HEC-RAS 2D Sediment Workshop: Sediment Transport Equations

Alex Sánchez, PhD
Stanford Gibson, PhD

Hydrologic Engineering Center,
Institute for Water Resources,
U.S. Army Corps of Engineers, U.S.A.

US Army Corps of Engineers

December 13, 2022

1

Mass Conservation

- Assuming a constant water density

$$\frac{\partial h}{\partial t} + \nabla \cdot (hV) = q$$

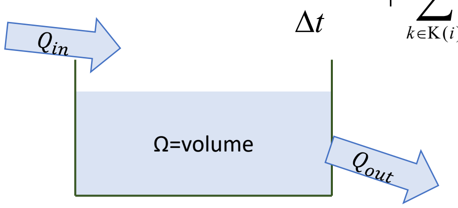
- Integrating over a computational cell

$$\frac{\partial}{\partial t}\iiint_{\Omega} d\Omega + \iint_{S} (V \cdot n) dS = Q$$

- Finite-Volume Discretization

$$\frac{\Omega_i^{n+1} - \Omega_i^n}{\Delta t} + \sum_{k \in K(i)} (V_k \cdot n_{ik}) A_k = Q_i$$

h : Water depth
q : Water souce/sink
Ω_i : Cell water volume
A_k : Face area
V_k : Face velocity
n_{ik} : Outward face-normal unit vector
Δt : Time step



Change in volume in a system balances with flow through boundaries

2

2

Momentum Conservation

- Momentum Equation (non-conservative form)

$$\underbrace{\frac{\partial \boldsymbol{V}}{\partial t}}_{\text{Temporal}} + \underbrace{(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}}_{\text{Advection}} + \underbrace{f_c \boldsymbol{k}\times\boldsymbol{V}}_{\text{Coriolis}} = \underbrace{-g\nabla z_s}_{\text{Pressure gradient}} + \underbrace{\frac{1}{h}\nabla\cdot(\boldsymbol{v}_t h\nabla\boldsymbol{V})}_{\text{Diffusion}} - \underbrace{\frac{\boldsymbol{\tau}_b}{\rho R}}_{\text{Bottom Friction}} + \underbrace{\frac{\boldsymbol{\tau}_s}{\rho h}}_{\text{Wind Stress}}$$

- From Newton's 2nd Law of motion
- Assumes constant water density, small vertical velocities, hydrostatic pressure, etc.
- Non-linear and a function of both velocity and water levels
- Continuity and Momentum Equations are the Shallow Water Equations or sometimes referred to as the "Full Momentum" equations in HEC-RAS

V : Velocity
z_s : Water level
g : Gravity
v_t : Turbulent eddy viscosity
h : Water depth
R : Hydraulic Radius
f_c : Coriolis Parameter
τ_b : Bed shear stress
τ_s : Surface stress

3

3

Acceleration and Total Derivative

- **Eulerian:** Frame of reference fixed in space and time

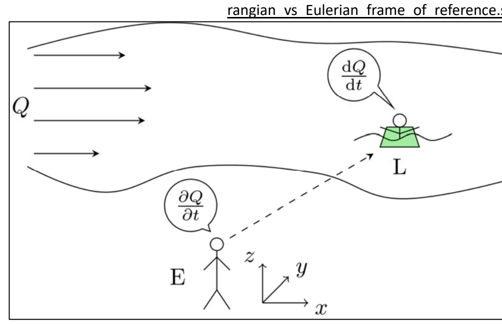
$$\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\cdot\nabla)\boldsymbol{V}$$

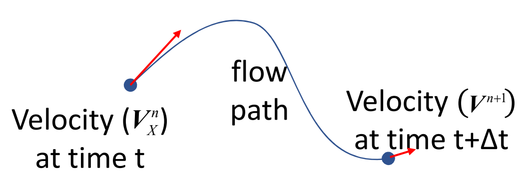
  - Easier to compute
  - Time-step restricted by Courant condition
- **Lagrangian:** Frame of reference moves with total derivative along flow path

$$\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}\cdot\nabla)\boldsymbol{V} = \frac{D\boldsymbol{V}}{Dt} = \frac{\boldsymbol{V}^{n+1} - \boldsymbol{V}_X^n}{\Delta t}$$

  - More expensive to compute
  - Allows larger time-steps

https://commons.wikimedia.org/wiki/File:Lagrangian_vs_Eulerian_frame_of_reference.svg





4

4

Diffusion of Momentum

- Non-conservative Formulation
  - Only option in Version 5.0.7 and earlier,
  - Optional in Version 6.0

$$\frac{D\boldsymbol{V}}{Dt} = -g\nabla z_s + \boxed{\nu_t \Delta \boldsymbol{V}} - \frac{\boldsymbol{\tau}_b}{\rho R}$$

- Conservative Formulation
  - Default in Version 6.0
  - Only option for Eulerian SWE solver

$$\frac{D\boldsymbol{V}}{Dt} = -g\nabla z_s + \boxed{\frac{1}{h}\nabla\cdot\left(\boldsymbol{\nu}_t h \nabla \boldsymbol{V}\right)} - \frac{\boldsymbol{\tau}_b}{\rho R}$$

$\Delta = \nabla^2$: Laplacian

u_N : Face-normal velocity

ν_t : Turbulent eddy viscosity

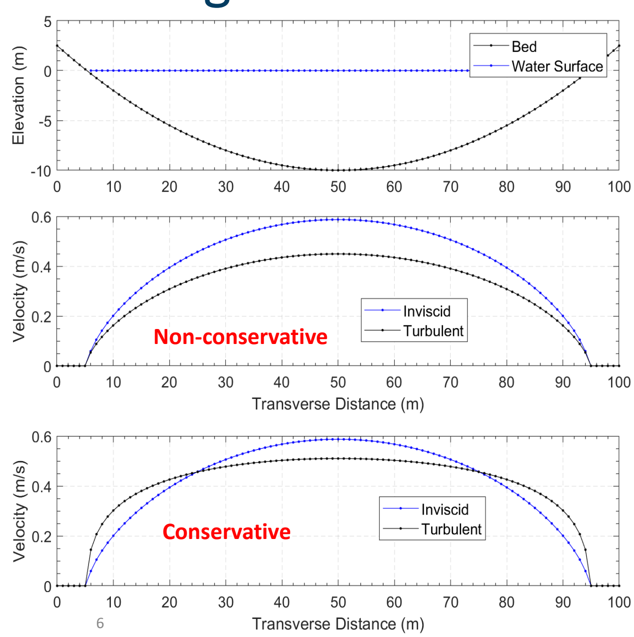
h : Water depth

c_f : Non-linear friction coefficient

5

5

Mixing Term Formulation Comparison



Bathymetry and water level

Produces a net dissipation

Decreases velocities in middle of channel but increases velocities near banks

6

6

Eddy Viscosity: Turbulence Model

- Old: Parabolic $\nu_t = D u_* h$
  - Versions 5.0.7 and earlier
  - Isotropic (same in all directions)
  - 1 parameter: mixing coefficient D
- New: Parabolic-Smagorisnky

$$\boldsymbol{\nu}_t = \boldsymbol{D} u_* h + (C_s \Delta)^2 |\bar{S}|$$

$$|\bar{S}| = \sqrt{2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2} \qquad \boldsymbol{D} = \begin{bmatrix} D_{xx} & 0 \\ 0 & D_{yy} \end{bmatrix}$$

$$D_{xx} = D_L \cos^2\theta + D_{,T} \sin^2\theta$$
$$D_{yy} = D_L \sin^2\theta + D_T \cos^2\theta$$

  - Default method in Version 6.0
  - Non-Isotropic (not the same in all directions)
  - 3 parameters: D_L, D_T, and C_s

u_* : Shear velocity
h : Water depth
D : Mixing coefficient
D_L : Longitudinal mixing coefficient
D_T : Transverse mixing coefficient
C_s : Smagorinsky coefficient

7

7

Bottom Friction

- Resisting force due to relative motion of fluid against the bed
- Bed Shear Stress

$$\boldsymbol{\tau}_b = \rho C_D |\boldsymbol{V}| \boldsymbol{V}$$

- Drag Coefficient

$$C_D = \frac{g n^2}{R^{1/3}}$$

- Friction coefficient

$$c_f = \frac{C_D}{R}|\boldsymbol{V}| = \frac{g n^2}{R^{4/3}}|\boldsymbol{V}|$$

n :Manning coefficient
ρ: water density
g: gravity acceleration constant
$|\boldsymbol{V}|$: velocity magnitude
R: hydraulic radius

8

8

Diffusive-Wave Approximation

- Ignoring the following terms

$$\underbrace{\cancel{\frac{\partial \boldsymbol{V}}{\partial t}}}_{\text{Temporal}} + \underbrace{\cancel{(\boldsymbol{V}\cdot\nabla)\boldsymbol{V}}}_{\text{Advection}} + \underbrace{\cancel{f_c \boldsymbol{k}\times\boldsymbol{V}}}_{\text{Coriolis}} = \underbrace{-g\nabla z_s}_{\text{Pressure gradient}} + \underbrace{\cancel{\frac{1}{h}\nabla\cdot\left(\boldsymbol{\nu}_t h\nabla\boldsymbol{V}\right)}}_{\text{Diffusion}} - \underbrace{\frac{\boldsymbol{\tau}_b}{\rho R}}_{\text{Bottom Friction}} + \underbrace{\cancel{\frac{\boldsymbol{\tau}_s}{\rho h}}}_{\text{Wind Stress}}$$

- Expanding and dividing both sides by the square of its norm leads to

$$\boldsymbol{V} = -\frac{\beta}{h}\nabla z_s \qquad \beta = \frac{R^{2/3}h}{n\left|\nabla z_s\right|^{1/2}}$$

- Inserting the above equation into the Continuity Equation leads to the Diffusion-Wave Equation (DWE)

$$\frac{\partial h}{\partial t} = \nabla\cdot\left(\beta\nabla z_s\right) + q$$

9

9

SWE vs. DWE

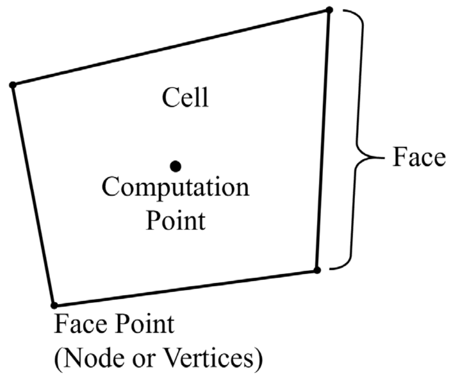
- Use SWE for:
  - Flows with dynamic changes in acceleration
  - Studies with important wave effects, tidal flows
  - Detail solution of flows around obstacles, bridges or bends
  - Simulations influenced by Coriolis, mixing, or wind
  - To obtain high-resolution and detailed flows
- Use DWE for:
  - Flow is mainly driven by gravity and friction
  - Fluid acceleration is monotonic and smooth, no waves
  - To compute approximate global estimates such as flood extent
  - To assess approximate effects of dam breaks
  - To assess interior areas due to levee breeches
  - For quick estimations or preliminary runs

10

10

Computational Mesh

- Mesh/grid can be unstructured
- Polygonal cells of up to 8 sides
- Cells must be concave
- Multiple 2D mesh can be run together or independently
- Grid Notation
  - Cells, Faces, Face Points (i.e. nodes or vertices), Computational Points, etc.
- State Variables
  - Cell Water levels
  - Face-normal Velocities



11

11

Numerical Methods

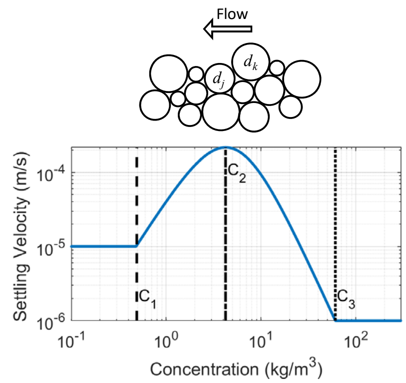
- Both DWE and SWE solvers are **Semi-implicit**
- Terms treated as:
  - Explicit: acceleration and diffusion terms
  - Semi-implicit: friction, flow divergence terms, and water level gradient
  - Fully-Implicit: pressure gradient term (for θ = 1)
- By treating the "fast" pressure gradient term implicitly, the time step limitation based on the wave celerity can be removed
- Both DWE and SWE use **Finite-Difference** and **Finite-Volume** Methods
- Time integration: **Finite-Difference**
- Continuity Equation: **Finite-Volume**
- Momentum Equation: **Finite-Difference** (no control volume)

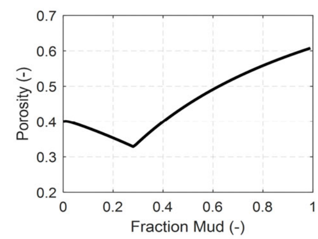
12

12

HEC-RAS 2D Sediment

- Key Features
  - Mixed cohesive and non-cohesive
  - Variable particle and bed bulk density
  - Quasi-3D effects
  - Hiding and exposure corrections
  - Bed-slope effects
  - Bed roughness predictors
  - Packing model for bed porosity
  - Avalanching
  - Hindered settling
  - Flocculation $f(C,T)$
  - Consolidation
  - Vertically varying cohesive properties
  - Sheet and splash erosion
  - Morphologic Acceleration
  - **Subgrid bathymetry, bed sorting, erosion, deposition, and hydrodynamics**

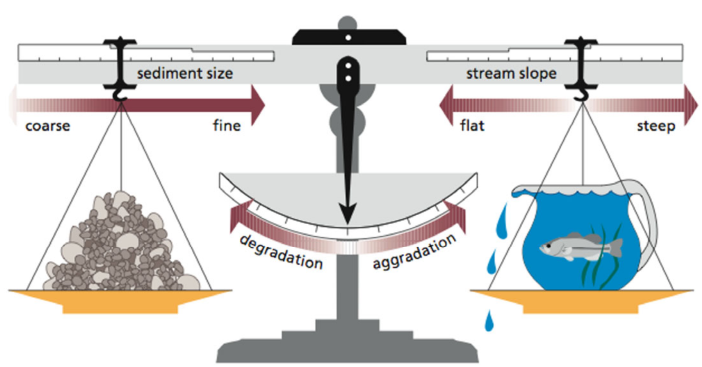




13

13

Lane’s Sediment Balance

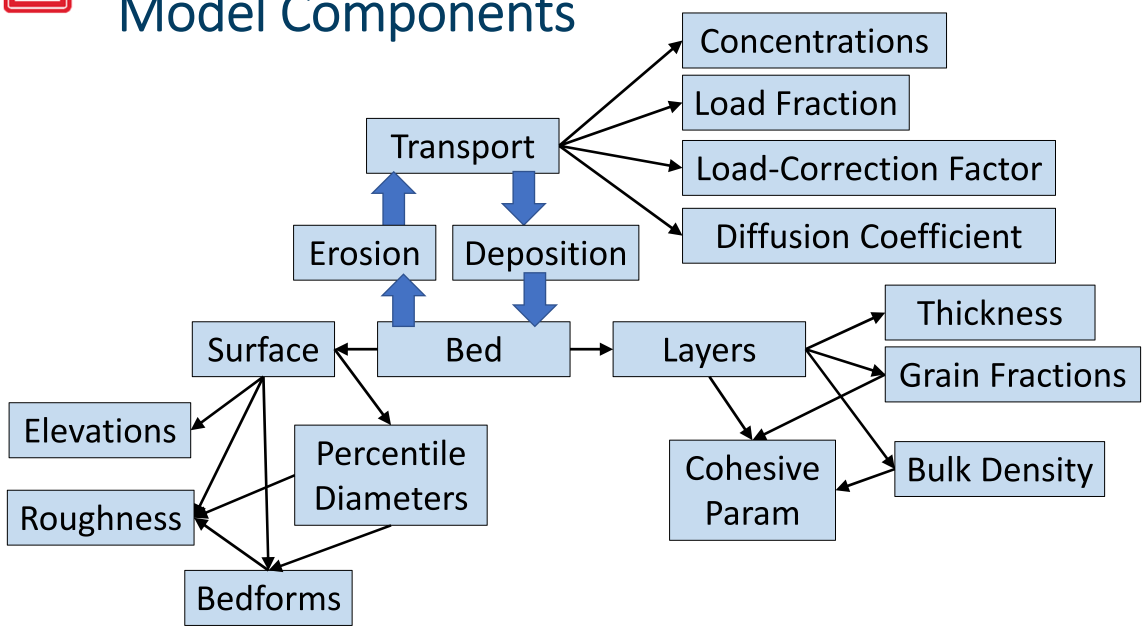


$$Q_s\, D_{50} \propto Q_w\, S$$

14

14

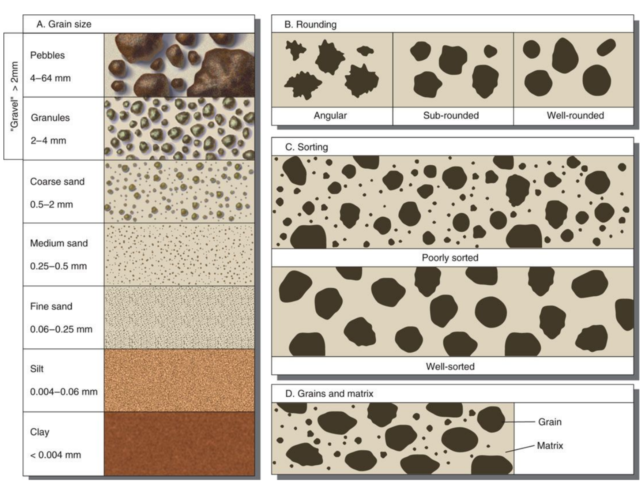
Model Components

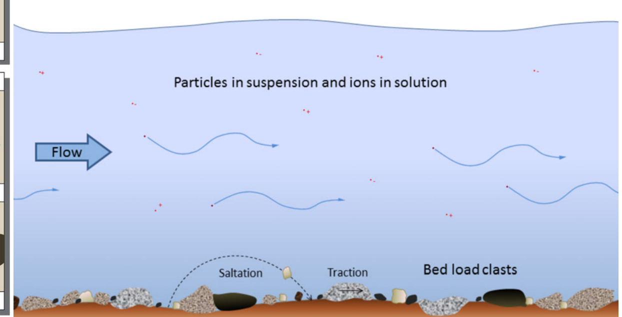


15

15

Sediment Transport

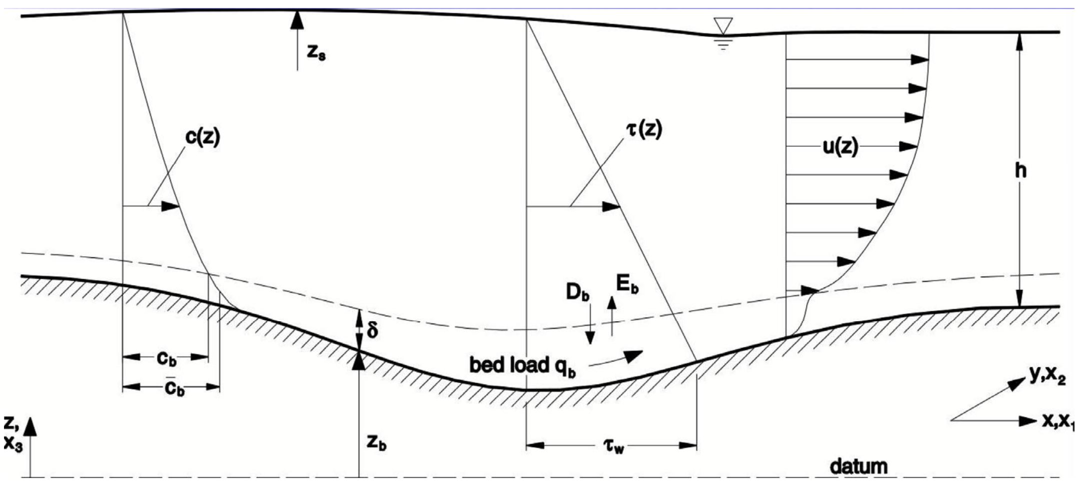




16

16

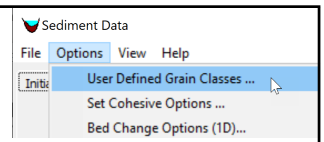
Sediment Transport



17

17

Grain Classes



Define Grain Classes and Sediment Properties

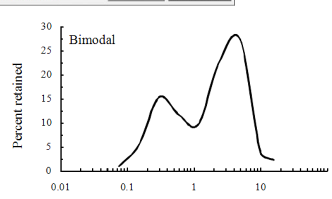
Sediment Diameters (mm)

Class	Label	Min	Max	Mean	SG	n	UW	Coh?	De
1	Clay	0.002	0.004	0.003	2.65	0.82	30	1	1
2	VFM	0.004	0.008	0.006	2.65	0.61	65	1	1
3	FM	0.008	0.016	0.011	2.65	0.61	65	1	1
4	MM	0.016	0.032	0.023	2.65	0.61	65	1	1
5	CM	0.032	0.0625	0.045	2.65	0.61	65	1	1
6	VFS	0.0625	0.125	0.088	2.65	0.44	93	0	1
7	FS	0.125	0.25	0.177	2.65	0.44	93	0	0.4
8	MS	0.25	0.5	0.354	2.65	0.44	93	0	0.09
9	CS	0.5	1	0.707	2.65	0.44	93	0	0.09
10	VCS	1	2	1.41	2.65	0.44	93	0	0.09
11	VFG	2	4	2.83	2.65	0.44	93	0	0.09
12	FG	4	8	5.66	2.65	0.44	93	0	0.09
13	MG	8	16	11.3	2.65	0.44	93	0	0.09
14	CG	16	32	22.6	2.65	0.44	93	0	0.09
15	VCG	32	64	45.3	2.65	0.44	93	0	0.09
16	SC	64	128	90.5	2.65	0.44	93	0	0.09
17	LC	128	256	181	2.65	0.44	93	0	0.09
18	SB	256	512	362	2.65	0.44	93	0	0.09
19	MB	512	1024	724	2.65	0.44	93	0	0.09
20	LB	1024	2048	1448	2.65	0.44	93	0	0.09

Currently Default

Density Method: Unit Weight (All Classes) | Defaults

☐ Enforce Adjacent-Non-Overlapping Grain Classes and Geometric Mean | OK | Cancel



- Select grain classes carefully and only use them when necessary
  - Do not add a grain class which represents 0.1% of the bed material
  - Each grain class adds a transport equation, bed change, and sorting equation
- Active grain grasses detected from initial bed gradations and boundary conditions

18

18

Concentration Definition

- **Depth-averaged**

$$\hat{C}_{tk} = \frac{1}{h}\int_0^h c_{tk}\,dz \qquad q_{tk} = \beta_t U h \hat{C}_{tk} \qquad \beta_t = \frac{1}{Uh\hat{C}_{tk}}\int_0^h u c_{tk}\,dz$$

  - Coefficient for transport (advection term)
  - **Used in HEC-RAS 1D**
- **Velocity weighted** (Einstein definition)

$$C_{tk} = \frac{1}{Uh}\int_0^h u c_{tk}\,dz \qquad q_{tk} = UhC_{tk}$$

  - Simpler formula for transport (advection term)
  - **Used in HEC-RAS 2D**
  - Coefficient in temporal term

19

19

Total-load Transport Equation

- Unsteady Advection-Diffusion Coefficient

$$\underbrace{\frac{\partial}{\partial t}\left(\frac{hC_{tk}}{\beta_{tk}}\right)}_{\text{Temporal or Storage}} + \underbrace{\nabla\cdot\left(h\mathbf{U}C_{tk}\right)}_{\text{Advection}} = \underbrace{\nabla\cdot\left(\boldsymbol{\varepsilon}_{tk} h \nabla C_{tk}\right)}_{\text{Diffusion}} + \underbrace{E_{tk}}_{\text{Erosion}} - \underbrace{D_{tk}}_{\text{Deposition}}$$

- Simulating total-load instead of separate bed- and suspended-loads reduces computational costs because it requires half as many transport equations

k : Grain class
h : Water depth
C_{tk} : Total-load concentration
β_{tk} : Total-load correction factor
$\mathbf{U}$: Current velocity
$\boldsymbol{\varepsilon}_{tk}$: Total-load diffusion coefficient
E_{tk} : Total-load erosion rate
D_{tk} : Total-load deposition rate

20

20

Erosion Rates

- Potential Erosion Rates

$$E^*_{tk} = \phi^D E^{*SS}_{tk} + \phi^W E^{*HF}_{tk}$$

- Actual Erosion Rates

$$E_{tk} = f_{1k} E^*_{tk}$$

E_{tk} → Total-load erosion rate
HF → Hydraulic flow
SS → Splash and sheet flow
ϕ^{SS} → Fraction of area with SS erosion
ϕ^{HF} → Fraction of area with CF erosion
E^{CF}_{tk} → Total-load CF erosion rate
E^{SS}_{tk} → Total-load SS erosion rate

21

21

Bed Change

- Fractional Bed Change

$$\underbrace{\rho_{sk}(1-\phi_b)\frac{\partial z_{bk}}{\partial t}}_{\text{Bed change}} = \underbrace{D_{tk}}_{\text{Deposition}} - \underbrace{E_{tk}}_{\text{Erosion}} + \underbrace{\nabla \cdot \left(\kappa_{bk} \left|q_{bk}\right| \nabla z_b\right)}_{\text{Bed slope term}}$$

- Total Bed Change

$$\frac{\partial z_b}{\partial t} = \sum_k \frac{\partial z_{bk}}{\partial t}$$

z_b : Bed elevation [L]
ρ_{sk} : Grain density [M/L^3]
ϕ_b : Bed Porosity [–]
D_{tk} : Fractional deposition rate [M/L^2/T]
E_{tk} : Fractional erosion rate [M/L^2/T]
κ_{bk} : Bedslope coefficient [–]

22

22

Bed Layering

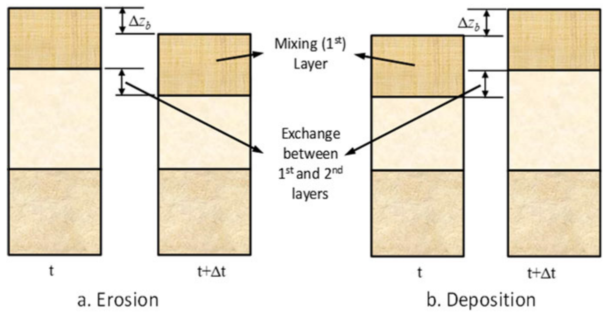
- Active Layer

$$\delta_1 = \max\left(f_{1,90} d_{90}, 0.5\Delta\right)$$

Δ : Bedform height

- Second Layer

$$\frac{\partial \delta_2}{\partial t} = \frac{\partial z_b}{\partial t} - \frac{\partial \delta_1}{\partial t}$$



23

23

Bed Sorting Model: Variable Bulk Density

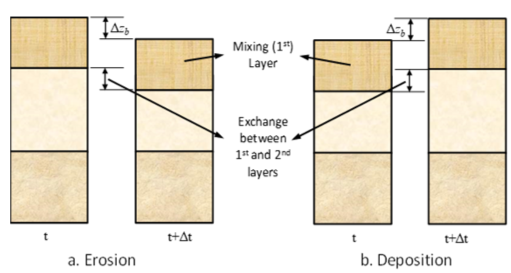
- Bed Mass Concentrations

$$\frac{\partial (m_{1k}\delta_1)}{\partial t} + m_{*k}\frac{\partial \delta_2}{\partial t} = \rho_{sk}(1-\phi_b)\frac{\partial z_{bk}}{\partial t} \qquad \frac{\partial (m_{2k}\delta_2)}{\partial t} = m_{*k}\frac{\partial \delta_2}{\partial t} \qquad m_{*k} = \begin{cases} m_{1k} & \text{for } \frac{\partial \delta_2}{\partial t} \geq 0 \\ m_{2k} & \text{otherwise} \end{cases}$$

- Dry Bulk Density and Mass Fractions

$$\rho_{d1} = \sum_k m_{1k} \qquad f_{1k} = \frac{m_{1k}}{\rho_{d1}}$$

- Advantages
  - ► Efficient
  - ► Preserves layer boundaries
  - ► Consistent



24

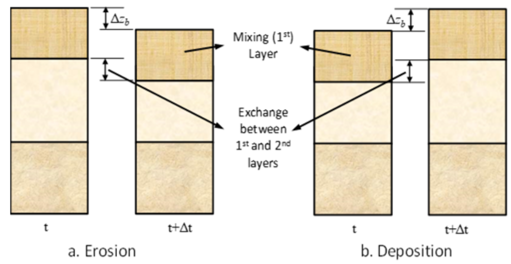
24

Bed Sorting Model: Constant Bulk Density

- Grain Fractions (by mass)

$$\frac{\partial (f_{1k}\delta_1)}{\partial t} + f_{*k}\frac{\partial \delta_2}{\partial t} = \frac{\partial z_{bk}}{\partial t} \qquad \frac{\partial (f_{2k}\delta_2)}{\partial t} = f_{*k}\frac{\partial \delta_2}{\partial t} \qquad f_{*k} = \begin{cases} f_{1k} & \text{for } \frac{\partial \delta_2}{\partial t} \geq 0 \\ f_{2k} & \text{otherwise} \end{cases}$$

- Advantages
  - ► Most efficient
  - ► Preserves layer boundaries
  - ► Consistent



Bed Sorting Model

- Model used automatically selected by engine based on the input
- Constant input bulk densities
  - ► Constant Bulk Density Model
- Variable input bulk densities or cohesives
  - ► Variable Bulk Density Model
- Constant bulk density model is simpler, faster, and more stable obviously because it has few

Fractions by Weight and Volume

- Fractions by weight/mass

$$m_k = f_k M \qquad M = \sum_k m_k$$

- Fractions by volume

$$v_k = \hat{f}_k V \qquad V = \sum_k v_k$$

- Conversion

$$m_k = \rho_{sk} v_k \qquad f_k = \frac{\rho_{sk} \hat{f}_k}{\sum_i \rho_{si} \hat{f}_i} \qquad \hat{f}_k = \frac{\rho_{sk}^{-1} f_k}{\sum_i \rho_{si}^{-1} f_i}$$

- Dry bulk density

$$\rho_d = (1-\phi) \sum_k \hat{f}_k \rho_{sk} = (1-\phi) \left(\sum_k \rho_{sk}^{-1} f_k \right)^{-1}$$

27

27

Bed Porosity (Newly Deposited Material)

- Noncohesives

$$\frac{\hat{f}_n}{1-\phi_n} = \sum_k \frac{\hat{f}_k}{1-\phi_k} \text{ for } k \in \text{noncohesive}$$

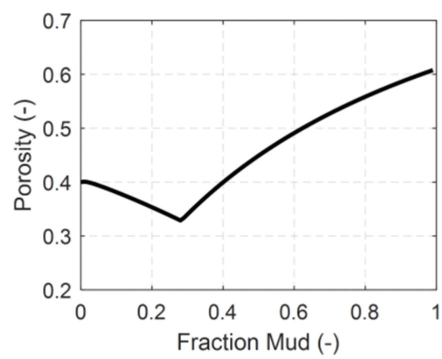
- Cohesives

$$\frac{\hat{f}_c}{1-\phi_c} = \sum_k \frac{\hat{f}_k}{1-\phi_k} \text{ for } k \in \text{noncohesive}$$

- Bimodal Mixture

$$\frac{1}{1-\phi_b} = (1-B) \frac{\hat{f}_n}{1-\phi_n} + \hat{f}_n B + \frac{\hat{f}_c}{1-\phi_c}$$

B : Packing parameter



28

28

Equilibrium vs. Non-Equilibrium Transport

$$\rho_s(1-\phi_b)\frac{\partial z_b}{\partial t}+\frac{\partial Q_{t*}}{\partial x}=0$$

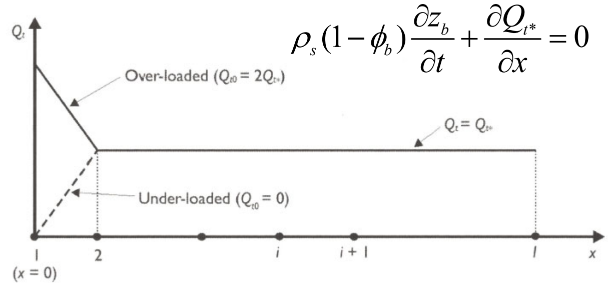


Figure 5.5 Sediment discharge profiles in equilibrium transport model.

From: Wu (2008)
Computational River
Dynamics

$$\frac{\partial(hC_t)}{\partial t}+\frac{\partial(hUC_t)}{\partial x}=\frac{\partial}{\partial x}\left(\varepsilon_t h\frac{\partial C_t}{\partial x}\right)+E_t-D_t$$

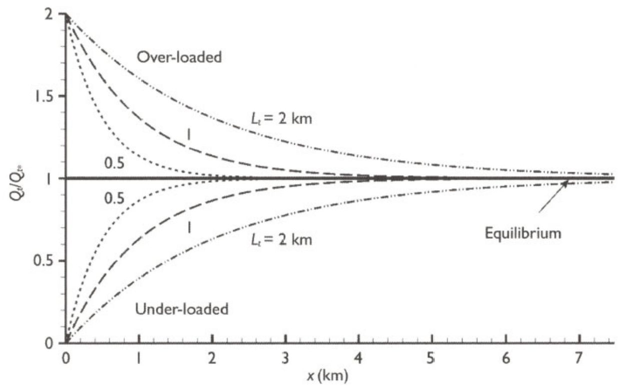


Figure 5.6 Sediment discharge profiles in non-equilibrium transport model.

29

Deriving Exner

- Starting with the transport and bed change equations and ignoring terms

$$\frac{\partial(hC_t)}{\partial t}+\frac{\partial(hUC_t)}{\partial x}=\frac{\partial}{\partial x}\left(\varepsilon_t h\frac{\partial C_t}{\partial x}\right)+E_t-D_t \qquad \rho_s(1-\phi_b)\frac{\partial z_b}{\partial t}=D_t-E_t$$

- Assuming the concentration is at equilibrium and defining the transport rate

$$C_t=C_{t*}=\frac{Q_{t*}}{hU}$$

- Combining the equations leads to the Exner equation

$$\rho_s(1-\phi_b)\frac{\partial z_b}{\partial t}+\frac{\partial Q_{t*}}{\partial x}=0$$

30

Equilibrium vs. Non-Equilibrium Transport

Equilibrium

- Pros
  - Simpler
  - Computationally efficient
  - Less parameters
  - Works well for large-scale models
- Cons
  - Less realistic
  - Breaks down for high-resolution models

Non-Equilibrium

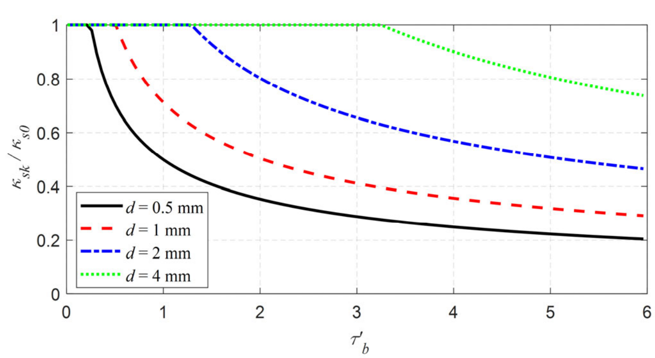
- Pros
  - Physically-based
  - Works for all scales
- Cons
  - Computationally expensive
  - More parameters to calibrate

Bed Slope Coefficient

- Accounts for down-slope movement of sediment during transport
- Larger grains more influenced by bed-slope
- Bed-slope effect less with increasing transport
- Tends to smooth bed elevations and thereby improve stability

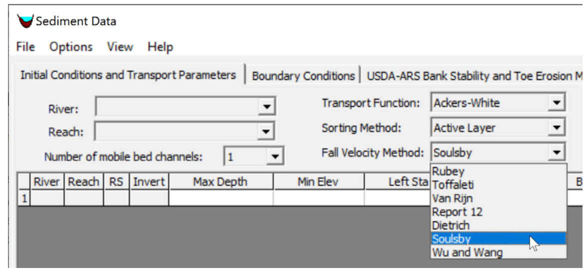
$$\kappa_{bk} = \kappa_{b0}\sqrt{\frac{\tau_{crk0}}{\max(\tau'_b, \tau_{crk0})}}$$

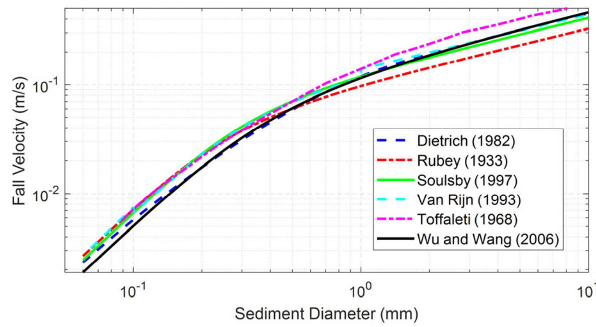
$$\kappa_{b0} \approx 0.1 - 0.5$$



Fall Velocity

- Represents free settling velocity
  - Does not account for hindered settling or flocculation
- Not a calibration parameter
- Use Soulsby except for specific conditions





33

33

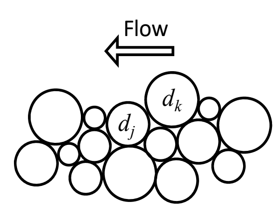
Hiding and Exposure

- Smaller particles "hidden" from flow and larger particles "exposed" to flow
- Hiding and exposure corrections
  - Incipient motion/mobility

$$\xi_k = \frac{\theta_{crk}}{\theta_{cr}} = \frac{\tau_{crk}}{\tau_{cr}} = \frac{U_{crk}^2}{U_{cr}^2}$$

  - Transport

$$\eta_k = \frac{q_k^*}{q^*} \qquad \eta_k \approx \frac{1}{\xi_k^a} \qquad a: \text{Coefficient}$$



- Correction on incipient motion done whenever possible unless formula does not have a threshold for incipient motion
- Because finer grain sizes represent a larger portion of the transported material, **applying** hiding/exposure corrections will generally **reduce** the overall transport
- **ALWAYS** use hiding and exposure corrections (default is off)

34

34

Hiding and Exposure Functions

- None (default) – Not recommended
- Ashida and Michiue (1971)

$$\xi_k = \begin{cases} \left[\dfrac{\log_{10}(19)}{\log_{10}(19 d_k / d_m)}\right]^2 & \text{for } d_k / d_m \geq 0.4 \\ d_m / d_k & \text{for } d_k / d_m < 0.4 \end{cases}$$

- Day (1980)
  - Developed for Ackers and White formula

$$\eta_k = \frac{1}{0.4(d_k / d_A)^{-0.5} + 0.6}$$

$$d_A = 1.6 d_{50} \left(\frac{d_{84}}{d_{16}}\right)^{-0.28}$$

- Egiazaroff (1965)
  - Used in AdH

$$\xi_k = \left[\frac{\log_{10}(19)}{\log_{10}(19 d_k / d_m)}\right]^2$$

- Hayashi (1980)

$$\xi_k = \begin{cases} \left[\dfrac{\log_{10}(8)}{\log_{10}(8 d_k / d_m)}\right]^2 & \text{for } d_k / d_m \geq 1 \\ d_m / d_k & \text{for } d_k / d_m < 1 \end{cases}$$

- Parker et al. (1982)

$$\xi_k = \left(\frac{d_k}{d_{50}}\right)^{-m}$$

35

35

Hiding and Exposure Functions

- Proffitt and Sutherland (1983)
  - Developed for Ackers and White

$$\eta_k = \begin{cases} 0.4 & \text{for } d_k / d_u \leq 0.075 \\ 0.53 \log_{10}(d_k / d_u) + 1 & \text{for } 0.075 < d_k / d_u \leq 3.7 \\ 1.3 & \text{for } d_k / d_u > 3.7 \end{cases}$$

- Wilcock and Crowe
  - Developed for Wilcock and Crowe

$$\xi_k = \frac{\tau_{r,k}}{\tau_{r,sm}} = \left(\frac{d_k}{d_{sm}}\right)^{b_k}$$

$$b_k = \frac{0.67}{1 + \exp\left(1.5 + \dfrac{d_k}{d_{sm}}\right)}$$

- Wu et al. (2000)
  - Developed for Wu et al.

$$\xi_k = \left(\frac{P_{ek}}{P_{hk}}\right)^{-m}$$

  - Most physically based
  - Uses entire grain size distribution
  - Hiding and exposure probabilities

$$P_{hk} = \sum_{j=1} \hat{f}_{1j} \frac{d_j}{d_k + d_j}$$

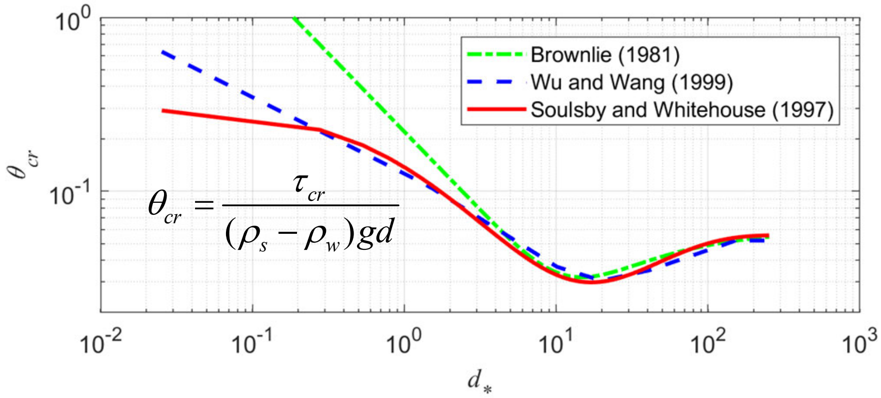
$$P_{ek} = \sum_{j=1} \hat{f}_{1j} \frac{d_k}{d_k + d_j}$$

36

36

Critical Shear for Erosion of Noncohesives

Shields Number



$$\theta_{cr} = \frac{\tau_{cr}}{(\rho_s - \rho_w) g d}$$

37

37

Critical Velocity for Erosion of Noncohesives

- Van Rijn (1984)

$$U_{cr} = \begin{cases} 0.19 d_{50}^{0.1} \log_{10}\left(\frac{4h}{d_{90}}\right) & \text{for } 0.1 \le d_{50} \le 0.5\,\text{mm} \\ 8.5 d_{50}^{0.6} \log_{10}\left(\frac{4h}{d_{90}}\right) & \text{for } 0.5 \le d_{50} \le 2.0\,\text{mm} \end{cases}$$

- Yang (1973)

$$\frac{U_{cr}}{\omega_{sd}} = \begin{cases} \dfrac{2.5}{\log_{10}\left(\frac{u_* d}{\nu}\right) - 0.06} + 0.66 & \text{for } 1.2 < \frac{u_* d}{\nu} < 70 \\ 2.05 & \text{for } \frac{u_* d}{\nu} \ge 70 \end{cases}$$

38

38

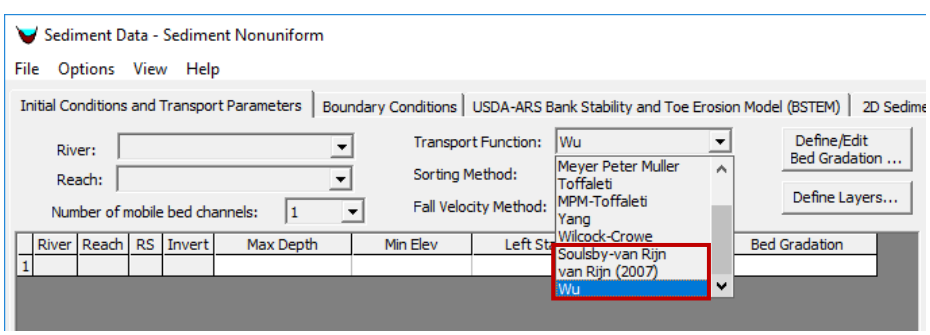
Transport Potential Formulas

- Most important model setting by far
- Available formulas
  - Ackers and White (1973)
  - Engelund-Hansen (1967)
  - Lausen-Copeland (1989)
  - Meyer-Peter-Muller (1948)
  - Soulsby-van Rijn (1997)
  - Toffaleti (1968)
  - Van Rijn (1984ab, 2007ab)
  - Wilcock and Crowe (2003)
  - Wu et al. (2000)
  - Yang (1984)

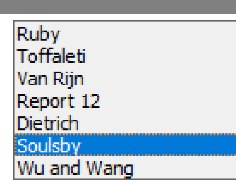
39

39

Transport Function



Fall Velocity Methods:



40

40

Ackers and White (1973)

- Total-load Formula
- Excess mobility based on stream power Fractional load formulation adopted for nonuniform sediments by Day (1980) and Proffitt and Sutherland (1983) for nonuniform sediments
- Works well for nonuniform sands and gravels

$$q_{tk}^{*} = \rho_w g h U X_{tk}^{*} \qquad \frac{X_{tk}^{*} h \rho_w}{d_k \rho_{sk}} \left(\frac{u_*}{U} \right)^n = \Lambda \left(\frac{F_{grk}}{A_c} - 1 \right)^m$$

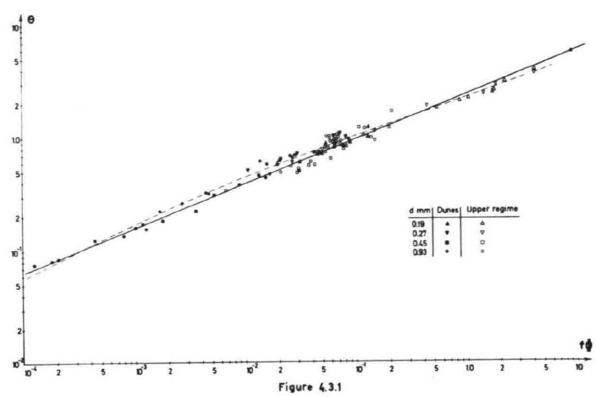
$$F_{grk} = \eta_k \frac{u_*^n}{\sqrt{R_k g d_k}} \left[\frac{U}{\sqrt{32} \log_{10}(10h/d_k)} \right]^{1-n}$$

41

41

Engelund-Hansen (1967)

- Total-load Formula
- Threshold for transport
  - Originally did not use one
  - Included in HEC-RAS 2D Sediment
- Uniform sediments dominated by suspended load



$$q_{tk}^{*} = \begin{cases} 0.05 \eta_k \rho_{sk} U^2 \sqrt{\dfrac{d_k}{g R_k}} \left(\dfrac{\tau_b}{g(\rho_{sk} - \rho_w) d_k} \right)^{3/2} & \text{for } \tau_b > \tau_{crk} \\ 0 & \text{otherwise} \end{cases}$$

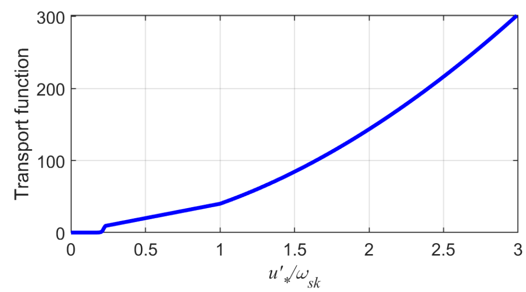
42

42

Laursen-Copeland (1968, 1989)

- Total-load Formulas (continued)
- Nonlinear excess shear formulation with empirical transport function
- Based on flume experiments and data from Arkansas River.
- Copeland extended Laursen equation to gravel

$$q_{tk}^{*} = a\rho_w Uh\left(\frac{d_k}{h}\right)^{7/6}\left(\frac{\theta_b'}{\theta_{crk}}-1\right)^{n} f_{tk}^{LC}\left(\frac{u_*'}{\omega_{sk}}\right)$$



43

Meyer-Peter-Müller (1948)

- Bed-load Formula
- Excess shear formulation
- Recalibrated several times in literature
- Most appropriate for uniform gravel
- Tends to under-predict for sands and silts

$$q_{bk}^{*} = A_M \rho_{sk}\sqrt{R_k g d_k^3}\left(\theta_b' - \theta_{crk}\right)^{E_M}$$

$$MPM \Rightarrow A_M = 8, E_M = 2/3, \text{ and } \theta_{crk} = 0.047$$

$$\text{Wong and Parker (2006)} \Rightarrow A_M = 3.97, E_M = 1.6, \text{ and } \theta_{crk} = 0.0495$$

44

44

Soulsby-van Rijn (1997)

- Total-load Formulas
- Developed by curve-fitting to a 2DV sediment transport model SEDTRANS by van Rijn
- Uses depth-averaged threshold current velocity
- Originally proposed for well sorted sands and extended here for nonuniformly sized sediments

$$q_{bk}^{*} = 0.005Uh\left(\frac{U - U_{crk}}{\sqrt{R_k g d_k}}\right)^{2.4}\left(\frac{d_k}{h}\right)^{1.2}$$

$$q_{sk}^{*} = 0.012Uh\left(\frac{U - U_{crk}}{\sqrt{R_k g d_k}}\right)^{2.4}\left(\frac{d_k}{h}\right)d_{*k}^{-0.6}$$

45

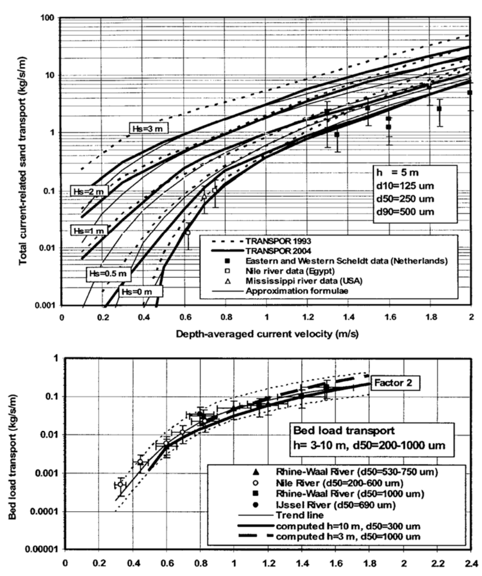
45

Van Rijn (1984ab, 2007ab)

- Total-load Formula
- Developed by curve-fitting to lab and field measurements
- Uses depth-averaged threshold current velocity
- Originally proposed for well sorted sands and extended here for nonuniformly sized sediments
- Suspended-load formula the same as the Soulsby-van Rijn

$$q_{bk}^{*} = 0.015Uh\left(\frac{U - U_{crk}}{\sqrt{R_k g d_k}}\right)^{1.5}\left(\frac{d_k}{h}\right)^{1.2}$$

$$q_{sk}^{*} = 0.012Uh\left(\frac{U - U_{crk}}{\sqrt{R_k g d_k}}\right)^{2.4}\left(\frac{d_k}{h}\right)d_{*k}^{-0.6}$$



46

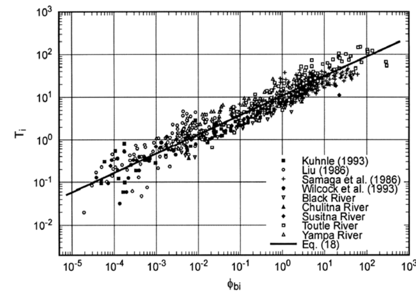
46

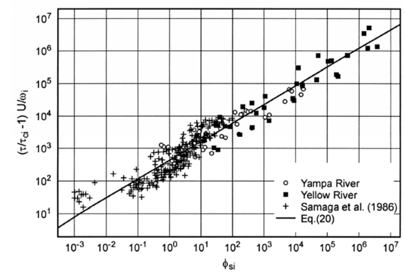
Wu et al. (2000)

- Total-load Formula
- Developed for nonuniform sediments
- Based on extensive lab and field measurements
- Nonlinear excess shear formulation for bed-load
- Stream-power formulation for suspended-load

$$q_{bk}^{*} = 0.0053\sqrt{R_k g d_k^3}\left(\frac{\tau_b'}{\tau_{crk}} - 1\right)^{2.2}$$

$$q_{sk}^{*} = 2.62\times10^{-5}\sqrt{R_k g d_k^3}\left[\left(\frac{\tau_b}{\tau_{crk}} - 1\right)\frac{U}{\omega_{sk}}\right]^{1.74}$$



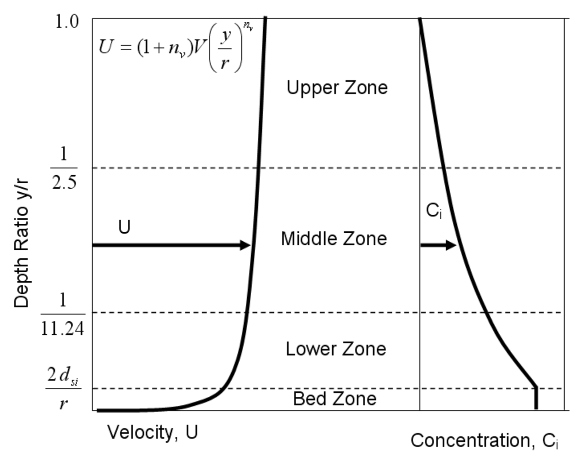


47

47

Toffaleti (1968)

- Total-load formulas
- Developed primarily for sand
- Splits the water column into 3 zones
- Assumes Rouse concentration profile
- Originally developed for bulk transport but here it is applied to individual grain classes
- Usually applied at "large" rivers since most of the data used to develop it were from large suspended-load dominant rivers
- Bed-load formula does not perform well for gravel and can be replaced with MPM



48

48

Wilcock and Crowe (2003)

- Bed-load Formula
- Developed for graded beds with sand and gravel
- No critical shear for transport
- However, it quickly goes to zero

$$q_{bk}^{*} = \frac{u_*^3 W_k^*}{R_k g}$$

$$W_k^* = \begin{cases} 0.002\phi^{7.5} & \text{for } \phi < 1.35 \\ 14\left(1 - \dfrac{0.894}{\phi^{0.5}}\right)^{4.5} & \text{for } \phi \geq 1.35 \end{cases} \qquad \phi = \frac{\tau_b}{\tau_{r,k}}$$

49

49

Yang (1979, 1984)

- Total-load formula
- Regression of potential energy dissipation
- Best for fine to medium sands
- Overestimates for coarse sands and gravel
- Sharp discontinuity at diameter of 2 mm

$$\log_{10}\left(C_{tk}^*\right) = M + N \log_{10}\left[\frac{S_f}{\omega_{sk}}\left(U - U_{crk}\right)\right]$$

$$M = M\left(\frac{\omega_{sk} d_k}{\nu}, \frac{u_*}{\omega_{sk}}\right), \; N = N\left(\frac{\omega_{sk} d_k}{\nu}, \frac{u_*}{\omega_{sk}}\right)$$

50

50

Thank You!

HEC-RAS Website:
https://www.hec.usace.army.mil/software/hec-ras/

Online Documentation:
https://www.hec.usace.army.mil/confluence/rasdocs

US Army Corps of Engineers®

51

51