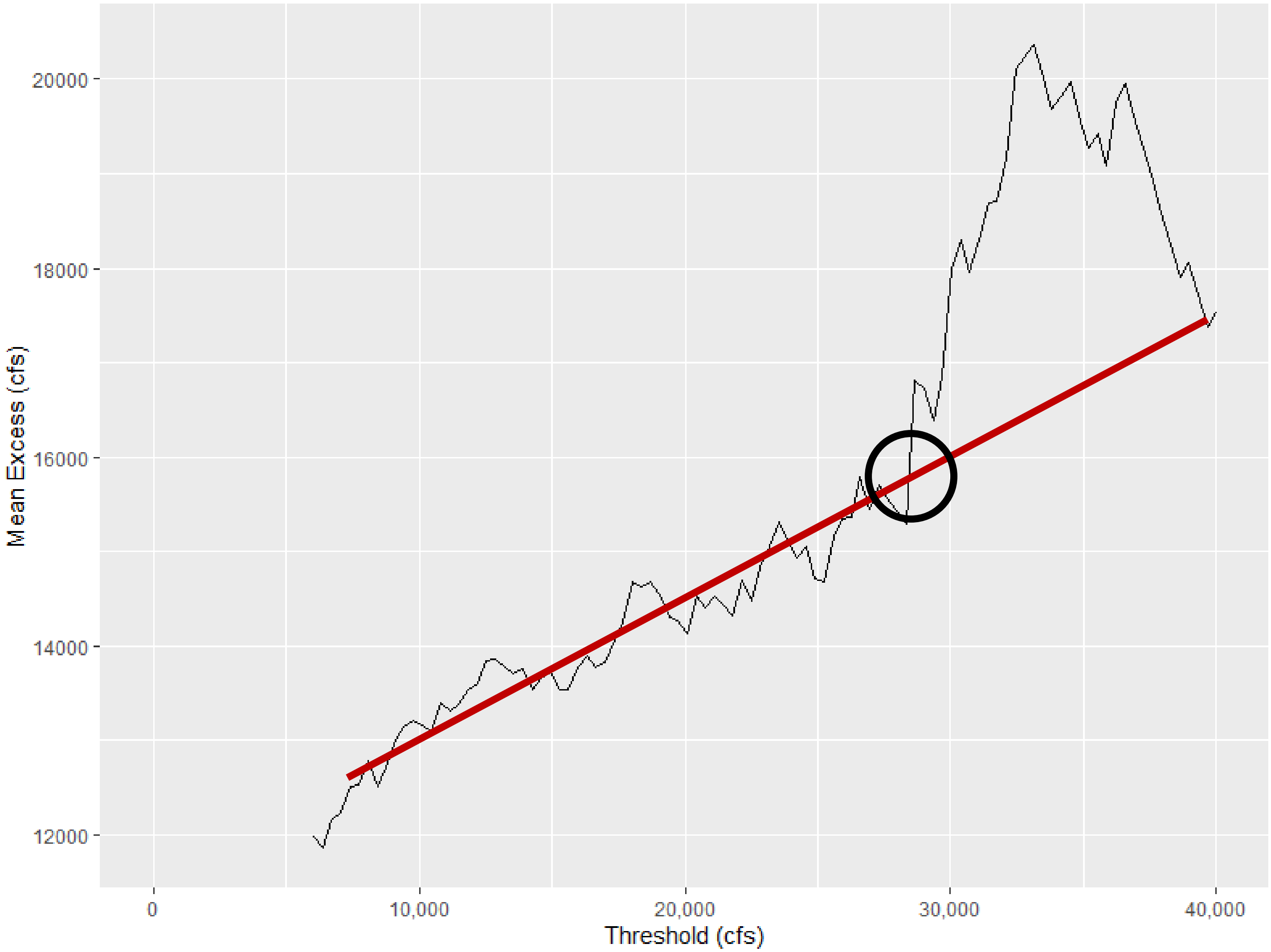
Other Diagnostics: Pareto Shape Plot

- Compute PDS for varying thresholds
- Fit generalized Pareto distribution to each
- Plot threshold vs. Pareto shape parameter
- Check for flat spot or change in shape



Other Diagnostics: Mean Excess Plot

- Compute PDS for varying thresholds
- Compute mean amount peaks exceed threshold
- Plot threshold vs. mean
- Check for linearity or change in shape



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Result

- Collection of IID flood events
- Variable number of events per year
- All events greater than a specified threshold
- ***Partial duration series (PDS)***

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Models for Partial Duration Series

Flood Frequency Guidance: Review

Annual maximum series (AMS) +

Log-Pearson type III distribution (LP3) +

Expected moments algorithm (EMA) =

Bulletin 17C

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Partial Duration Series

- Distribution of independent values exceeding a threshold
- *Probability of exceeding a value, given it's already exceeded threshold*
  - $P(X > x | X > x_0)$
- Need two things:
  - Distribution of those values
  - Distribution of counts per year

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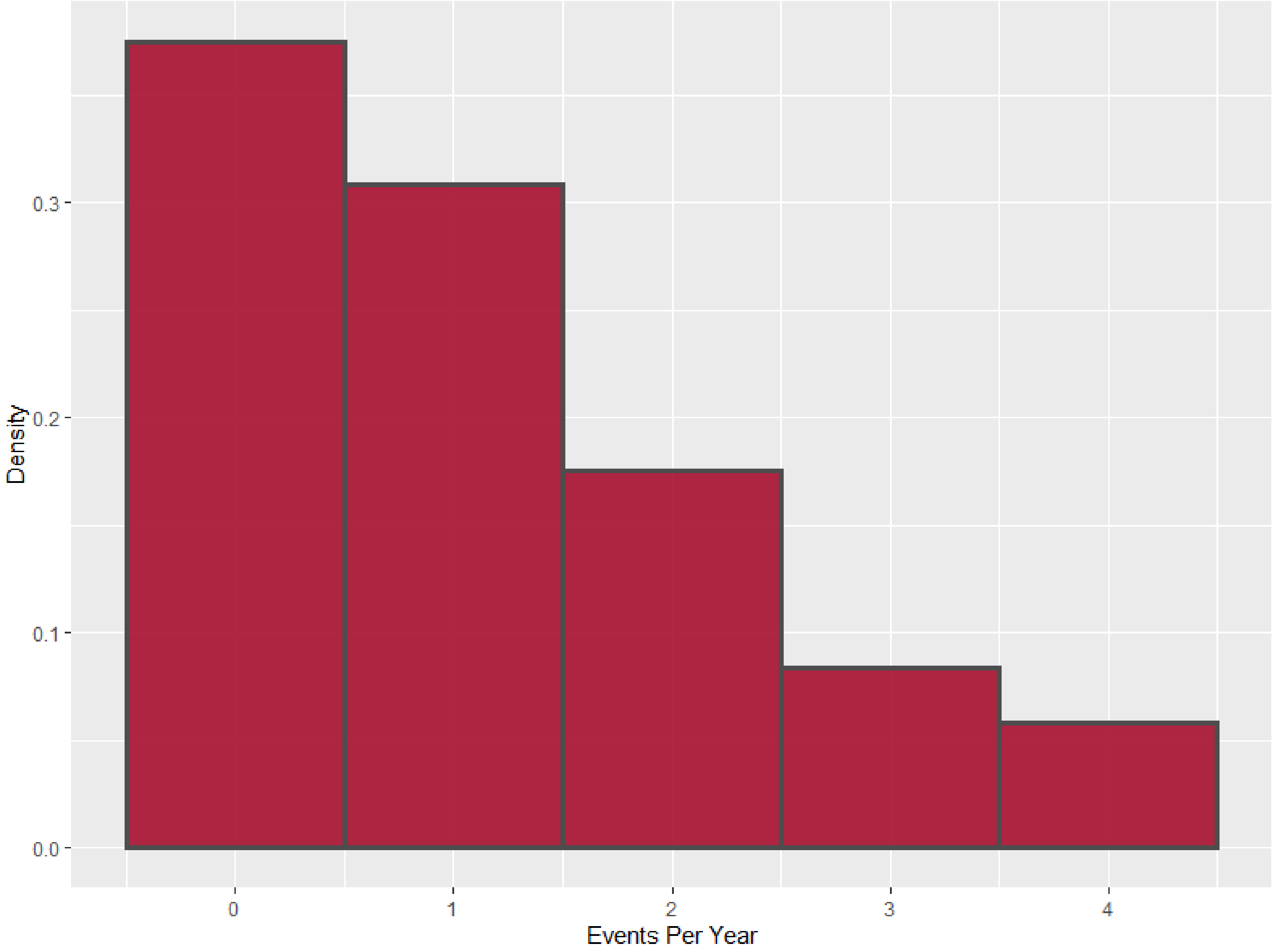
Counts

- 0 or more events per year in PDS
- Two key questions:
  - Probability of at least one event in a year?
  - Typical number of events in a year?



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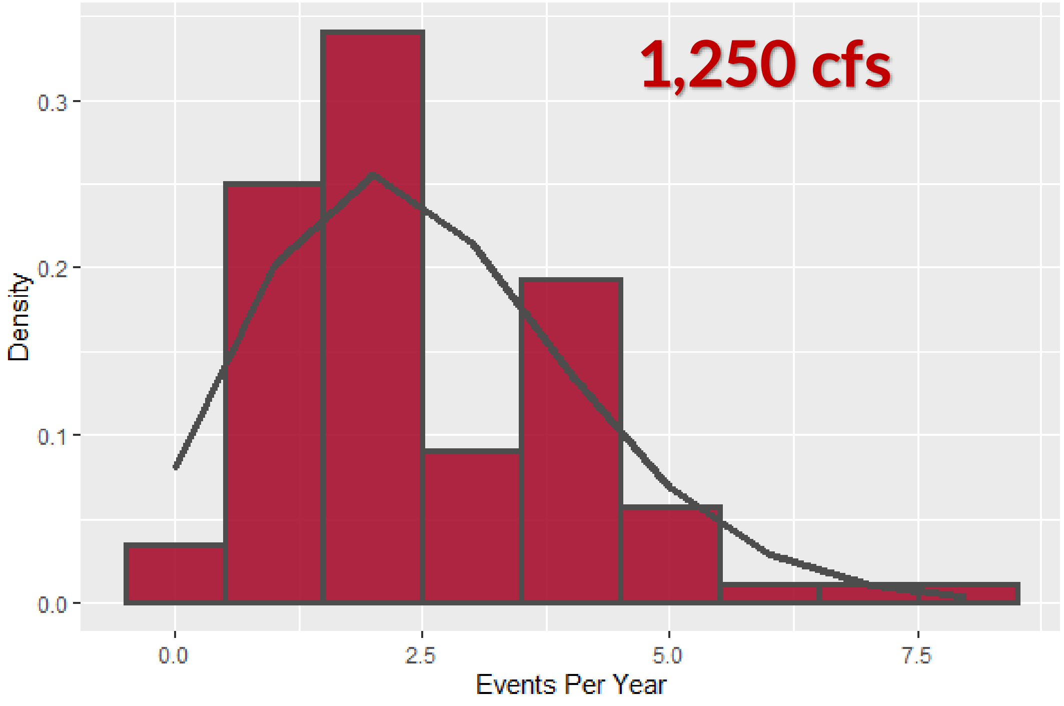
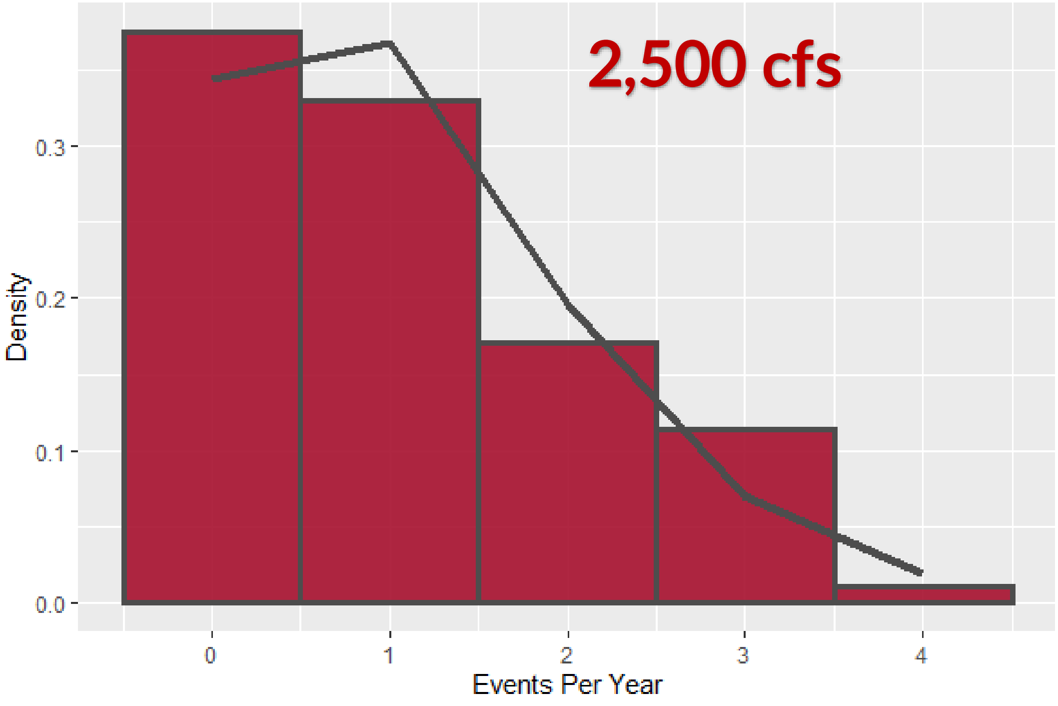
Counts

- Threshold choice affects count distribution
- Usually assumed to have a *Poisson Distribution*





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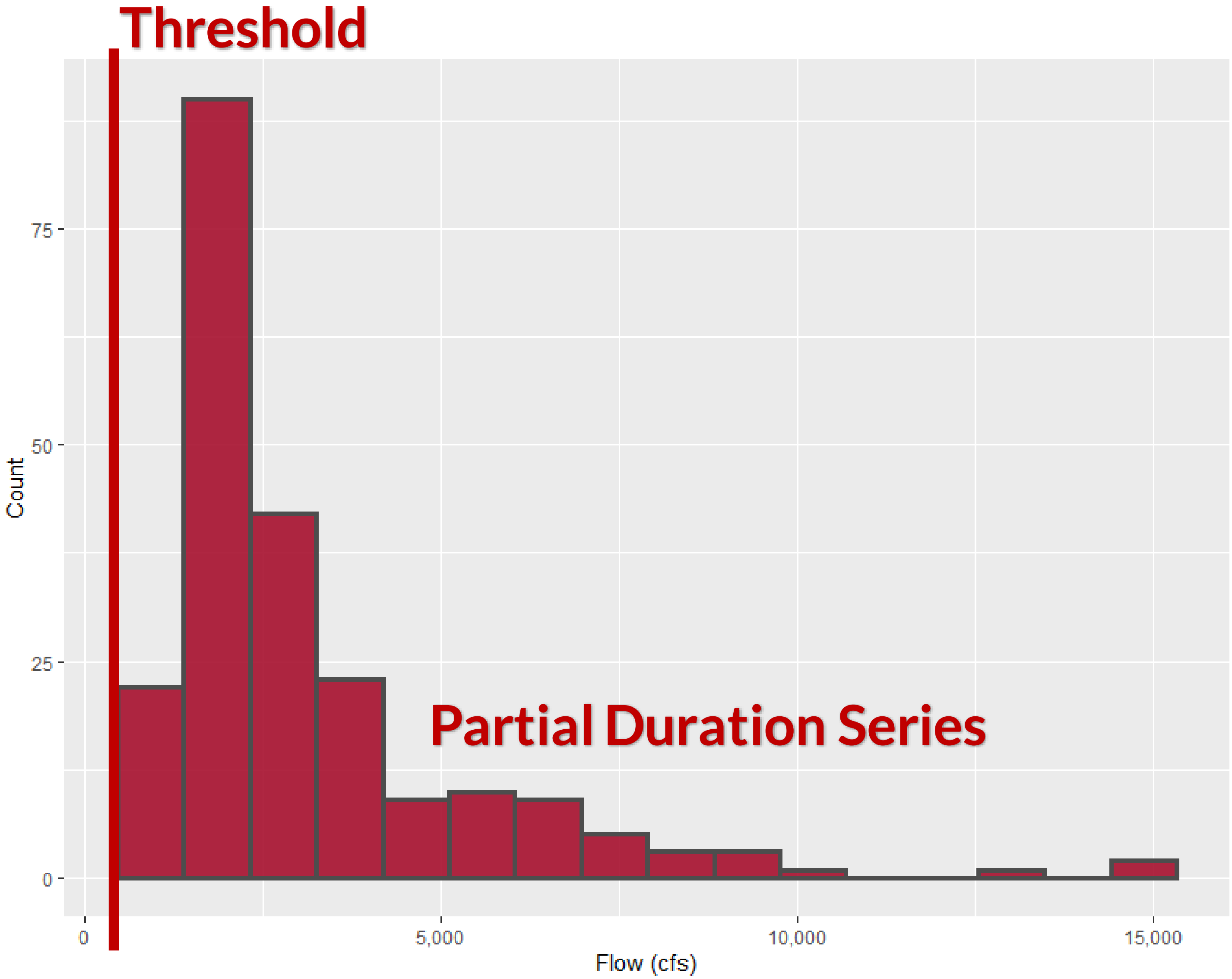
Distribution of Values

- Model magnitude of values greater than threshold
- Tend to cluster close to threshold



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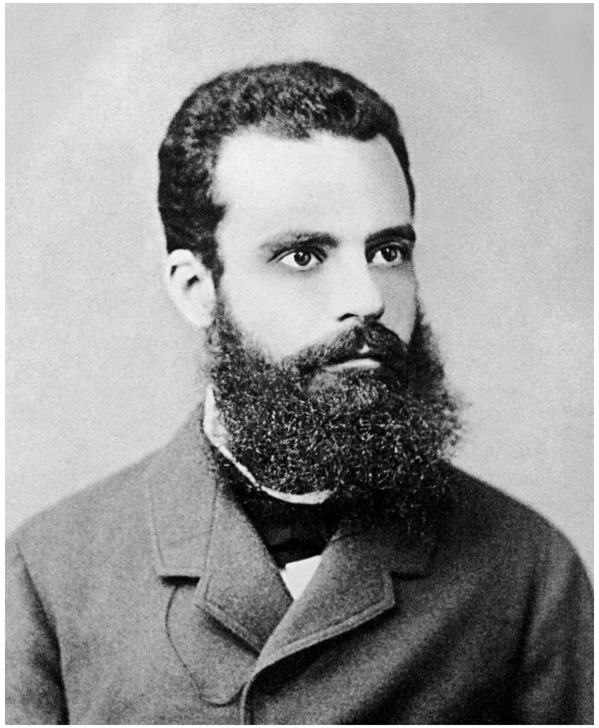
Distribution of Values

- Lean on extreme value theory
  - We have a model for this situation!
- Generalized Pareto Distribution (GPD)
  - 3 parameters: ξ (location) α (scale) κ (shape)

For more information, come to the Statistical Methods class!



Vilfredo Pareto

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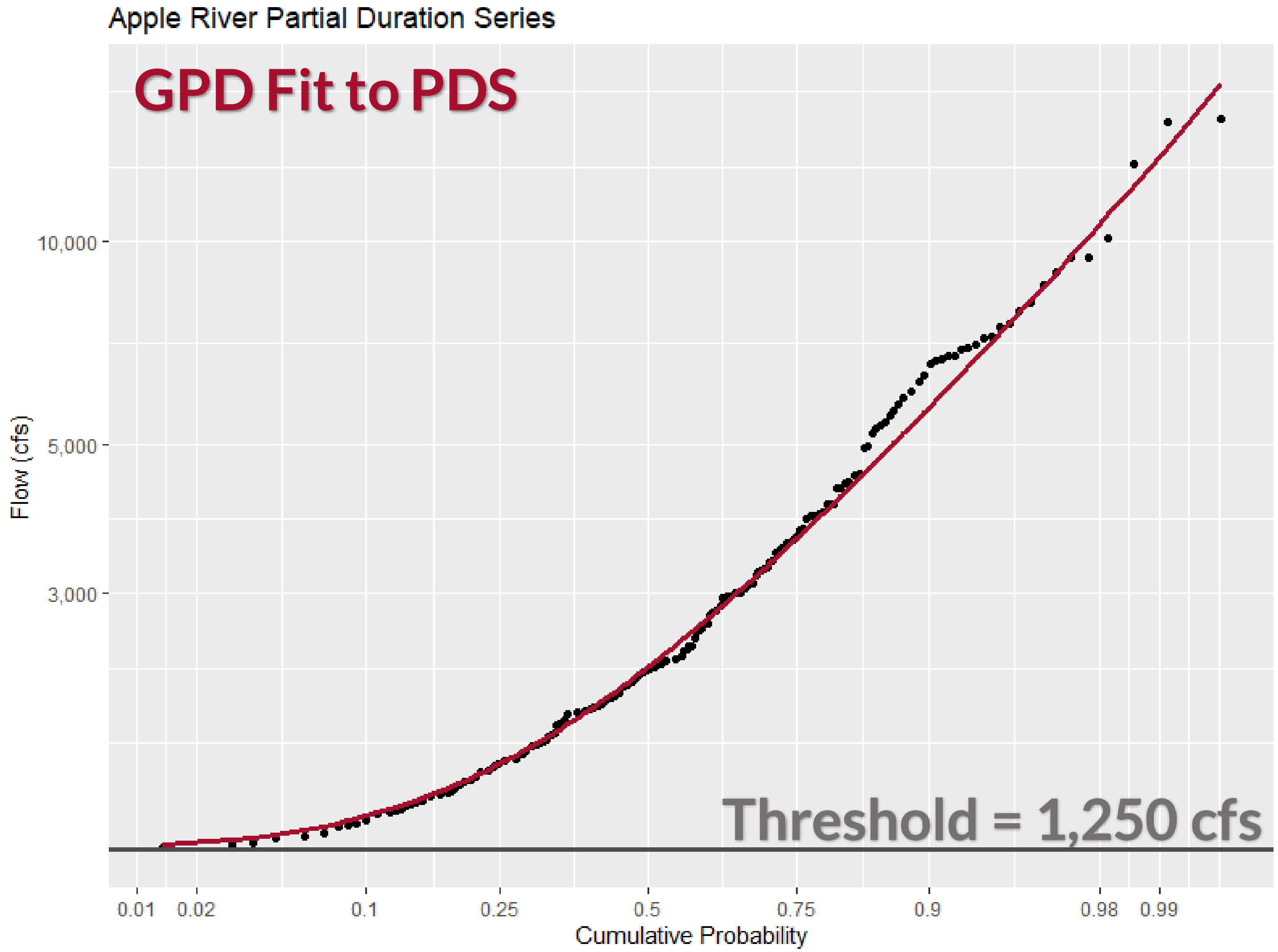
Distribution of Values

- Cumulative probability, given the values exceed the threshold
- ***NOT AN AEP***



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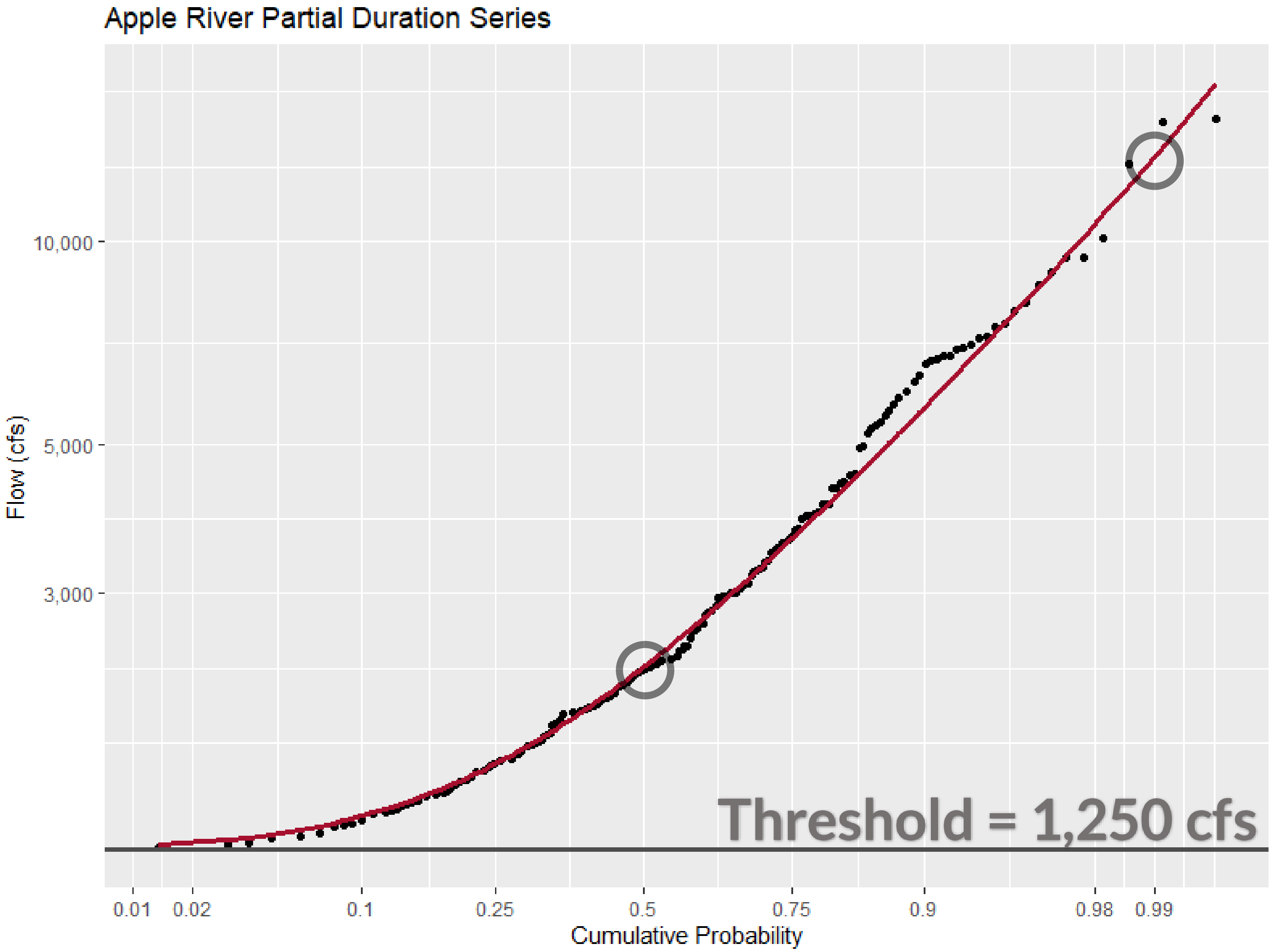
Interpreting the Distribution

- 50% of floods that exceed 1,250 cfs are greater than 2,330 cfs
- 1% of floods that exceed 1,250 cfs are greater than 13,400 cfs



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Computing AEP from PDS

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Annual Exceedance Probability

- AMS analysis provides AEP events
- PDS analysis provides conditional exceedance events

- We need AEP for things like annualized damages!

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Computing AEP from PDS

Two general approaches:

- Empirical
- Analytical

- “Annualization”

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Empirical Annualization

- Langbein (1949) and Takeuchi (1984)
- Assume storm per year counts have a Poisson distribution
  - One parameter λ
  - PDS CDF $G(x)$, AMS CDF $F(x)$
- $F(x) = e^{[-\lambda\{1-G(x)\}]}$
- $G(x) = \frac{ln[F(x)]+\lambda}{\lambda}$

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Applying Empirical Annualization

- Get AEPs you want to compute
- Convert them to PDS CDF values
- Plug them into your GPD distribution quantile function

- Can also be used to plot observed peaks
  - Convert the plotting positions

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Analytical Annualization

- Stedinger et al. (1993), Madsen et al. (1997)
  - Generalized Pareto + Poisson → Generalized Extreme Value
- Have:
  - Poisson parameter λ
  - Generalized Pareto parameters ξ, α, κ
- Get:
  - GEV parameters ξ^*, α^*, κ^*
- $\xi^* = \xi + \frac{\alpha}{\kappa}(1 - \lambda^{-\kappa})$ $\quad \alpha^* = \alpha\lambda^{-\kappa}$ $\quad \kappa^* = \kappa$

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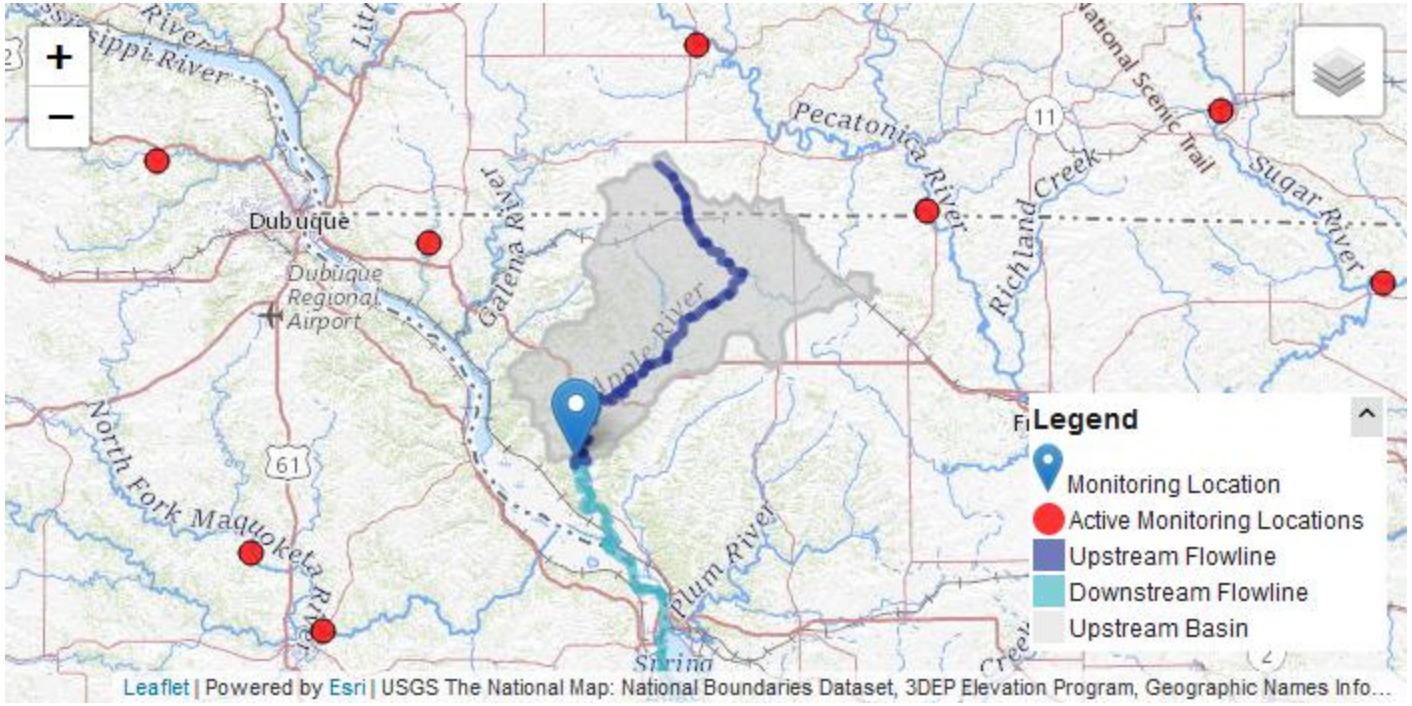
Applying Analytical Annualization

- Compute equivalent GEV parameters from GPD/Poisson
- Plug AEP values into GEV quantile function

Fit and Annualization Example

- Apple River at Hanover, IL
  - 246 mi^2
  - WY1935-2021
  - Daily average flows
  - Unregulated
  - Little urbanization



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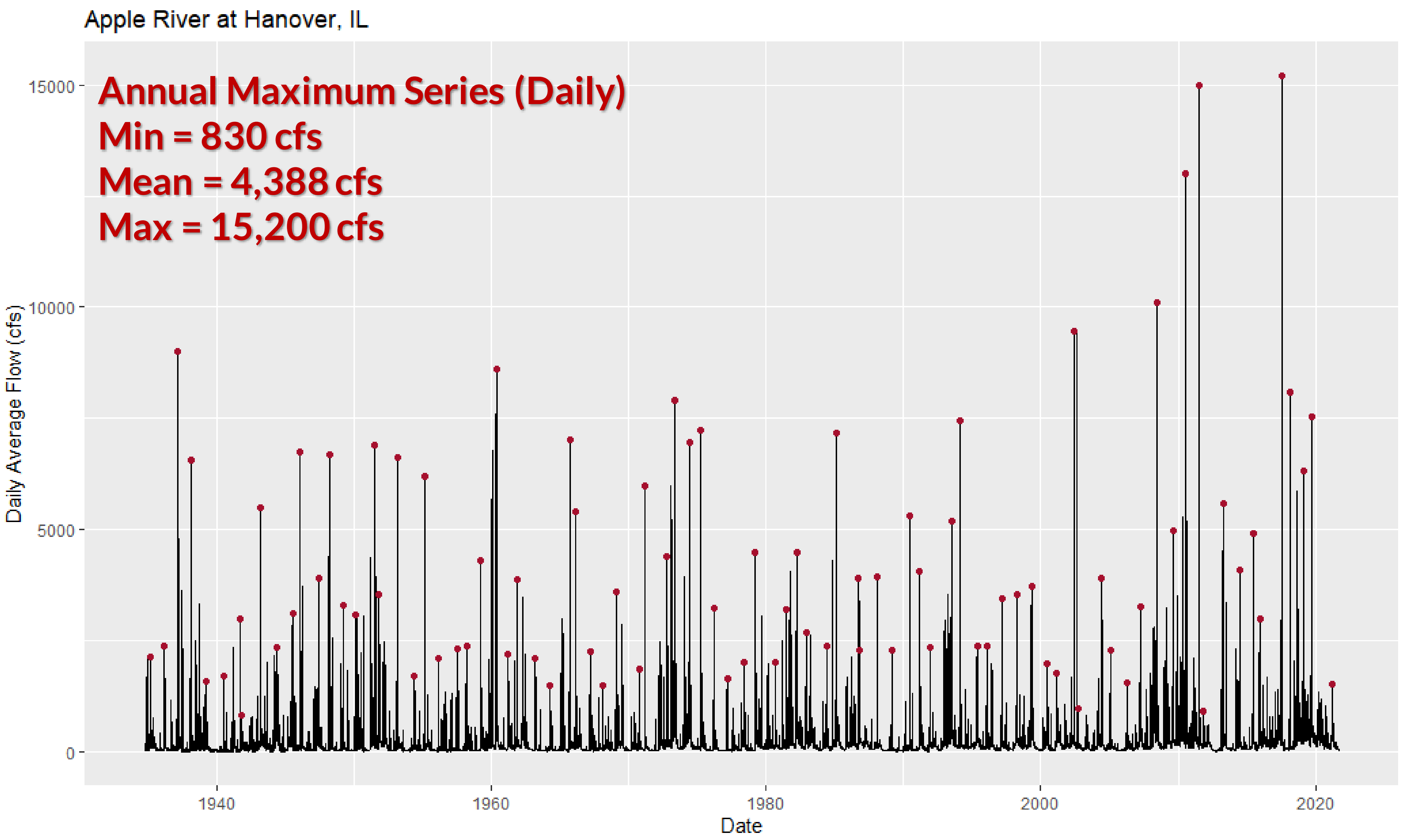

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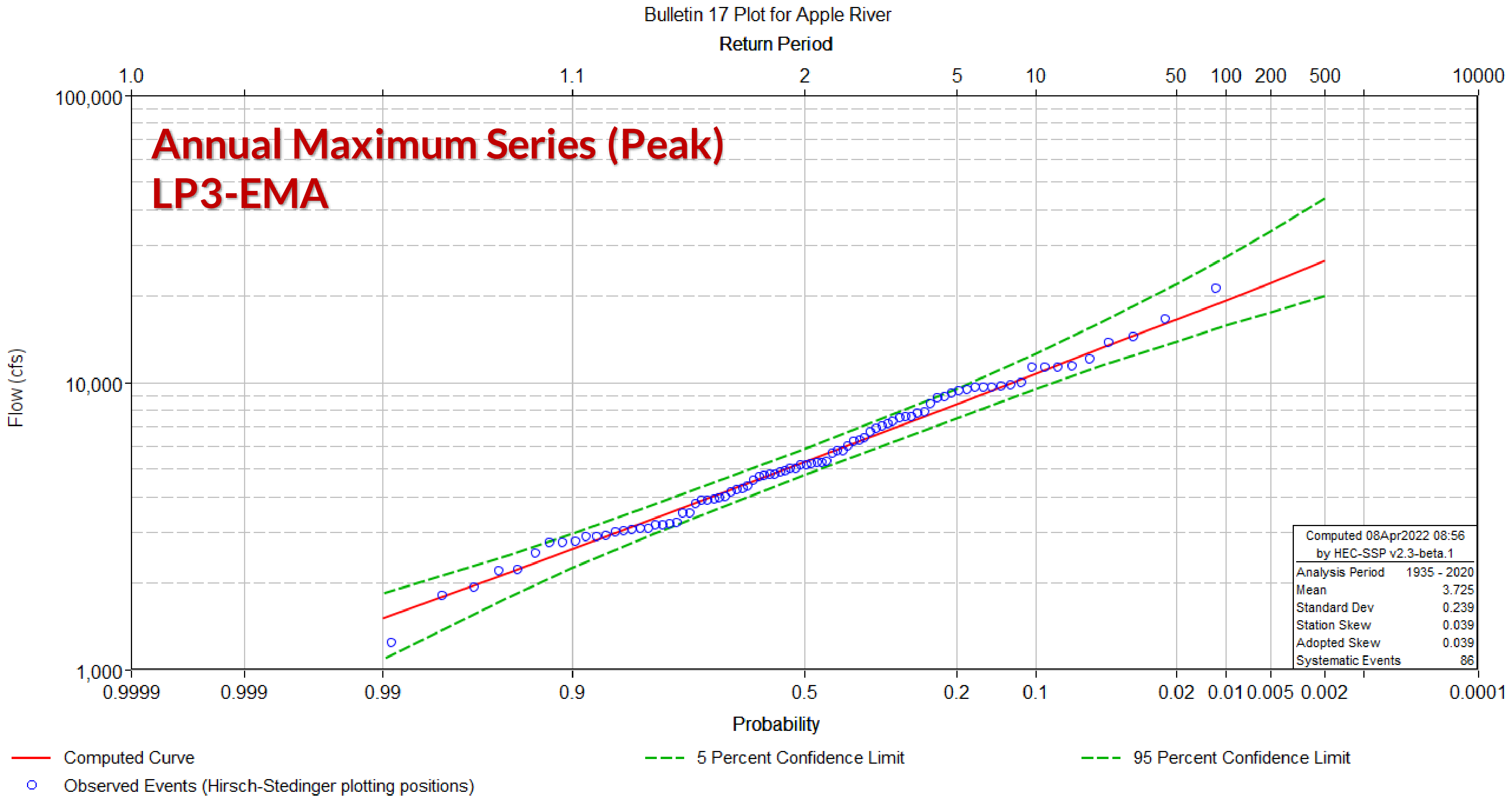
Annual Maximum Series (Peak)
LP3-EMA



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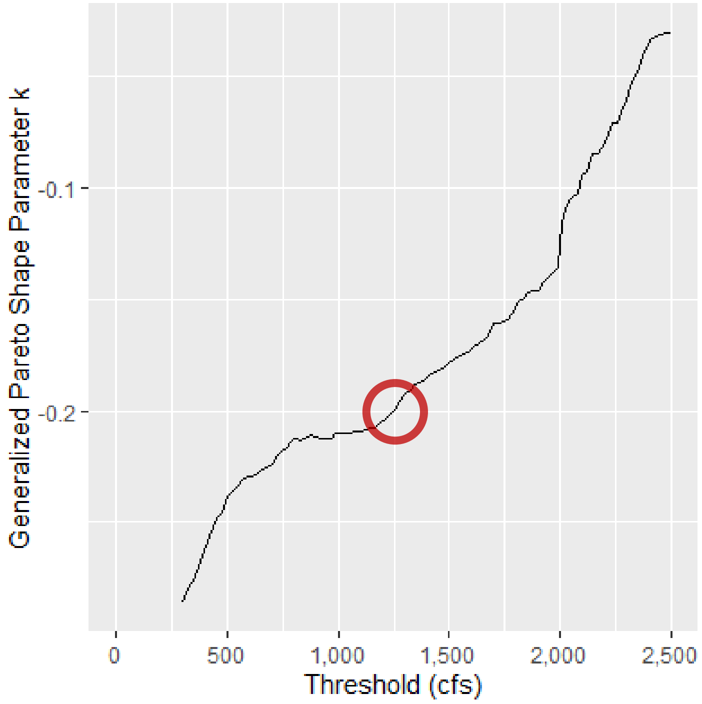
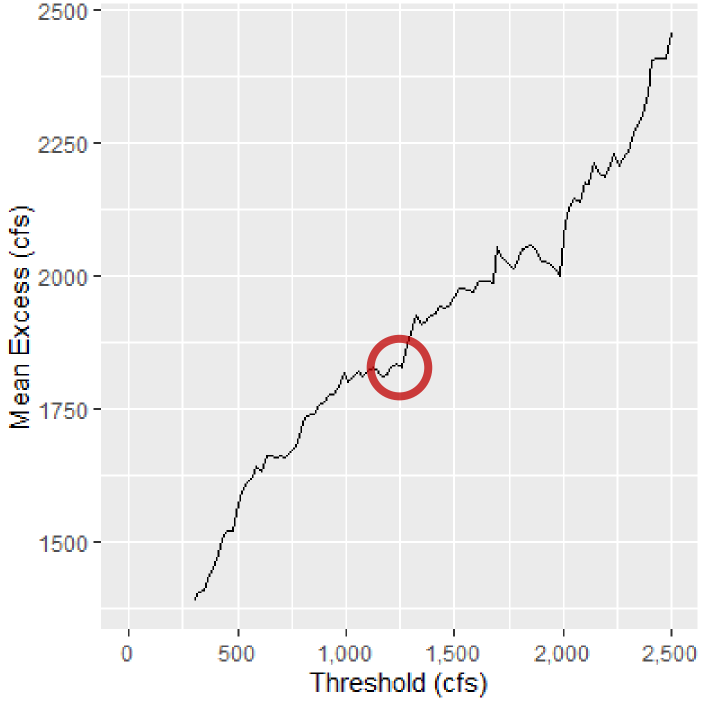
Filtering for PDS

- Threshold = 1,250 cfs
  - Based on shape plot, mean excess plot
- Time between events 11 days
  - $[5 + \ln(DA)]$ guideline
- Recession threshold 250 cfs
  - Flow must go below threshold between events





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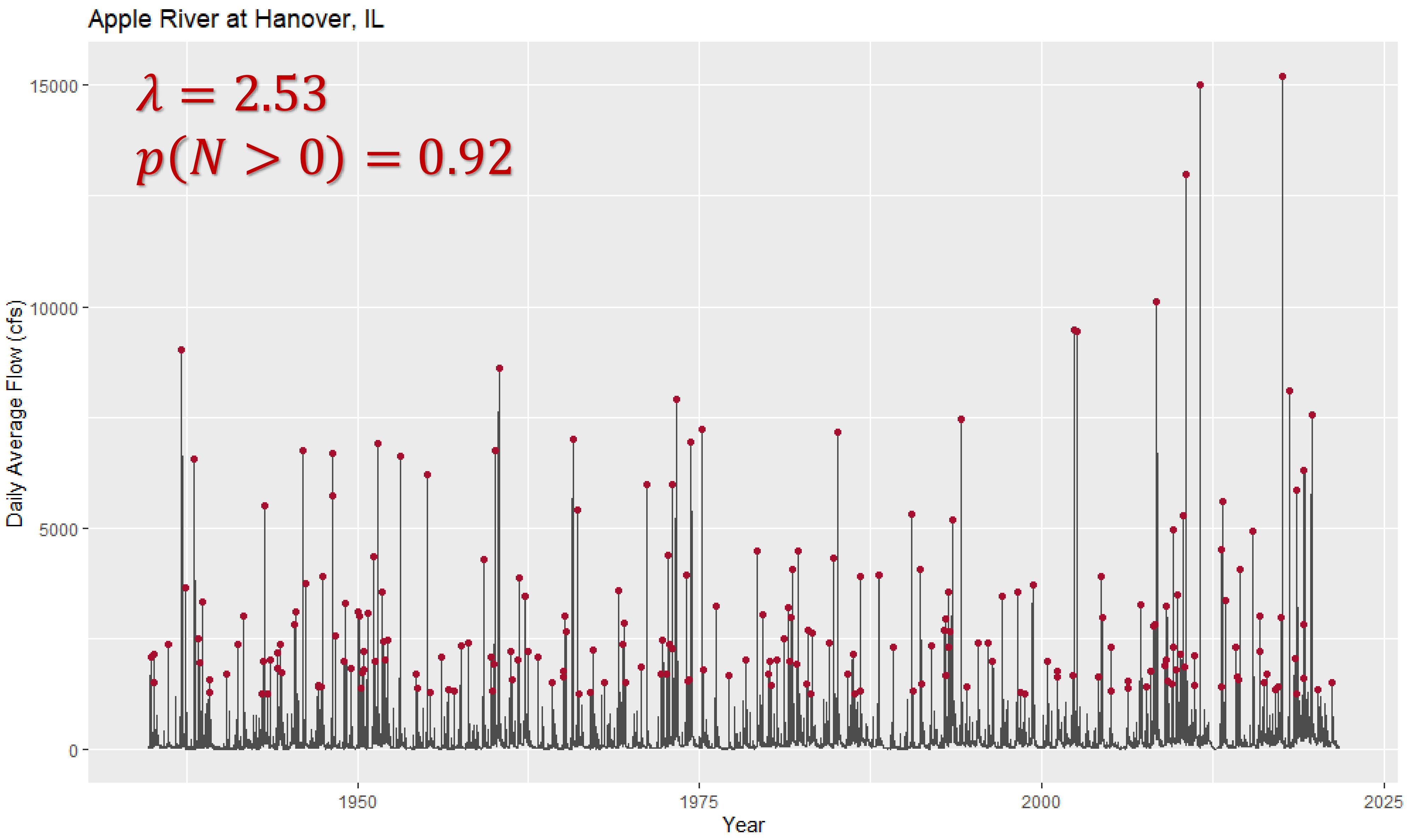

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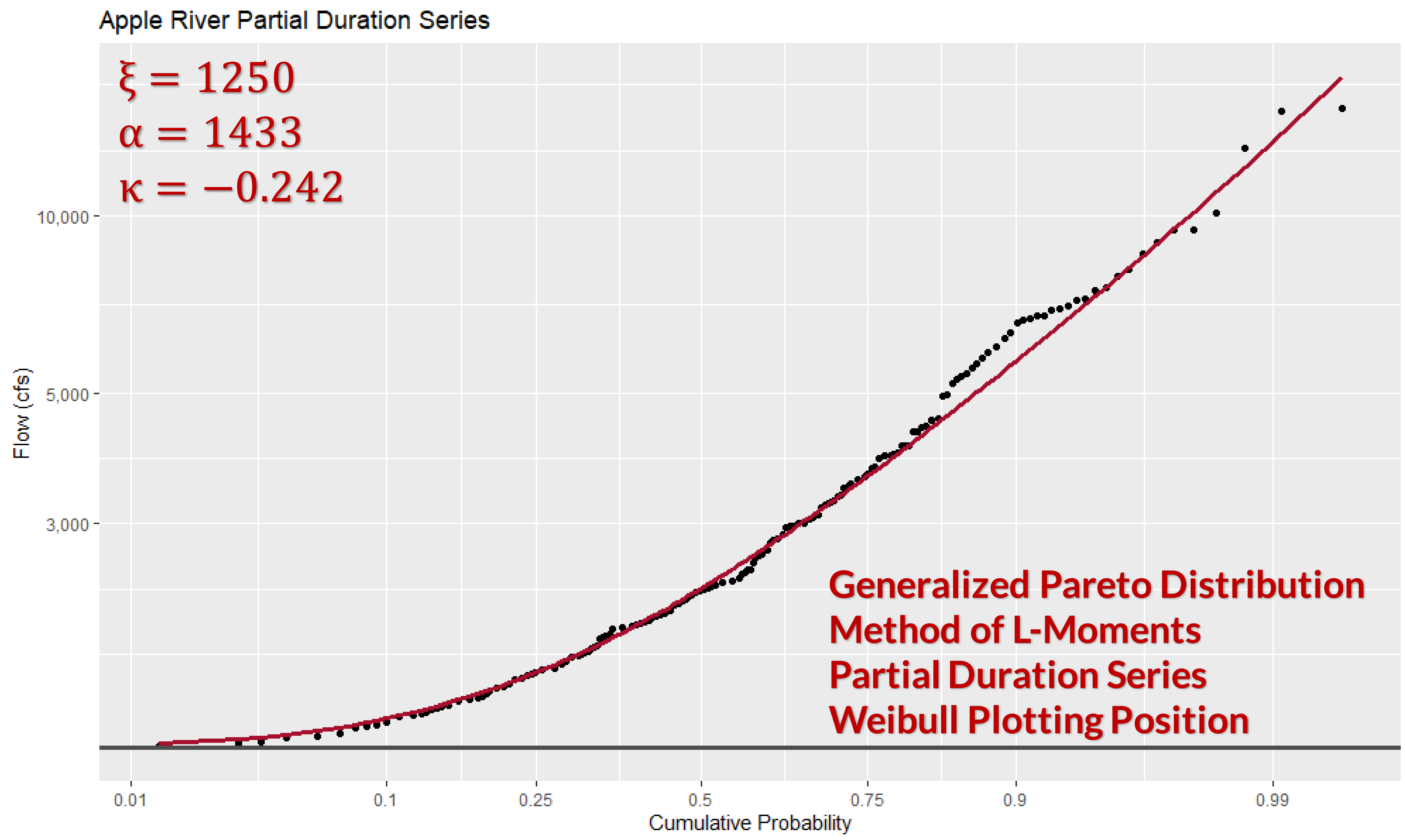

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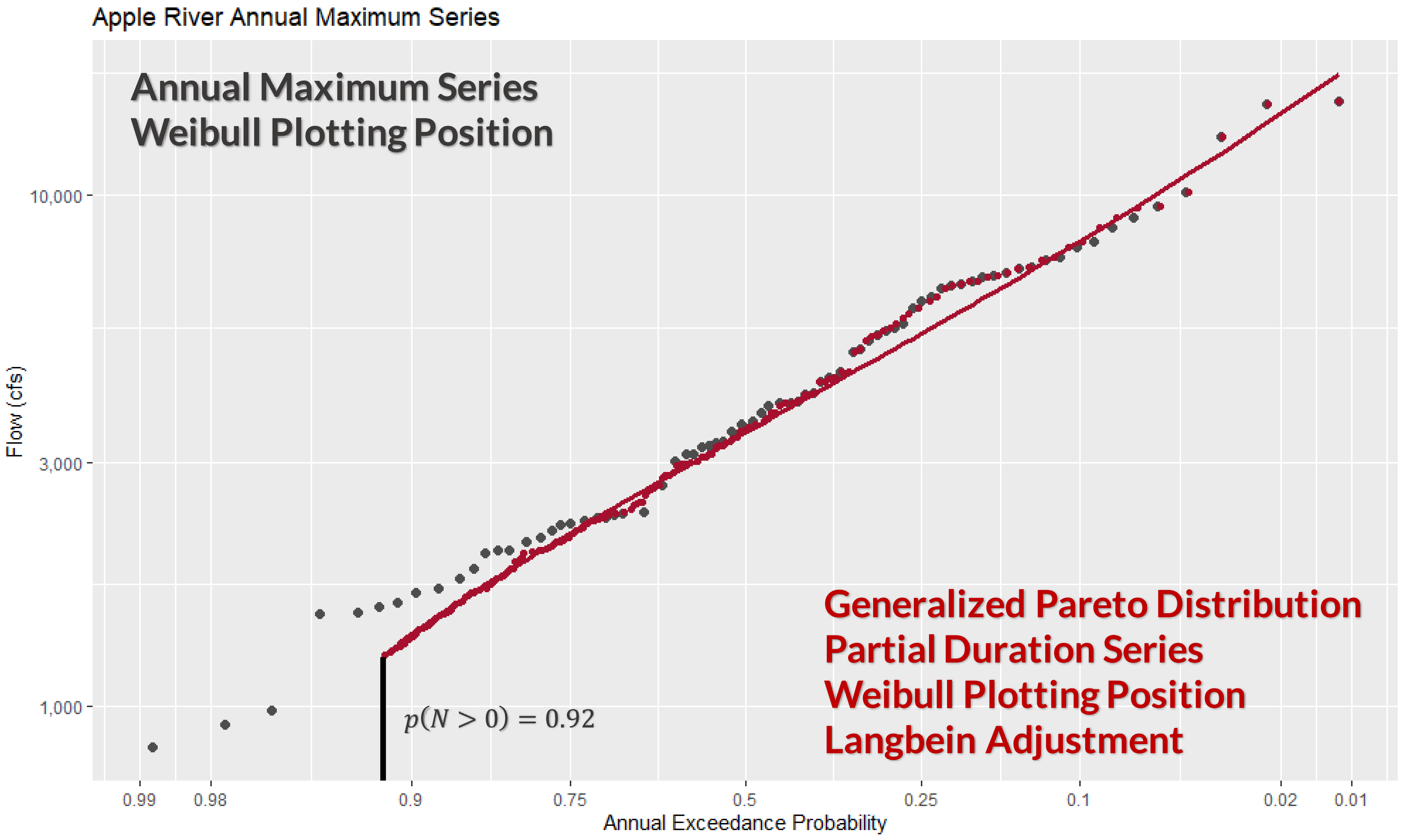

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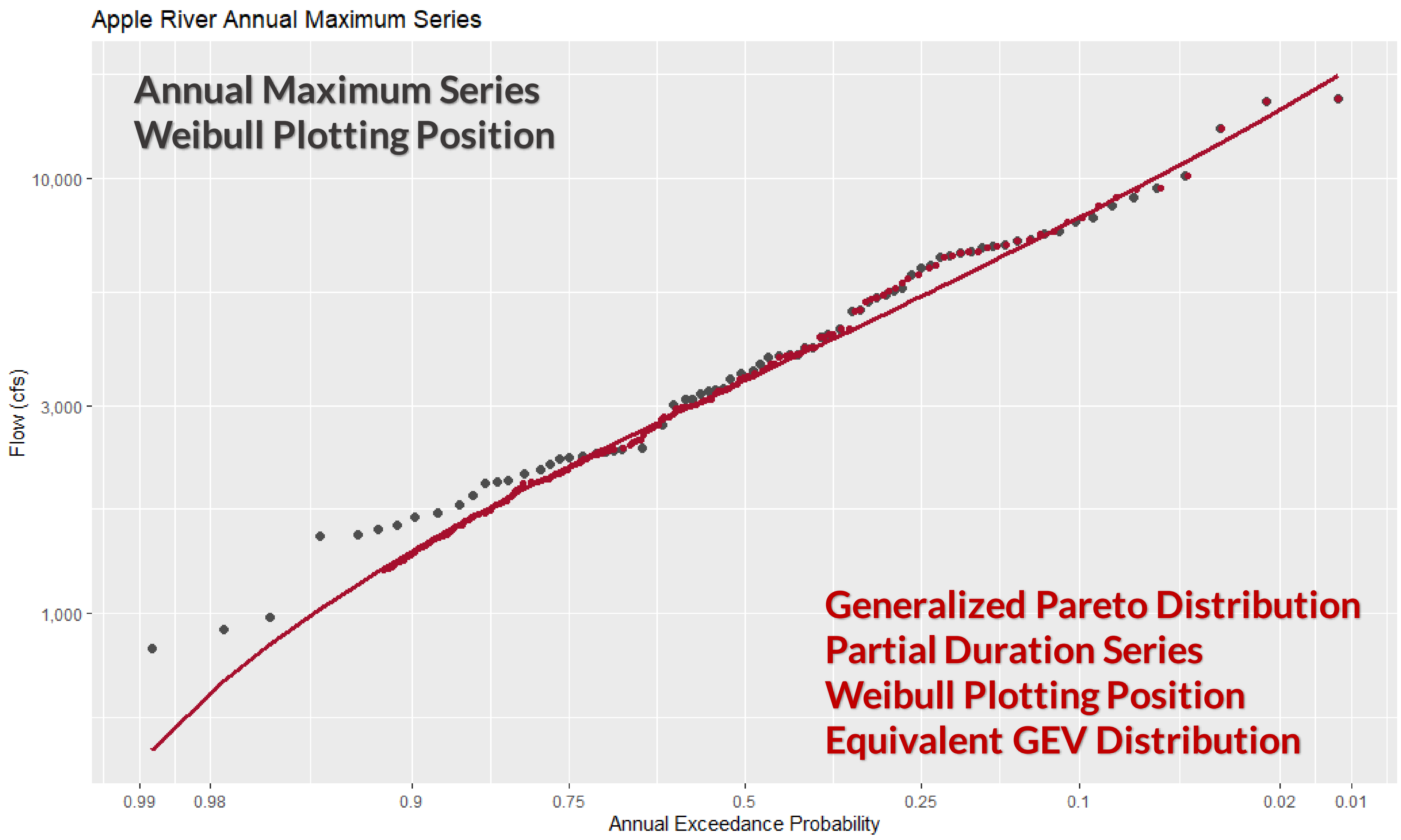

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Why PDS for Streamflow?

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Partial Duration Series Limitations

- Usually has to be done on daily average data
  - Instantaneous data have a short record
  - Requires approximation of peaks
- Takes extra work to ensure independence
- Some subjectivity involved
  - Choosing a threshold and separation criteria can be hard!

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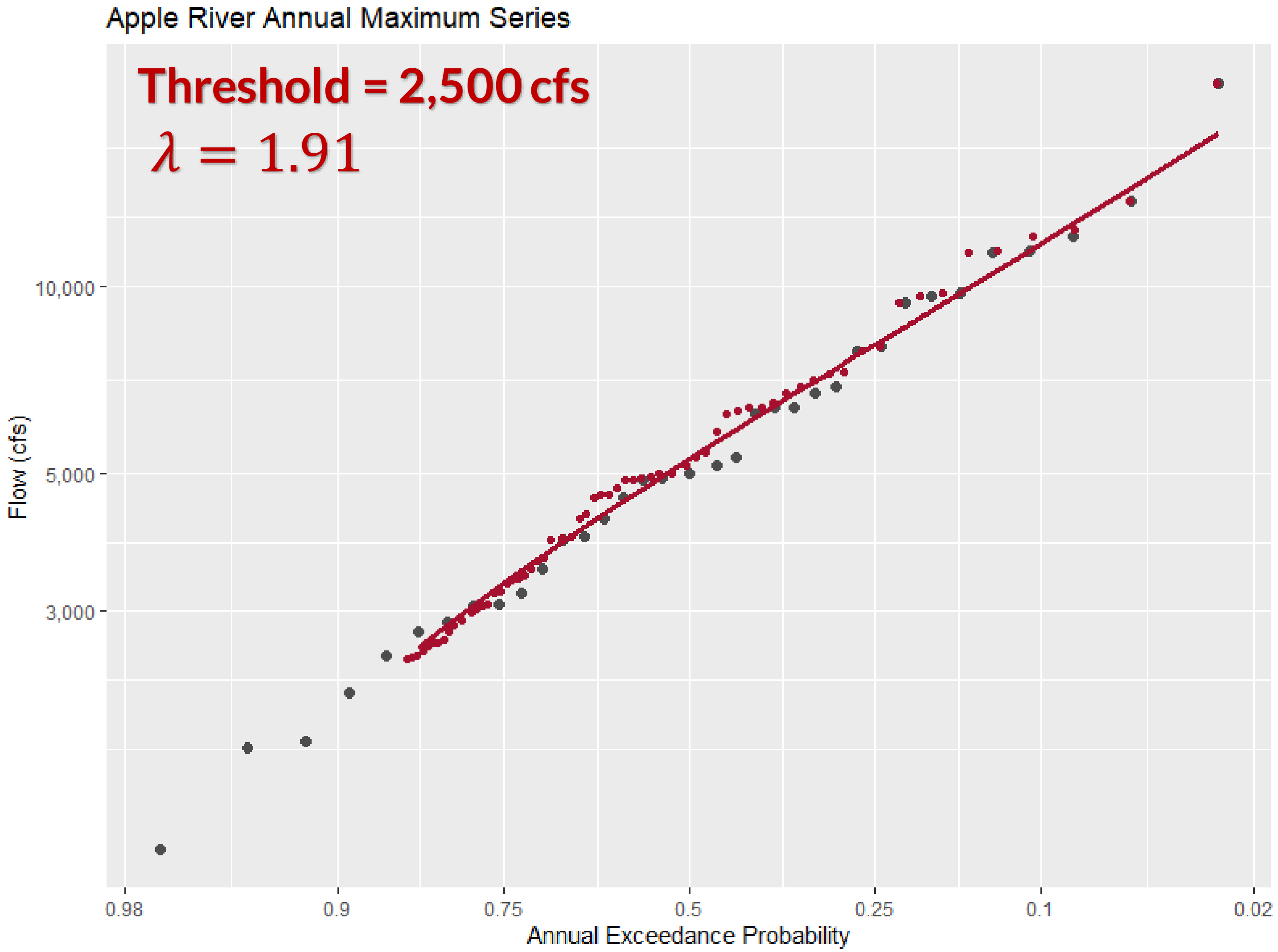
PDS with Instantaneous Data

- Shorter record, but
- No conversion



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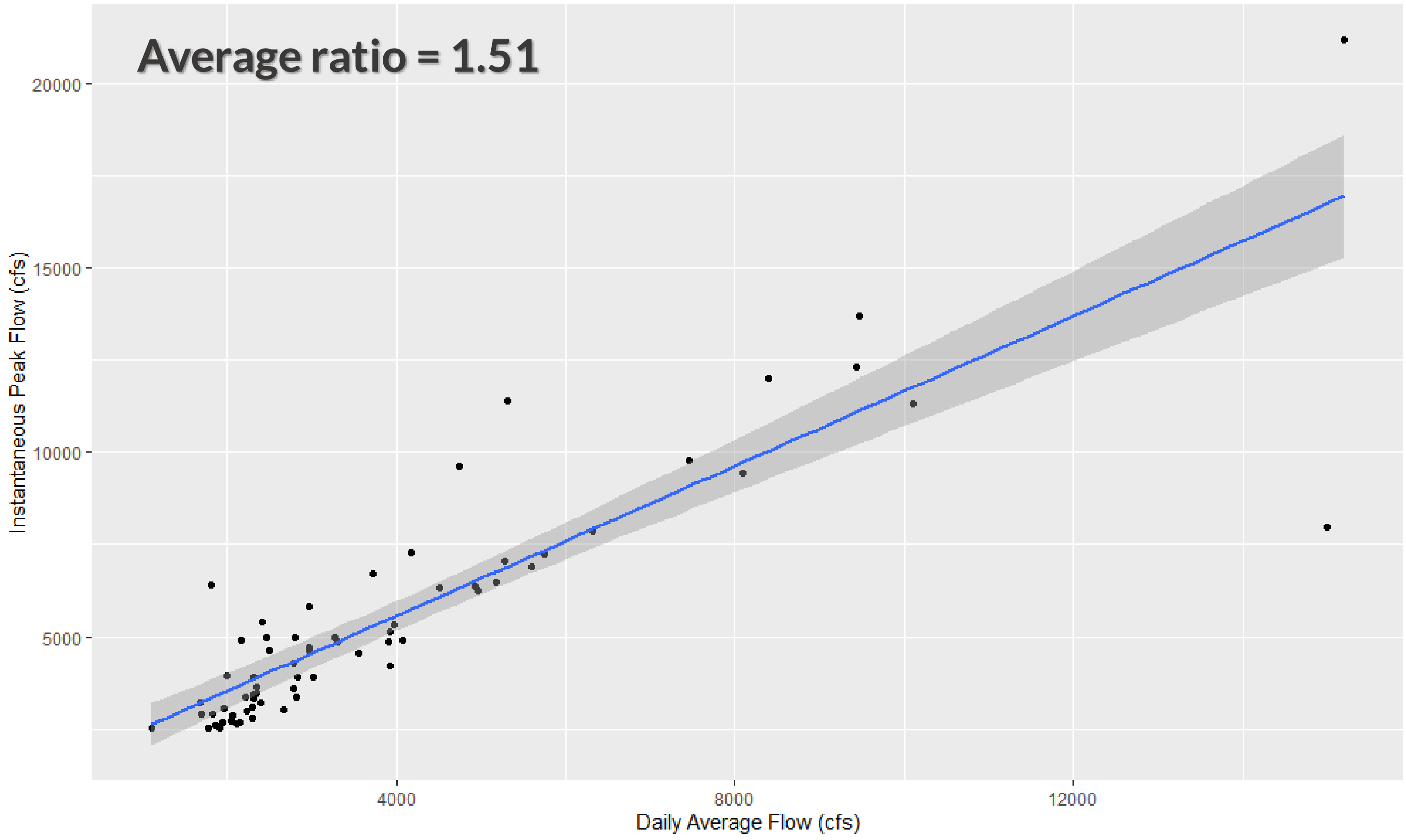
Approximating Peaks



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Partial Duration Strengths

- Sample sizes often larger than AMS
- More samples of floods
- Automatic rejection of small annual maxima
- Lower error in estimating rare quantiles in many circumstances
- Samples are easier to sub-divide
  - Good for mixed population analysis

Mixed Population

- Split the PDS by flood-causing mechanisms
- Bigger sample size for each part compared to splitting AMS

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Seasonal Analysis Example (Apple River)

Season 1

- Nov – Apr
- 1.75 events/year
- Mean event = 2,800 cfs
- Snow/synoptic meteorology

Season 2

- May – Oct
- 1.10 events/year
- Mean event = 3,500 cfs
- Convective meteorology

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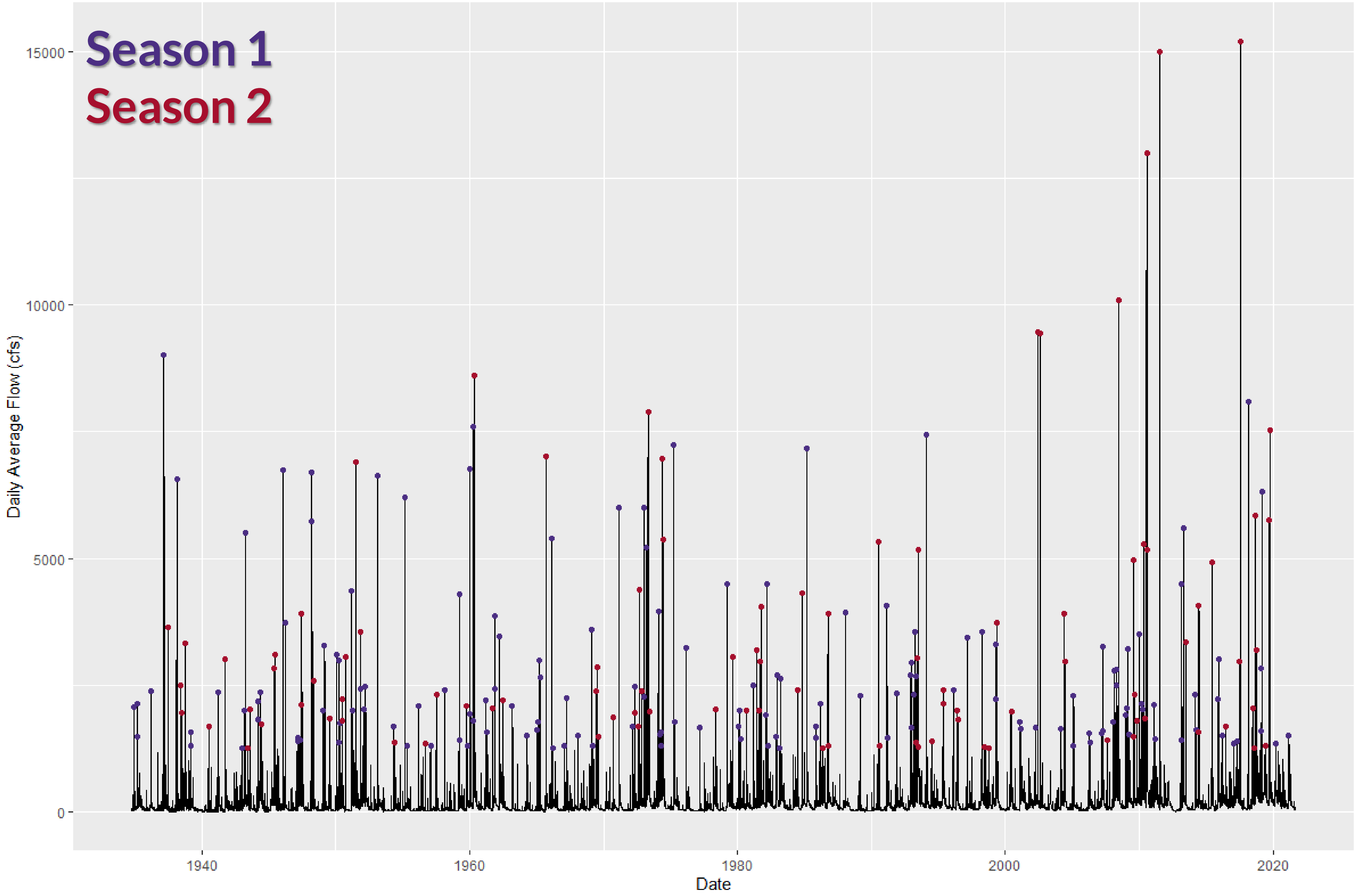

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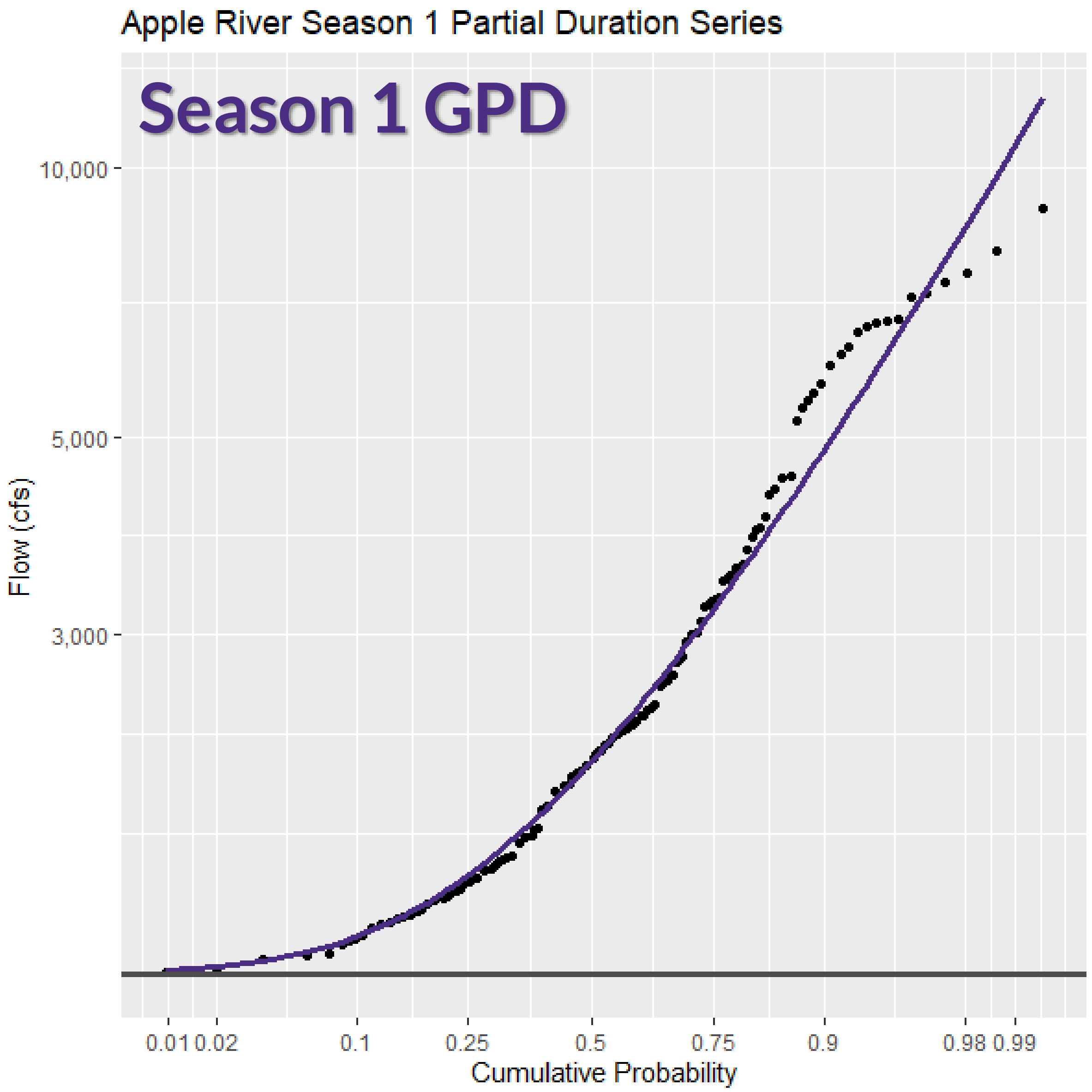

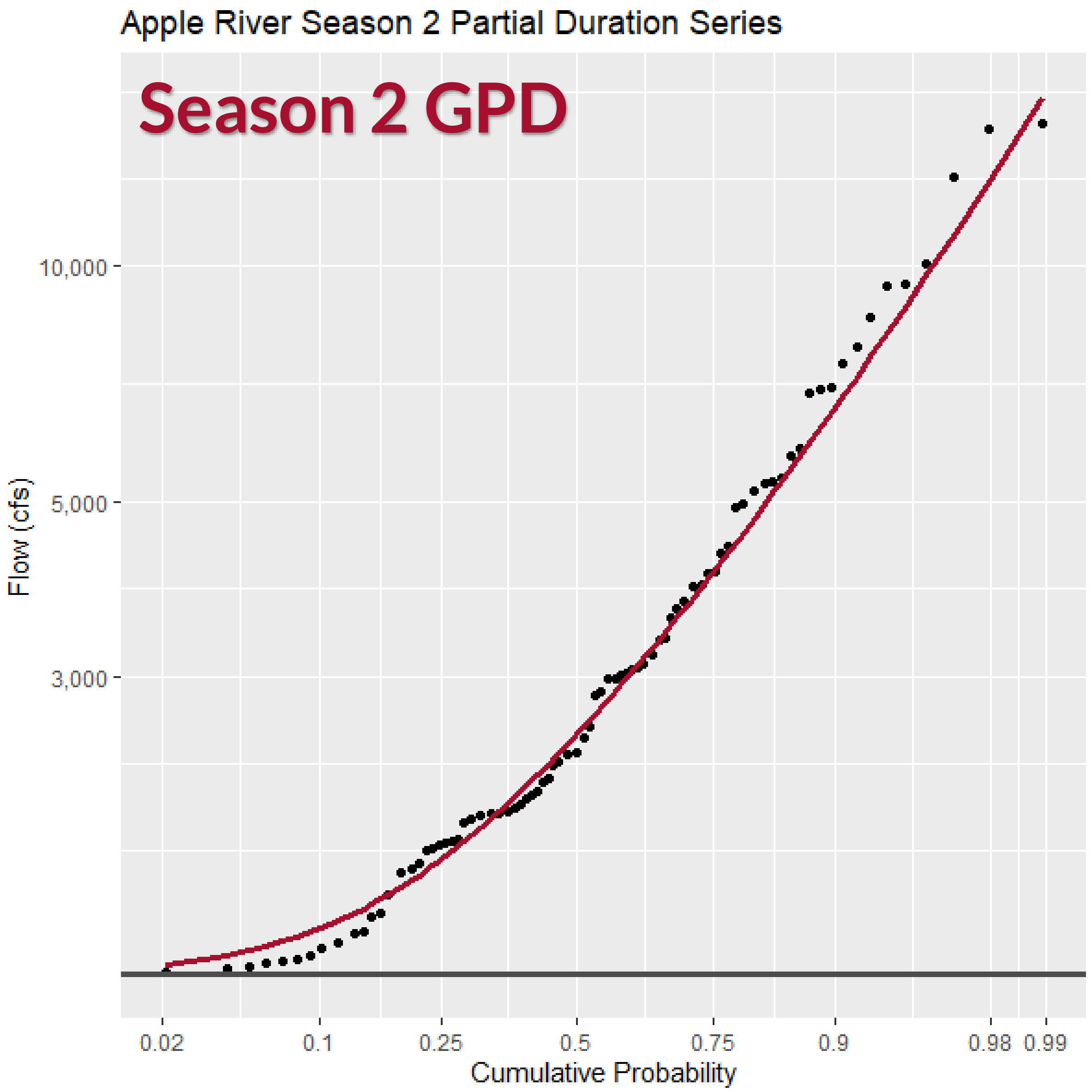

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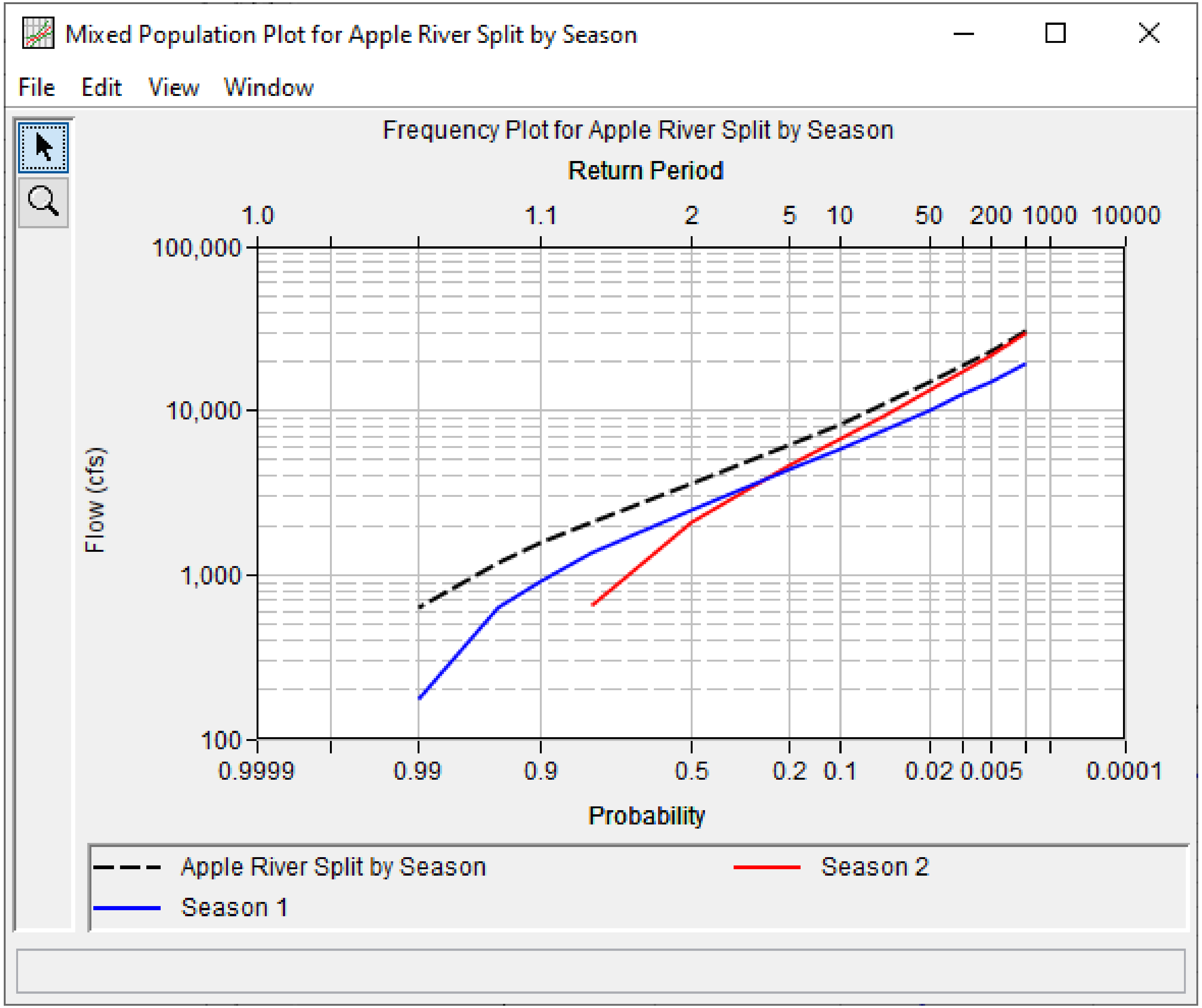
- Analytical annualization
- Compute mixed curve from equivalent GEV



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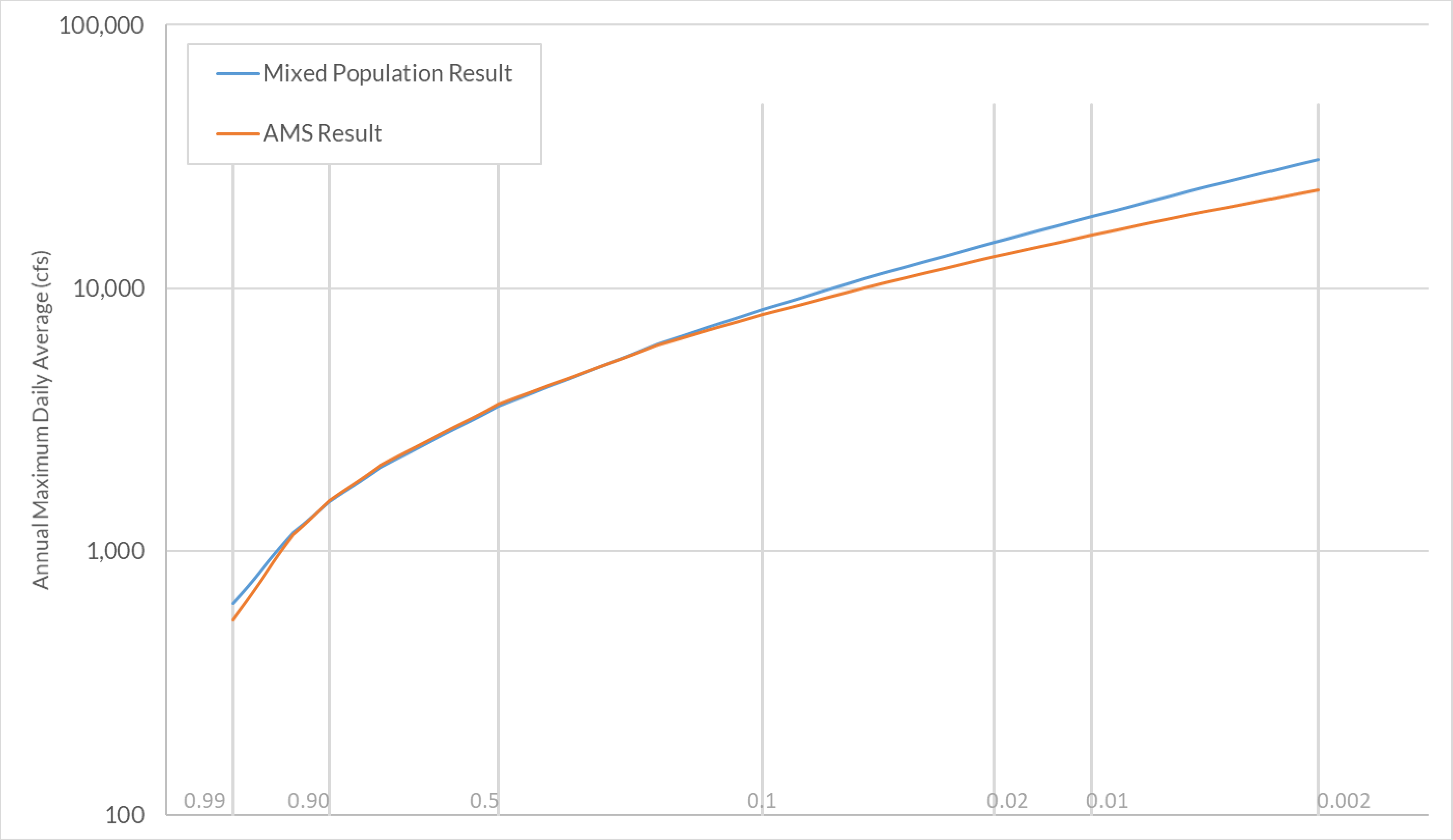

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Questions?

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