Diffusion Wave vs Full Momentum (SWE)

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Objectives

- Overview of the Diffusion Wave and Full Shallow Water Equations (SWE).
- Learn the positive and negative attributes of the Diffusion Wave Equations.
- Learn the positive attributes of the Full SWE.
- Understand the impacts through examples

Hydraulic Equations

- **Full Equations (SWE)**
  - **Mass Conservation** (Continuity)
  - **Full SWE Momentum Equation**
    - Gravity
    - Friction
    - Hydrostatic pressure
    - Acceleration (local and convective)
    - Turbulent eddy viscosity (optional)
    - Coriolis term (optional)
    - Wind Forces

- **Diffusion Wave Equation**
  - **Mass Conservation** (Continuity)
  - **Diffusion Form Momentum Equation**
    - Gravity
    - Friction
    - Hydrostatic pressure

2D SWE Momentum Equations

$$\frac{Du}{Dt} - fv = -g\frac{\partial H}{\partial x} + \nu_t \nabla^2 u - c_f u + \frac{\tau_{sx}}{\rho h}$$

$$\frac{Dv}{Dt} + fu = -g\frac{\partial H}{\partial y} + \nu_t \nabla^2 v - c_f v + \frac{\tau_{sy}}{\rho h}$$

2D Diff Wave form of Momentum Eqns.

$$\cancel{\frac{Du}{Dt}} - \cancel{fv} = -g\frac{\partial H}{\partial x} + \cancel{\mu \nabla^2 u} - c_f u + \cancel{\frac{\tau_{sx}}{\rho h}}$$

$$\cancel{\frac{Dv}{Dt}} + \cancel{fu} = -g\frac{\partial H}{\partial y} + \cancel{\mu \nabla^2 v} - c_f v + \cancel{\frac{\tau_{sy}}{\rho h}}$$

Diffusion Wave Positive Attributes

- Flow is mainly driven by **gravity** and **friction**
  - Good for steep to moderate sloping streams (S > 2 ft/mi)
  - Hydrographs that rise and fall slowly
- **Very Stable Computationally**
  - Can handle larger time step Courant C > 2 (C = 5 max)
- Good for computing **rough global estimates**, such as flood extent
- Good for assessing **rough effects of dam breaks**
- Good for assessing **interior areas due to levee breeches**
- Good for **quick estimations before a full momentum (SWE)** run
  - Often used to get model up and running stable before use SWE

Diffusion Wave Negative Attributes

- **Not as good for fast rising and falling flood waves** due to lack of acceleration terms (Dambreak or flash floods).
- **Not good for sharp contractions and expansions**.
  - Will generally under compute water surface upstream due to no contraction force
  - Will not accurately predict expansion zones and recirculation patterns
- **Can't handle tidal boundary conditions accurately**
  - No wave propagation up stream (This requires acceleration terms)
- **Not good for sharp bends** – can't predict any super elevation
- **Note good for predicting detailed velocity distributions** in channels or around objects.
- **Does not work well for mixed flow regimes and hydraulic jumps**

Full Momentum (SWE) Should be used in the following situations

- **Highly Dynamic Flood Waves -** Rapidly rising and falling flood waves (dam break, flash floods, etc..)
- **Abrupt Contractions and Expansions -** flow with high velocities, as well as flow approaching structures on an angle.
- **Flat Sloping River Systems:** Slopes less than 2 ft/mile
- **Detailed Velocities and Water Surface Elevations:** (natural channels and around structurers)
- **Mixed Flow Regime**: sub to supercritical flow transitions, and hydraulic jumps (super to subcritical)
- **Tidal boundary conditions** (wave propagation upstream)
- **Super elevation around bends**
- **General Wave Propagation**: If the user needs to model wave propagation due to rapidly opening or closing of gated structures, or wave run-up on a wall or around an object
- **Simulations influenced by turbulence, wind, or Coriolis effects**

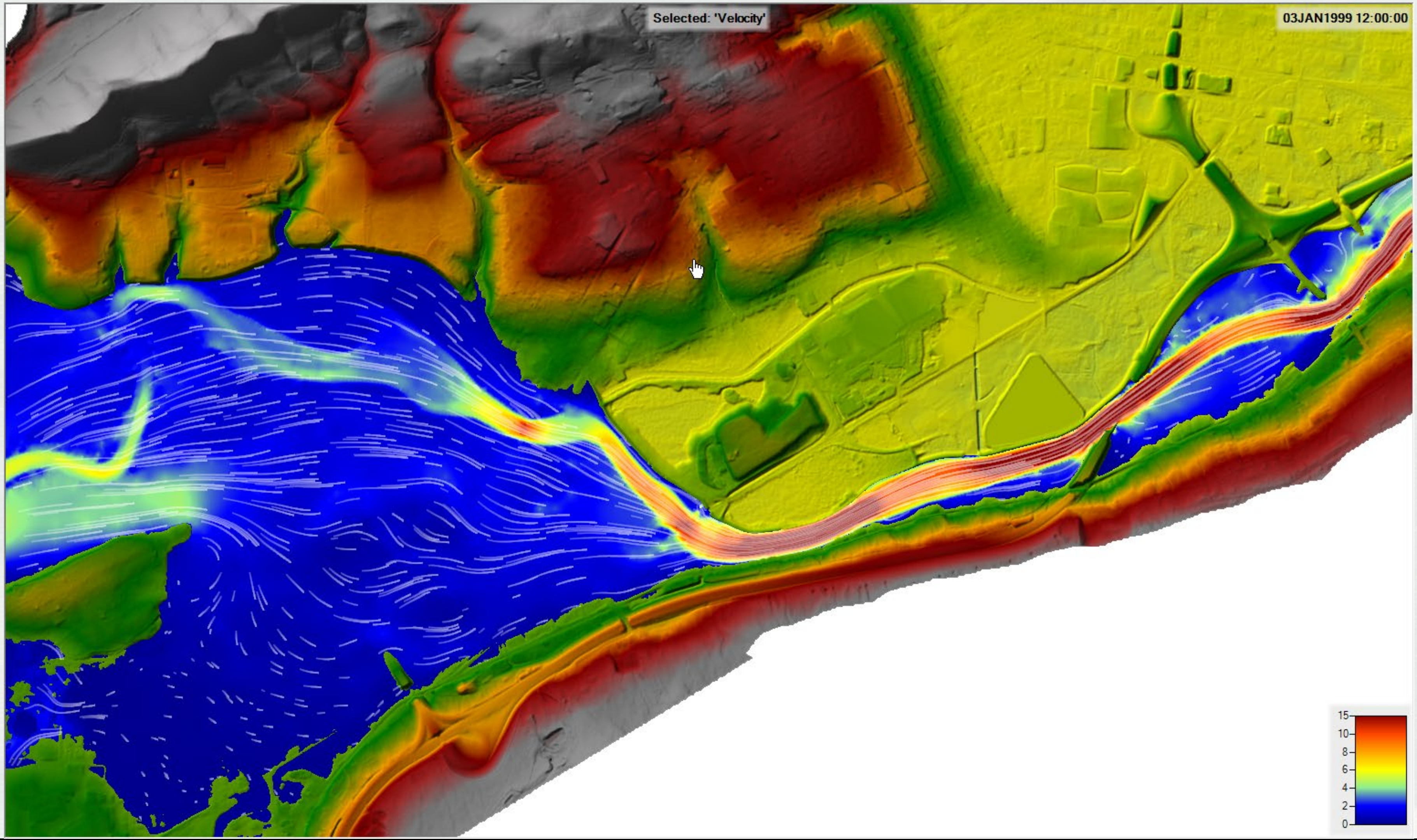
How Can you Test if Diffusion Wave is Adequate?

1. Create two Plans: Diffusion Wave and Full SWE.
2. Run both
3. Compare the Water surface, velocities, and flow rates
4. Where differences are significant, means you should be using the Full SWE.

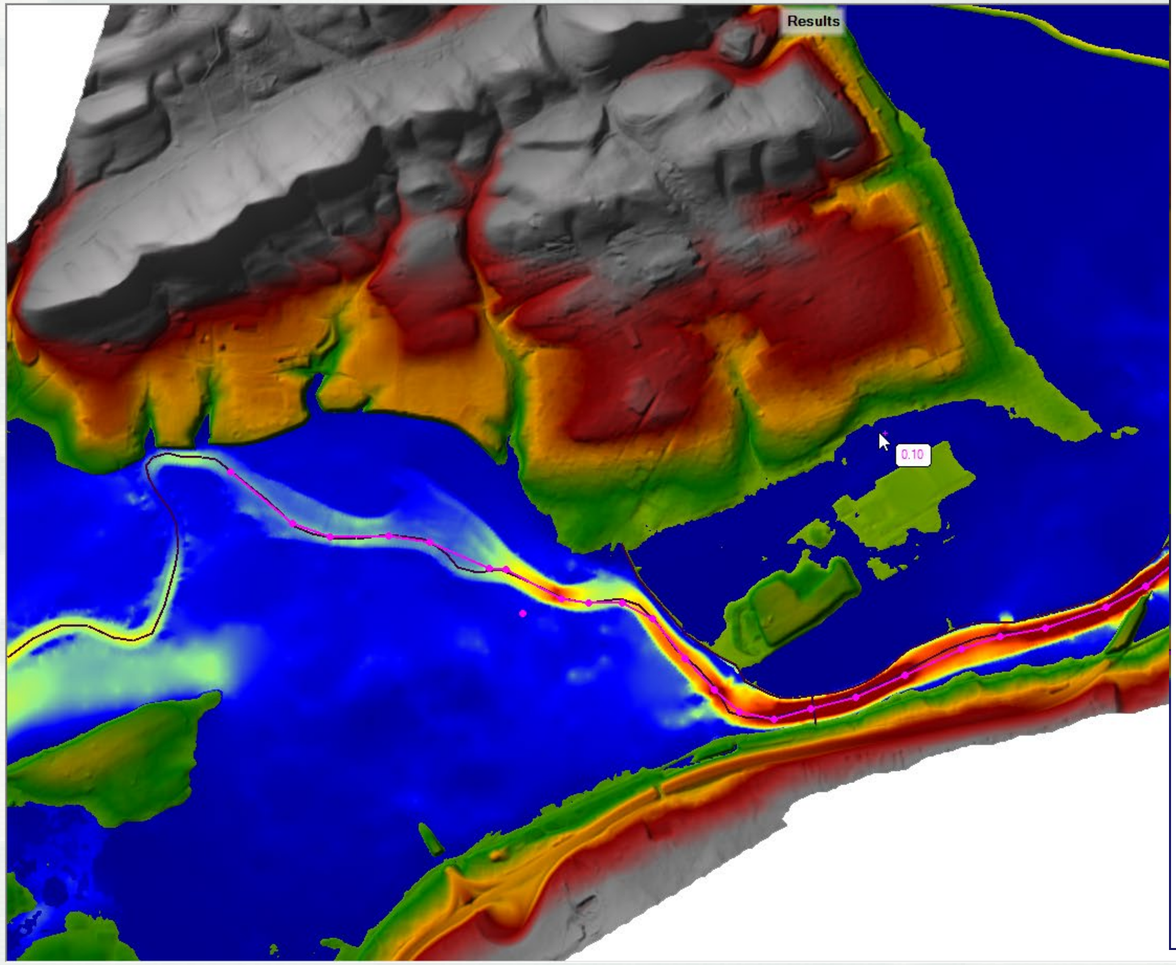
Example Applications for Diff Wave vs Full Momentum (SWE)

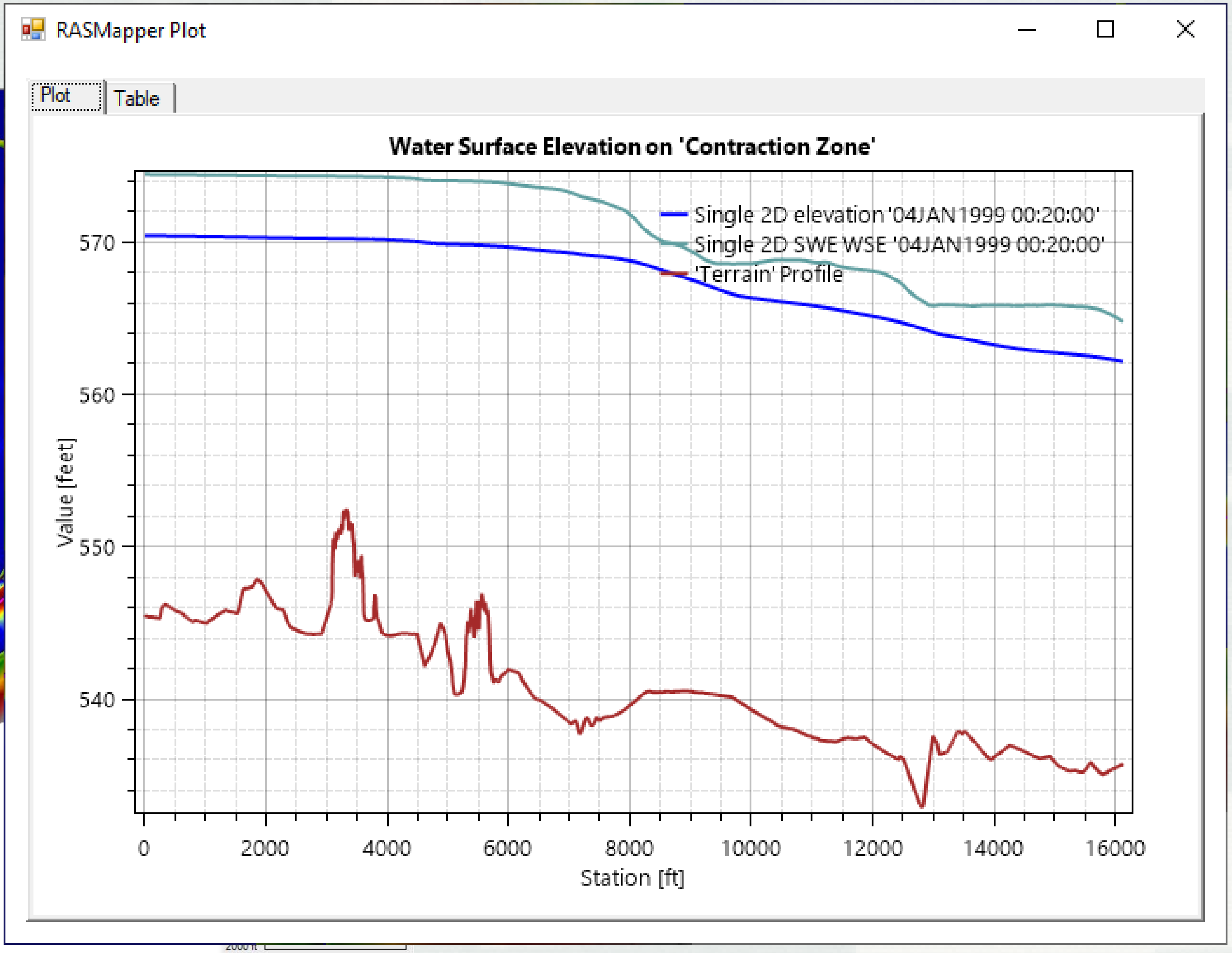
- Sharp contraction
- Dam break model run
- Tidal boundary condition

Sharp Contraction

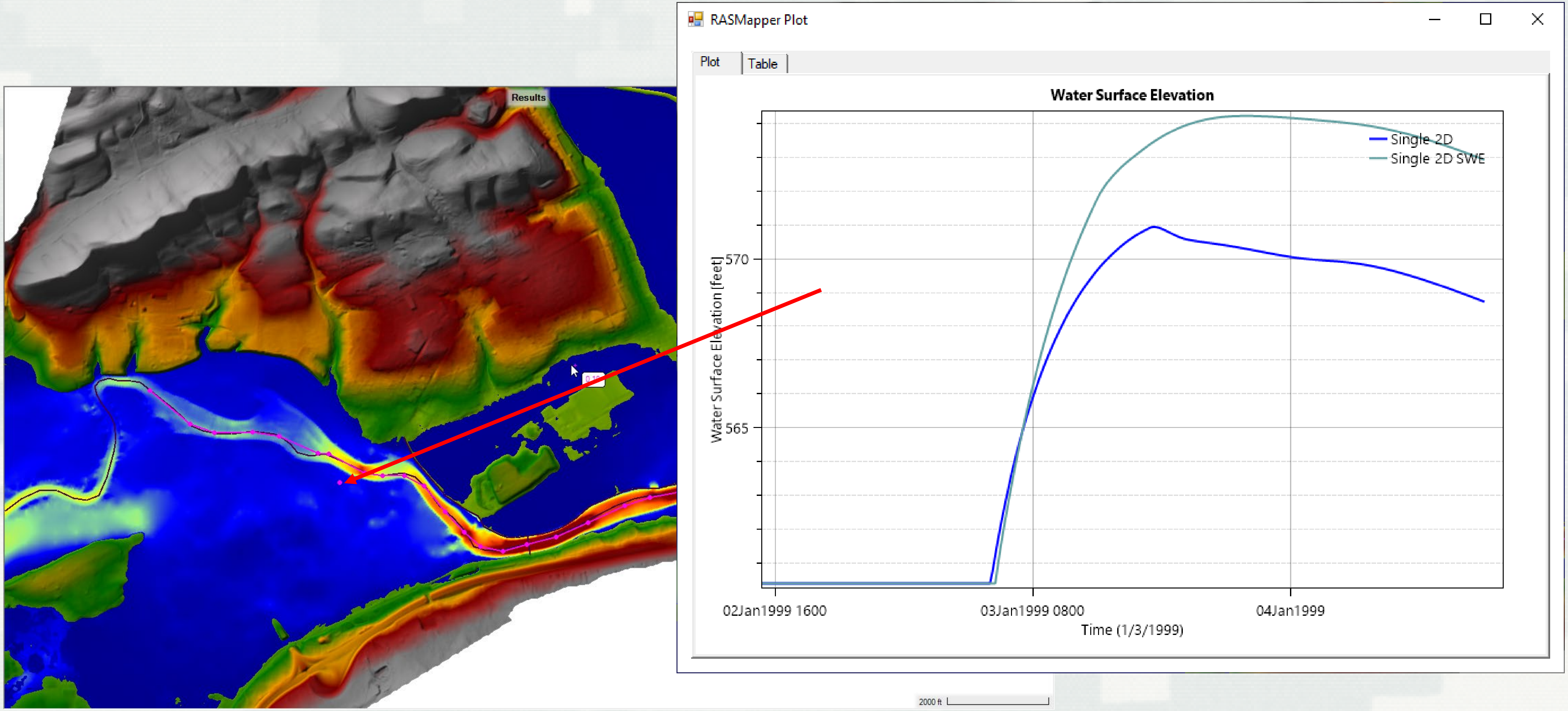


Sharp Contraction – WS Profiles





Sharp Contraction – WS Time Series

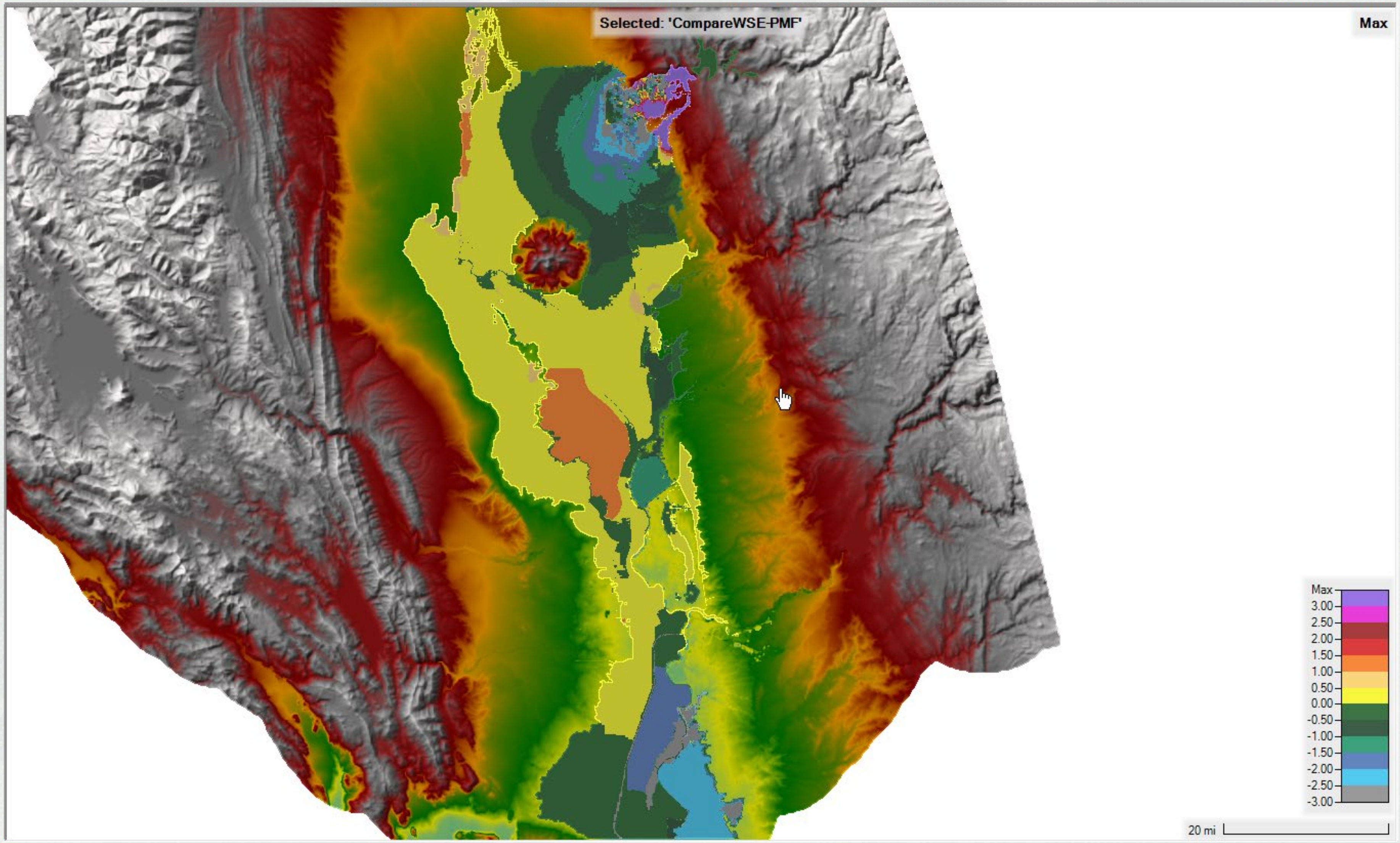


Inundation Map – SWE (Blue), Diff (Green to Red)

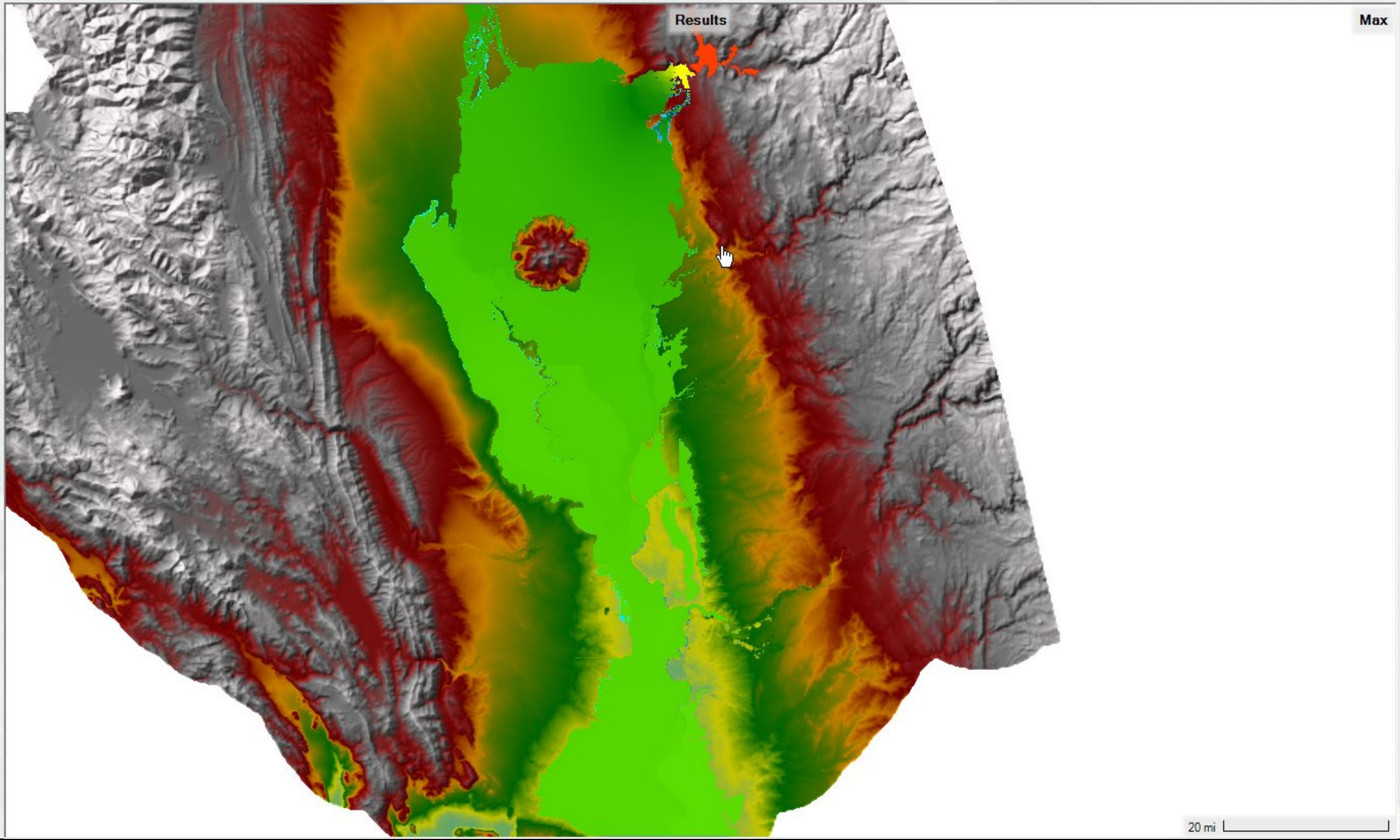
Dam Break – Oroville Dam, Sacramento Valley

Dam Break – Oroville Dam – WS Comparison

Difference = SWE WS – Diff Wave WS



Dam Break – Oroville Dam – WS Max



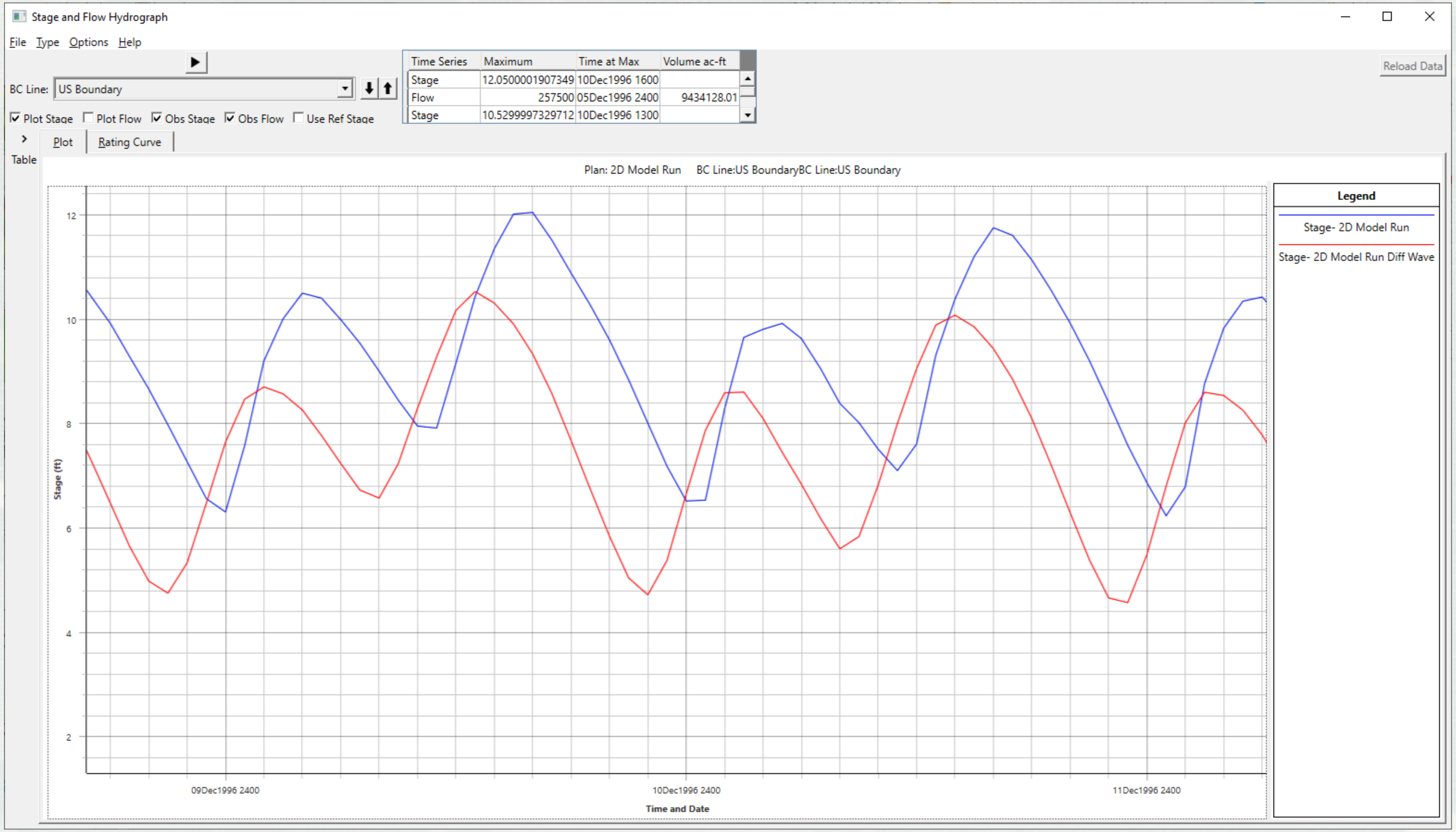
Tidal Boundary Condition – Lower Columbia

Tidal BC – Lower Columbia – WS Profiles

SWE (Dark Blue) and Diff Wave (light Blue)

Tidal BC – Lower Columbia – US Hydrograph

SWE (Blue) and Diff Wave (Red)



Questions?