HEC-RAS 2D Sediment Workshop: Adaptation Parameters

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Equilibrium vs. Non-Equilibrium Transport

$$\rho_s(1-\phi_b)\frac{\partial z_b}{\partial t}+\frac{\partial Q_{t*}}{\partial x}=0$$

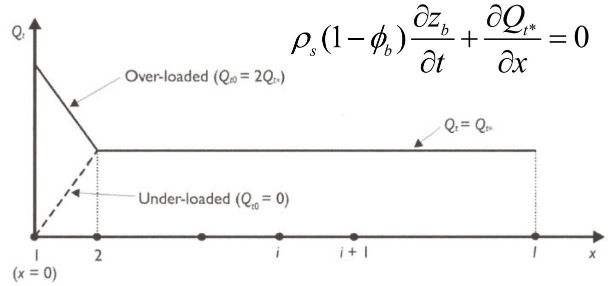

Figure 5.5 Sediment discharge profiles in equilibrium transport model.

From: Wu (2008)
Computational River
Dynamics

$$\frac{\partial(hC_t)}{\partial t}+\frac{\partial(hUC_t)}{\partial x}=\frac{\partial}{\partial x}\left(\varepsilon_t h\frac{\partial C_t}{\partial x}\right)+E_t-D_t$$

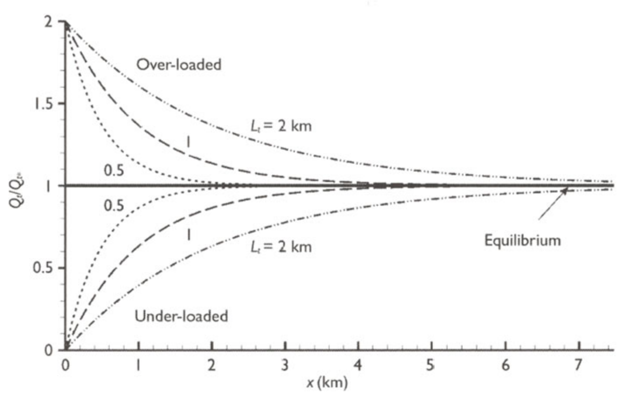

Figure 5.6 Sediment discharge profiles in non-equilibrium transport model.

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Sediment Boundary Conditions

- Water Surface Boundary Condition

$$\left(\omega_s c + \varepsilon_s \frac{\partial c}{\partial z}\right)_{z=z_s} = 0$$

- Near-Bed Boundary Condition

$$\left(\omega_s c + \varepsilon_s \frac{\partial c}{\partial z}\right)_{z=z_b+\delta} = D_s - E_s$$

$$E_s = \left(-\varepsilon_s \frac{\partial c}{\partial z}\right)_{z=z_b+\delta} = \omega_s c_{b*}$$

ε_s : Verticl diffusivity [L^2/T]

ω_s : Fall velocity [L/T]

c_{b*} : Near-bed equilibrium (capacity) concentration [M/L^3]

$$D = (\omega_s c)_{z=z_b+\delta} = \omega_s c_b$$

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Sediment Boundary Conditions

- “Gradient” Boundary Condition

$$\left(-\varepsilon_s \frac{\partial c}{\partial z}\right)_{z=z_b+\delta} = \omega_s c_{b*}$$

- Equilibrium Condition

$$\left(\omega_s c + \varepsilon_s \frac{\partial c}{\partial z}\right)_{z=z_b+\delta} = D_s - E_s = 0$$

$$\therefore c_b = c_{b*}$$

ω_s: Fall velocity [L/T]

ε_s : Verticl diffusivity [L^2/T]

$c(z)$: concentration [M/L^3]

E_s : Mass erosion rate [M/L^2/T]

D_s : Mass deposition rate [M/L^2/T]

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Noncohesive Erosion and Deposition of Suspended Load

- Near-bed Model

$$D_s = \omega_s c_b$$
$$E_s = \omega_s c_{b*}$$

ω_s: Fall velocity [L/T]

c_b : Near-bed concentration [M/L^3]

c_{b*} : Near-bed conc. capacity [M/L^3]

- Near-bed concentration and capacity difficult to estimate for depth-averaged models
- Values vary by several orders of magnitude
- Very few equations for near-bed concentration capacity
- Near-bed concentration capacity is very difficult to measure

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Noncohesive Erosion and Deposition of Suspended Load

- Relating near-bed values to depth-averaged values

$$c_b = \alpha_c C \qquad c_{b*} = \alpha_{s*} C_*$$

- Inserting into deposition and erosion rates

$$D = \alpha_s \omega_s C \qquad E = \alpha_{s*} \omega_s C_*$$

- Depth-average concentrations can easily be computed and are readily available

$$\alpha_s \approx \alpha_{s*}$$
$$\therefore D_s - E_s = \alpha_s \omega_s \left(C - C_* \right)$$

α_s : Adaptation coefficient (correction factor R_{cp}) [-]

α_{s*} : Adaptation coefficient under equilibrium conditions [-]

ω_s: Fall velocity [L/T]

C : Depth-averaged concentration [M/L^3]

C_* : Depth-averaged capacity [M/L^3]

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Noncohesive Erosion and Deposition of Suspended Load

- In fact, under equilibrium conditions defined as $D_s - E_s = 0$

  and the computed coefficients $\alpha_s \neq \alpha_{s*}$

  then $C \neq C_*$

  which is obviously incorrect
- This why forcing $\alpha_c = \alpha_{c*}$ is a good approximation

α_s : Adaptation coefficient (correction factor R_{cp}) [-]

α_{s*} : Adaptation coefficient under equilibrium conditions [-]

C : Depth-averaged concentration [M/L^3]

C_* : Depth-averaged capacity [M/L^3]

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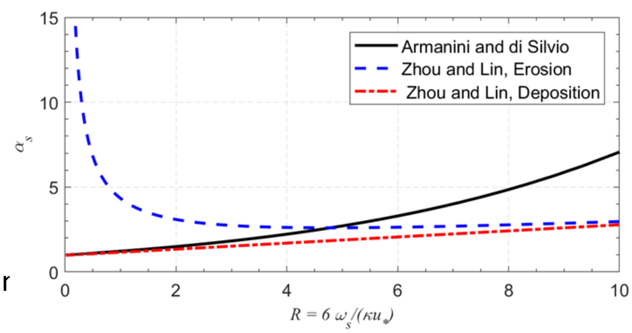
Suspended-load Adaptation Coefficient

- Armanini and di Silvio (1986)
  - Approximate analytical integration of the pure vertical 2D advection-diffusion equation with "gradient" near-bed BC
- Zhou and Lin (1998)
  - Approximate analytical integration of the pure vertical 2D advection-diffusion equation with "concentration" BC for erosion and "gradient" BC for deposition



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Complexity of Suspended-load Adaptation Coefficient

e.g. Zhou and Lin (1998)

$$\frac{\alpha_{s,1D}}{\alpha_s} = \frac{\int_0^W h^{r+1} dy \int_0^W h^{(3r-1)m} dy}{W \int_0^W h^{(3r-1)m+r+1} dy}$$

- Subgrid Bathymetry
  - 1D and 2D models with subgrid don't have a flat bed
- Subgrid Hydraulics and Sediment Dynamics
  - Non-equilibrium velocity and sediment profiles. Profiles highly influenced by advection and diffusion.
  - Bedform effects poorly understood
  - Fall velocity near bed affected by sediment concentrations and the Saffman (1965) lift force near the bed where velocity gradients are high

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Bed-load Adaptation

- Bed-load exchange models are typically formulated as

$$D_b - E_b = \frac{1}{L_b}\left(q_b - q_{b*}\right)$$

- Adaptation length is a measure of the distance it takes for the load to reach equilibrium
- Methods
  - Constant L_b
  - Depth-dependent $L_b = f_{bL} h \quad f_{bL} \approx 7.3$

L_b : Bed-load adaptation length [L]

q_b : Actual bed-load transport rate [M/L/T]

q_{b*} : Bed-load transport capacity [M/L/T]

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Complexity of Bed-load Adaptation Length

- Varies spatially and temporally
- Adaptation length is related to
  - Bedform length (from ripples to dunes)
  - Bed-load saltation length
  - Scour hole size
  - etc.
- Grid Resolution
  - For computational accuracy stability $\Delta x \ll L_b$
  - Can be limiting factor
  - Rule of thumb $\Delta x < L_b < 2\Delta x$

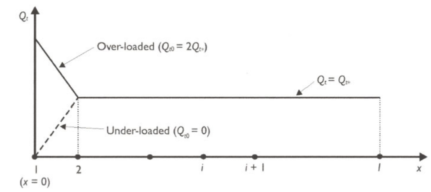

Figure 5.5 Sediment discharge profiles in equilibrium transport model.

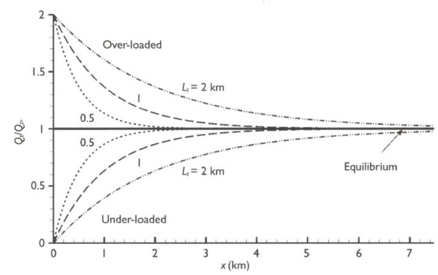

Figure 5.6 Sediment discharge profiles in non-equilibrium transport model.

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Total-load Adaptation Length

- Adaptation approach (Wu 2000)

$$D_t = \alpha_t \omega_s C_t \qquad E_t = \alpha_t \omega_s C_{t*}$$

- Total-load Adaptation Coefficient
  - Constant Adaptation Length

$$\alpha_t = \frac{hU}{L_t \omega_s}$$

  - Weighted Bed- and Suspended-lengths

$$\alpha_{tk} = r_{sk}\alpha_{sk} + (1 - r_{sk})\frac{hU}{L_b \omega_{sk}}$$

α_t : Total-load adaptation coefficient [-]

L_t : Total-load adaptation length [L]

C_t : Total-load concentration [M/L^3]

C_{t*} : Total-load capacity [M/L/T]

r_s : Ratio of suspended-load to total-load [-]

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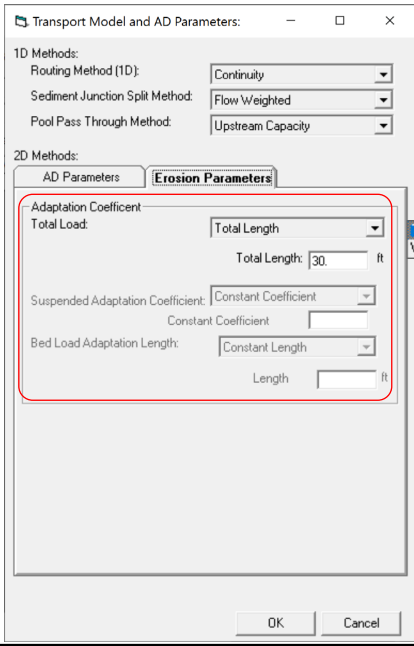
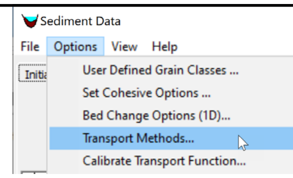
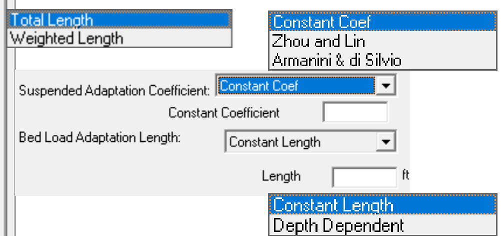
Adaptation Parameters





Transport equation

$$\frac{\partial}{\partial t}\left(\frac{hC_{tk}}{\beta_{tk}}\right)+\nabla\cdot(hUC_{tk})=\nabla\cdot(\varepsilon_{tk}h\nabla C_{tk})+E_{tk}-D_{tk}$$



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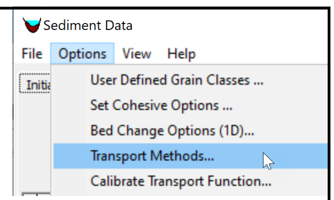
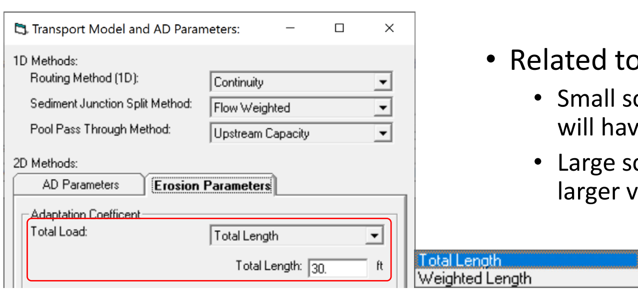
Total-load Adaptation Length





- Related to space time scales
  - Small scale high-resolution models will have a smaller value
  - Large scale coarse models will have a larger value

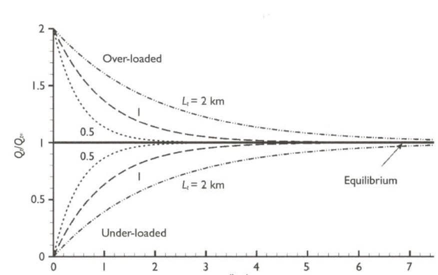


Figure 5.6 Sediment discharge profiles in non-equilibrium transport model.

- Increasing
  - Decreases bed change
  - Smooths bathymetry
  - Improves stability

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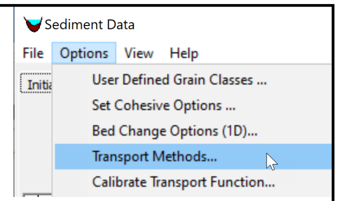
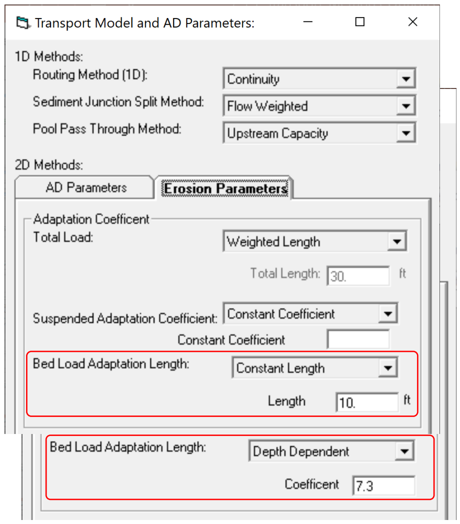
Bed-Load Adaptation Length





- Constant length
  - Most robust
  - Easiest to calibrate
  - Less accurate
- Depth-dependent
  - Less robust
  - Harder to calibrate
  - More accurate

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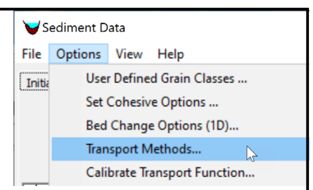
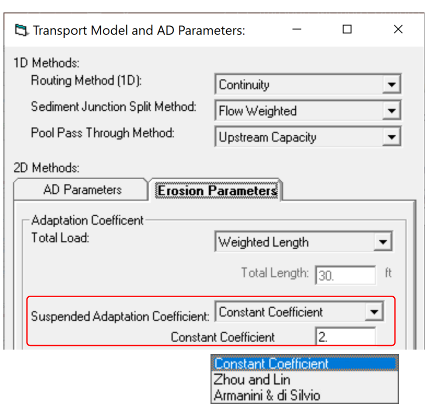
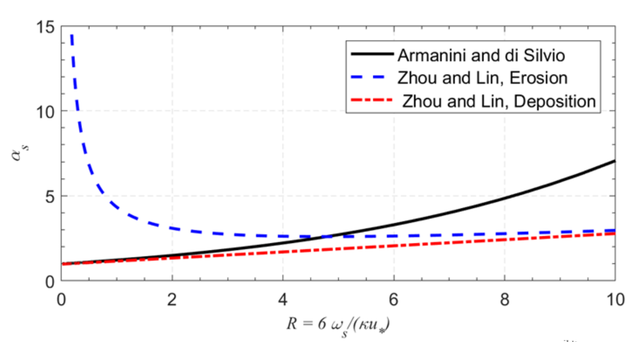
Suspended-Load Adaptation Coefficient





$$\alpha_s \approx 0.5 - 5$$

- Calibration parameter
- Many processes are lumped into parameter



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Discussion

- Total-load adaptation length
  - Use for single mode transport (e.g. well sorted fine sand)
  - At least 1-2x cell size
- Weighted bed and suspended load lengths
  - Use for mixed mode transport (e.g. poorly sorted sediments)
  - Try different formulations for suspended and bed-load adaptation parameters

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Thank You!

HEC-RAS Website:
https://www.hec.usace.army.mil/software/hec-ras/

Online Documentation:
https://www.hec.usace.army.mil/confluence/rasdocs

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