River Mechanics and Intro to Unsteady Flow Equations

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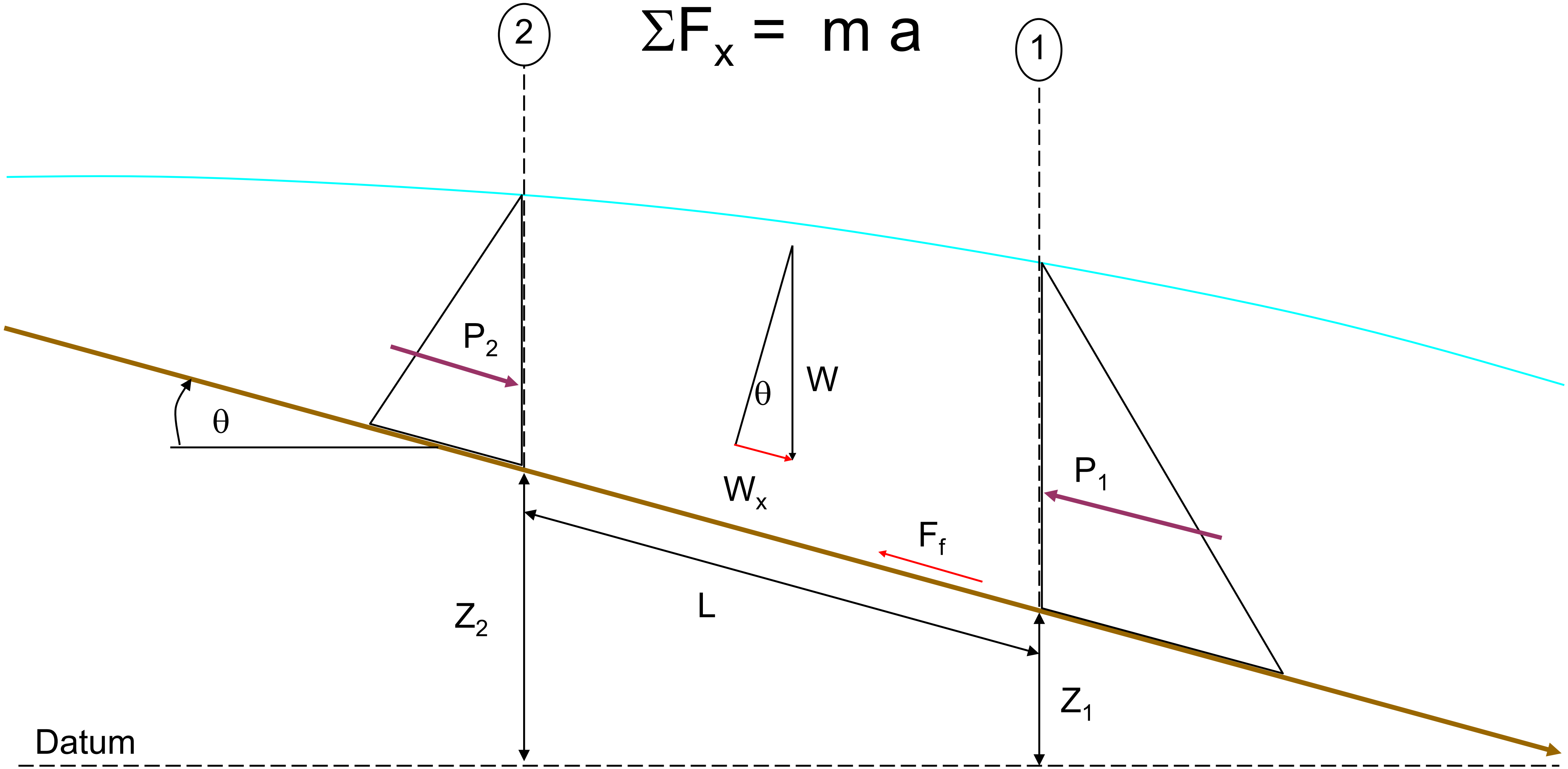
Momentum Equation



Momentum Equation

$$P_2 - P_1 + W_x - F_f = Q \rho \Delta V_x$$

Where:

P = Hydrostatic Pressure

W_x = Force due to weight of water in X direction

F_f = Force due to external friction from 2 to 1

Q = Discharge

ρ = Density of water

ΔV_x = Change in velocity from 2 to 1 in X direction

Momentum Equation – Forces

Pressure: $P = \gamma A \overline{Y}$

Weight: $W_x = W \sin\theta$ $\quad W_x = \gamma \left(\frac{A_1 + A_2}{2} \right) L\, S_0$

Friction: $F_f = \tau \, \overline{p} \, L$

Where: $\tau = \gamma \, \overline{R} \, \overline{S_f}$

$$F_f = \gamma \left(\frac{A_1 + A_2}{2} \right) \overline{S_f} \, L$$

Mass ∗ acceleration: $ma = \frac{Q\gamma}{g} \left(\beta_1 V_1 - \beta_2 V_2 \right)$

Unsteady Flow Equations

Momentum Equation:

$$\frac{\partial Q}{\partial t}+\frac{\partial(\boldsymbol{\alpha} Q^2/A)}{\partial x}+gA(\frac{\partial h}{\partial x}-S_o+S_f)=0$$

Continuity Equation:

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=0$$

Steady Flow Equations

Steady Flow form of Momentum Equation:

$$\frac{\partial(\boldsymbol{\alpha} Q^2/A)}{\partial x} + gA\left(\frac{\partial h}{\partial x} - S_o + S_f\right) = 0$$

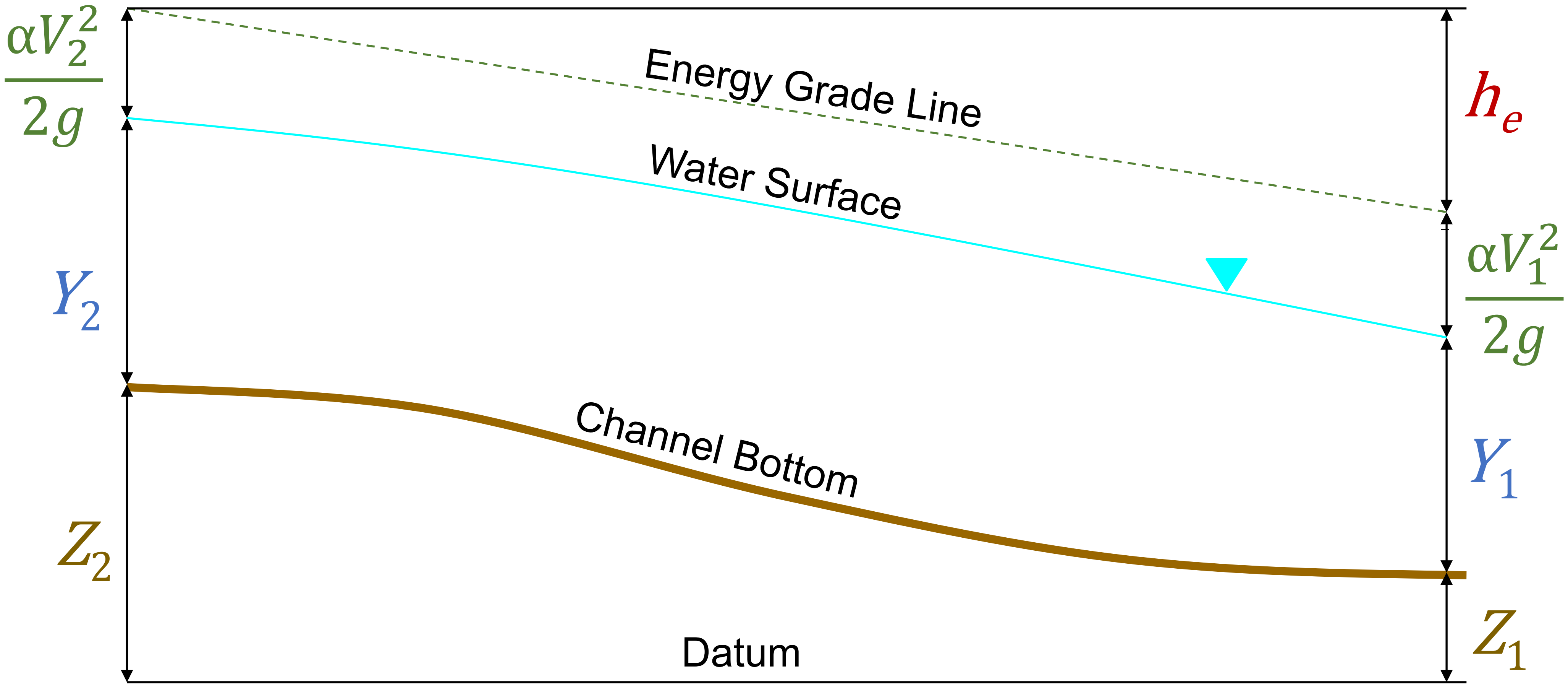
Continuity Equation:

$$Q = VA$$

Steady vs Unsteady Equations

	Momentum	Continuity
Unsteady	$\frac{\partial Q}{\partial t} + \frac{\partial(\boldsymbol{\alpha} Q^2/A)}{\partial x} + gA(\frac{\partial h}{\partial x} - S_o + S_f) = 0$	$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$
Steady	$\frac{\partial(\boldsymbol{\alpha} Q^2/A)}{\partial x} + gA(\frac{\partial h}{\partial x} - S_o + S_f) = 0$	$Q = VA$

Steady Flow Energy Principles



$$Z_2 + Y_2 + \frac{\alpha_2 V_2^2}{2g} = Z_1 + Y_1 + \frac{\alpha_1 V_1^2}{2g} + h_e$$

Unsteady Flow Routing (1D or 2D)
vs
Hydrologic Routing/Steady Flow Hydraulics?

- Considerations:
  - Physical description of the river channels, floodplain areas, bridges/culvert, levees, road, hydraulic structures
  - Size (area/length) and complexity of the system?
    - (1, 10, 50, 100, 500, or 1000's of sq. miles)
  - Model Objective: planning or real time
  - Events will be analyzed?
    - (2yr-500 yr, PMF)
  - Typical duration of a flood event?
    - (1/2 day, 1-3 days, week, month, or 6 months)

Unsteady Flow Routing (1D or 2D)
vs
Hydrologic Routing/Steady Flow Hydraulics?

- Unique aspects of the system that affect the choice :
  - Tidal Influence
  - Do wind speed and direction affect the water surface elev.
  - Is the river affected by floating ice or ice jams
  - Debris on bridges or hydraulic structures
  - Dam/levee overtopping and breaching
  - Bridges/culverts, dams, and weirs
  - Gates and pump stations

Unsteady Flow Routing (1D or 2D) vs Hydrologic Routing/Steady Flow Hydraulics?

- Availability and accuracy of the terrain data and hydraulic structure information?
- Accuracy of the hydrology used to drive the models?
- Model objectives – results?
  - Water surface elevations, water depths, arrival times, average velocities, detailed velocities at specific locations, etc…
- Required accuracy of the results?

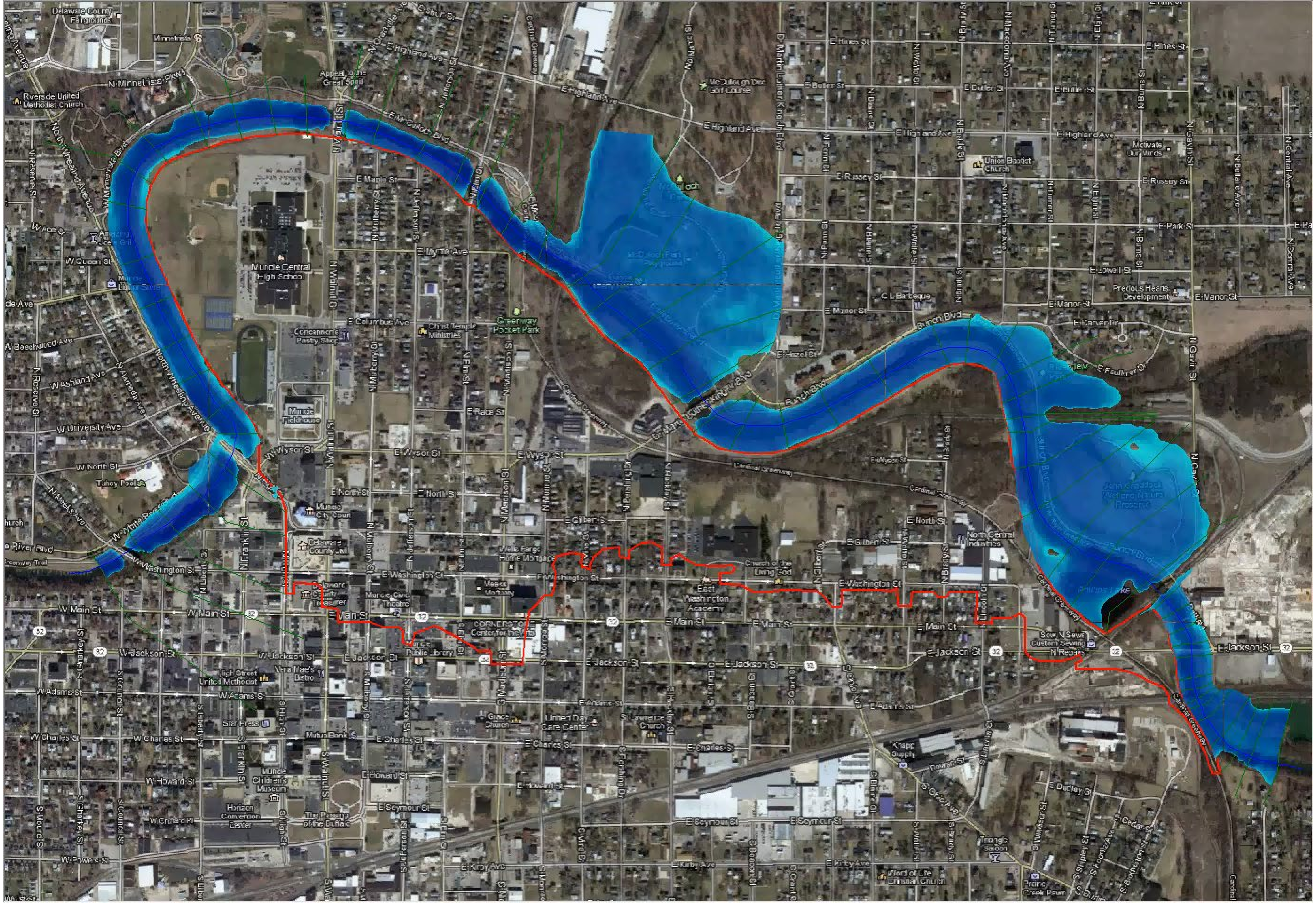
Steady vs. Unsteady Flow Modeling

- Steady Flow Models should generally **Not be used** for:
  - Tidally influenced rivers
  - Very dynamic events
    - (i.e. Dambreak flood waves; flash floods; etc…)
  - Complex Flow networks and/or flow reversals
  - Levee overtopping and breaching
  - Extremely flat river systems, where gravity is not necessarily the only significant driving force of the flow.
  - Structures with gates/gate operation
  - Pump stations

- Steady-flow models require accurate flows from hydrologic model or stream gages.

1D vs. 2D Hydraulic Modeling

- Some areas in which 2D models often outperform 1D models:
  - When modeling an area behind a leveed system, and the levee will be overtopped and/or breached, and the water can go in many directions.
  - Bays and Estuaries in which 2D velocities are needed
  - Alluvial Fans
  - Flow around abrupt bends in which a significant amount of super elevation will occur during the event.
  - Very wide and flat flood plains, such that when the flows goes out into the overbank area, the water will take multiple flow paths and have varying water surface elevations
  - Applications where it is very important to obtain detailed velocities for the hydraulics of flow in a channel, floodplain, or around an object, such as a bridge abutment or bridge piers, etc…



E Highland Ave
E Washington St
E Main St
E Jackson St
Muncie Central High School

1D vs. 2D Hydraulic Modeling

- Some applications where 1D results will be similar to 2D results:
  - Rivers and floodplains in which the dominant flow directions and forces follow the general river flow path.
  - Steep streams that are highly gravity driven and have small overbank areas.
  - River systems that contain a lot of bridges/culvert, weirs, dams, gated structures, levees, pump stations, etc….
  - Medium to large river systems (50 or more miles), and it is necessary to run longer time periods (2 week to 6 months).
  - Areas in which the basic data does not support the potential gain of using a 2D model.

Steady vs. Unsteady Computational Differences

- Flow and boundary condition data
- Numerical solution
- Computation of X-Sec properties
- Friction Equation
- Storage/ineffective areas
- Contraction and expansion losses
- Application strategy

Data Requirements
(Flow and Boundary Conditions)

Steady: Discharge (Q) at each cross section and stage/discharge relationship at D/S

Unsteady: Inflow hydrograph(s) at boundaries and interior locations, which are routed by the model, and stage at D/S.

Numerical Solution

Algorithms used in HEC-RAS:

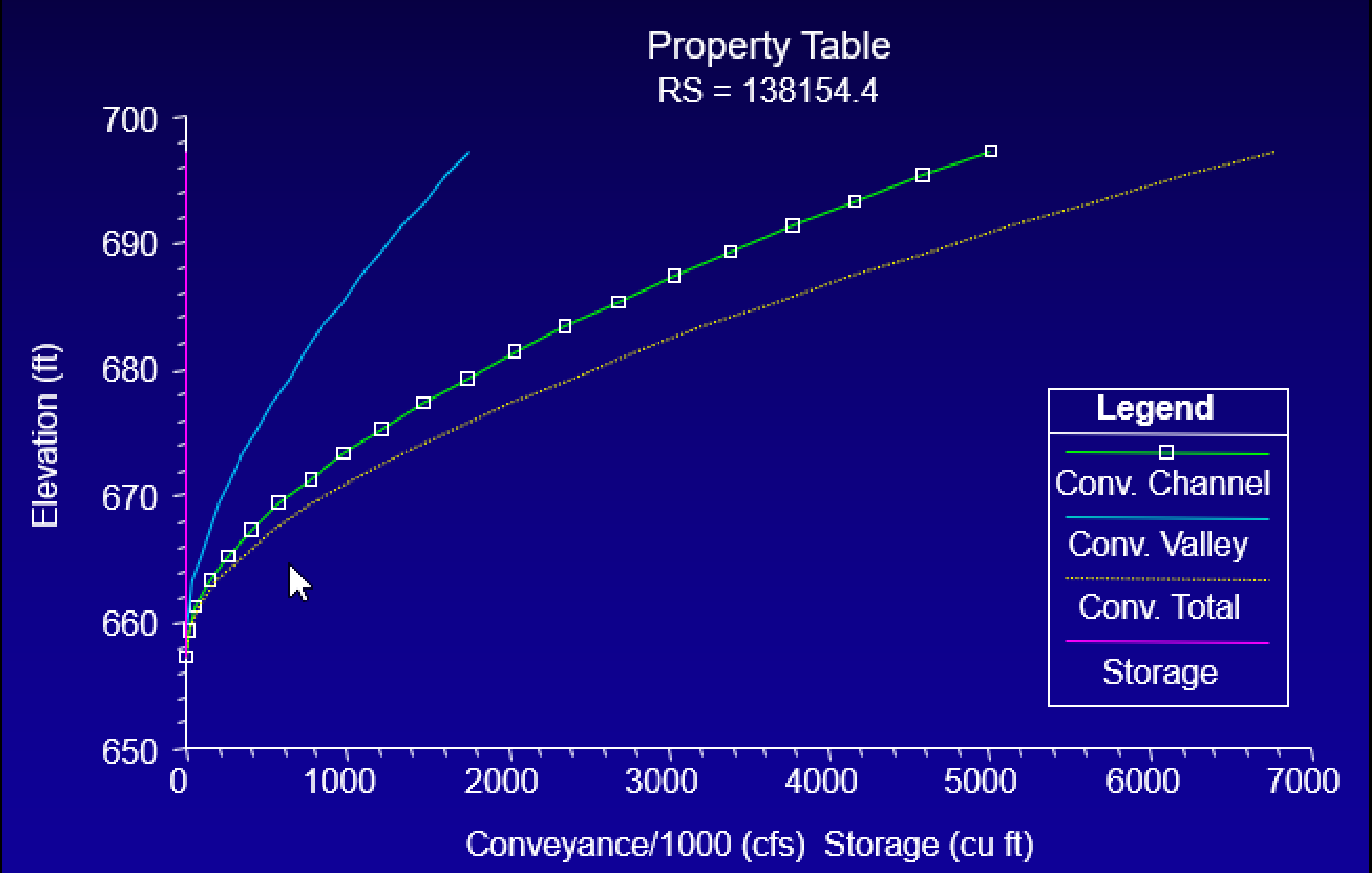
Steady: Iterative solution of the energy equation section-by-section for each flow.

Unsteady: Matrix solution of the continuity and momentum equations, for flow and stage, simultaneously at all sections each time step.

Hydraulic Properties
(cross sections, bridges, etc.)

Steady – Compute exact hydraulic properties at a section for each trial water surface elevation from the cross section points, *n*-values, etc., during the computations.

Unsteady – Hydraulic properties are pre-computed for a range of water surface elevations at each cross section or bridges and culverts. Hydraulic properties are interpolated from the curves during the unsteady flow computations.



Property Table
RS = 138154.4
Elevation (ft)
700
690
680
670
660
650
0
1000
2000
3000
4000
5000
6000
7000
Conveyance/1000 (cfs) Storage (cu ft)
Legend
Conv. Channel
Conv. Valley
Conv. Total
Storage

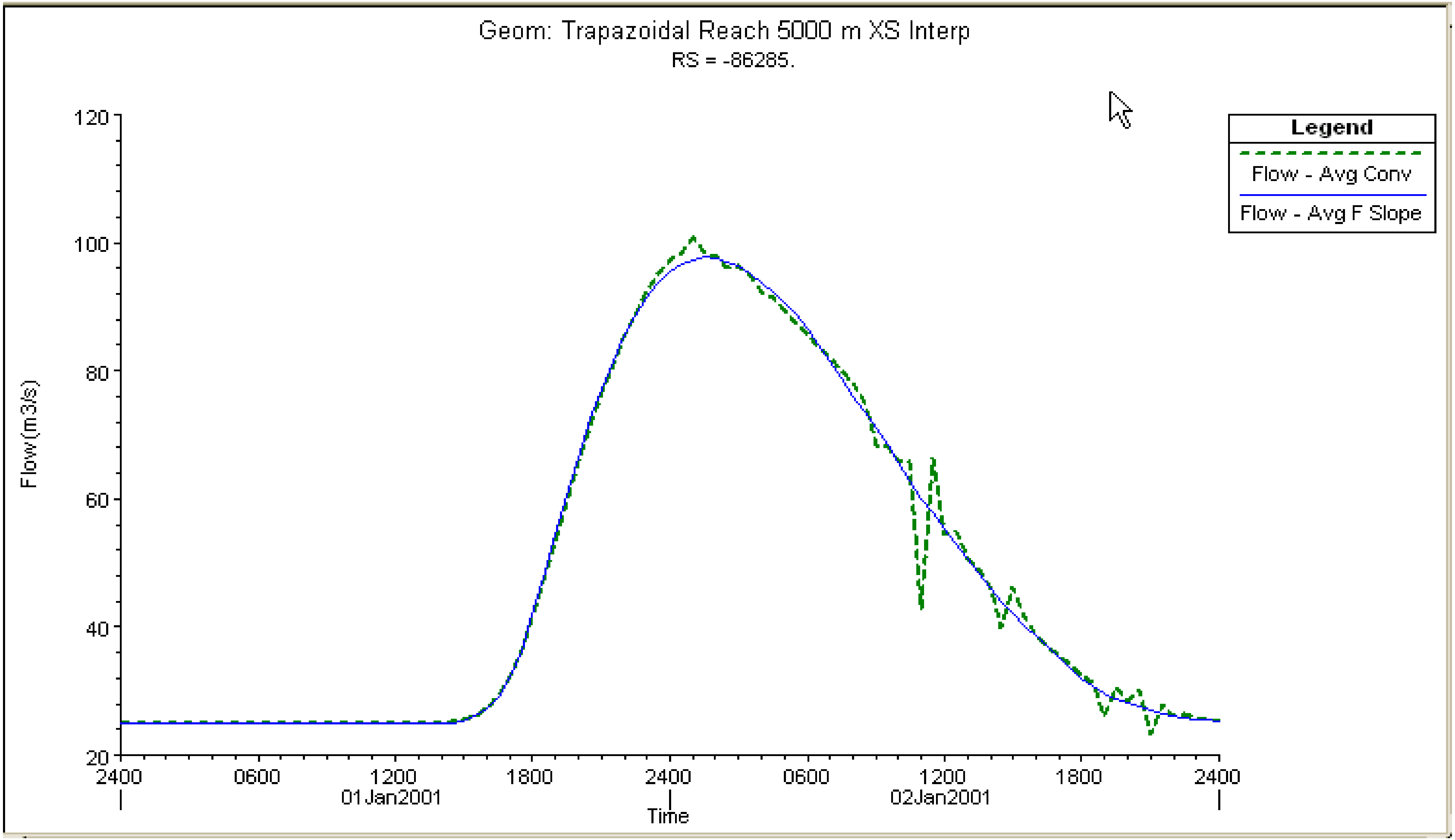
Friction Losses
Energy vs. Momentum

- **Energy** – Internal energy dissipation represented by loss term, S_f (Manning's *n*)

- **Momentum** – External boundary shear forces represented by friction term, S_f (Manning's *n*)

Friction Loss Calculations

Unsteady	Steady
Average conveyance	Average friction slope
$\overline{S}_f = \left(\dfrac{Q_1 + Q_2}{K_1 + K_2} \right)^2$	$\overline{S}_f = \dfrac{S_{f1} + S_{f2}}{2}$



Geom: Trapazoidal Reach 5000 m XS Interp
RS = -86285.
Legend
Flow - Avg Conv
Flow - Avg F Slope
Flow (m3/s)
120
100
80
60
40
20
2400
0600
1200
1800
2400
0600
1200
1800
2400
01Jan2001
02Jan2001
Time

Storage/Ineffective Flow Areas

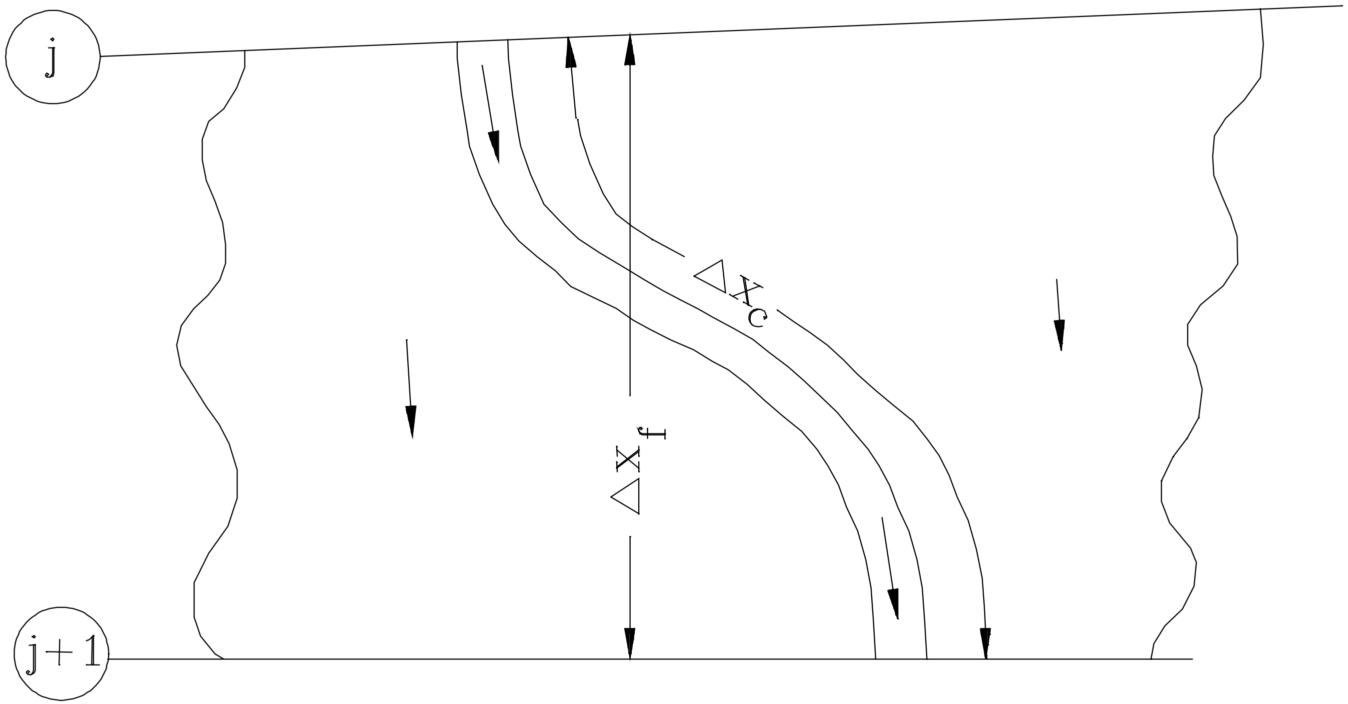
Steady – "Ineffective" areas only affect wetted perimeter and conveyance area. Water that does not move does not affect results.

Unsteady – All areas that contain water must be included even if it is not moving. Ineffective areas of the cross section is computed as storage.

Storage/Ineffective Flow Areas

Unsteady HEC-RAS combines the properties of the left and right overbank into a single flow compartment.

Hydraulic properties for the floodplain are computed by combining the left and right overbank elevation vs Area, conveyance, and storage into a single set of relationships.

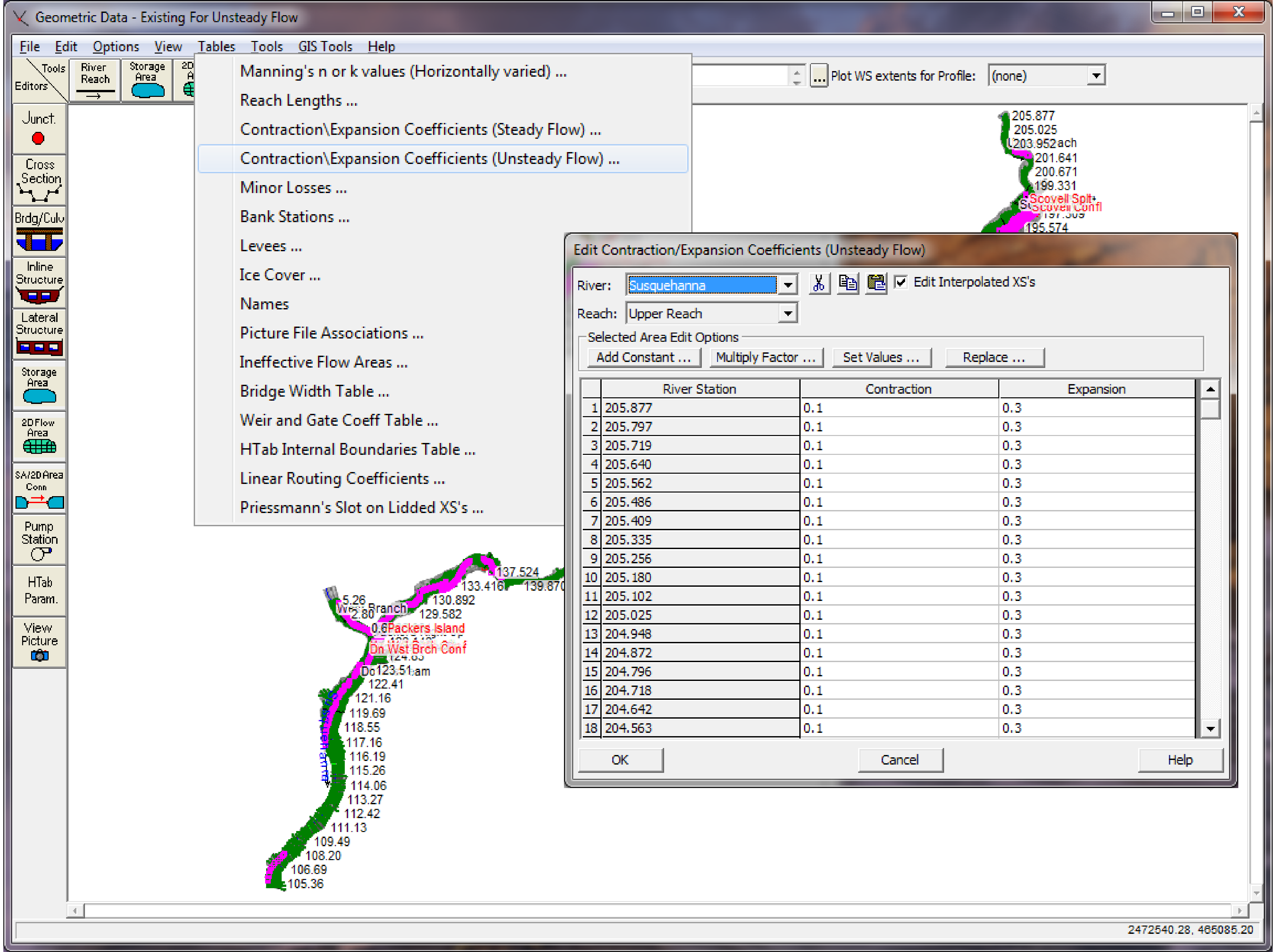


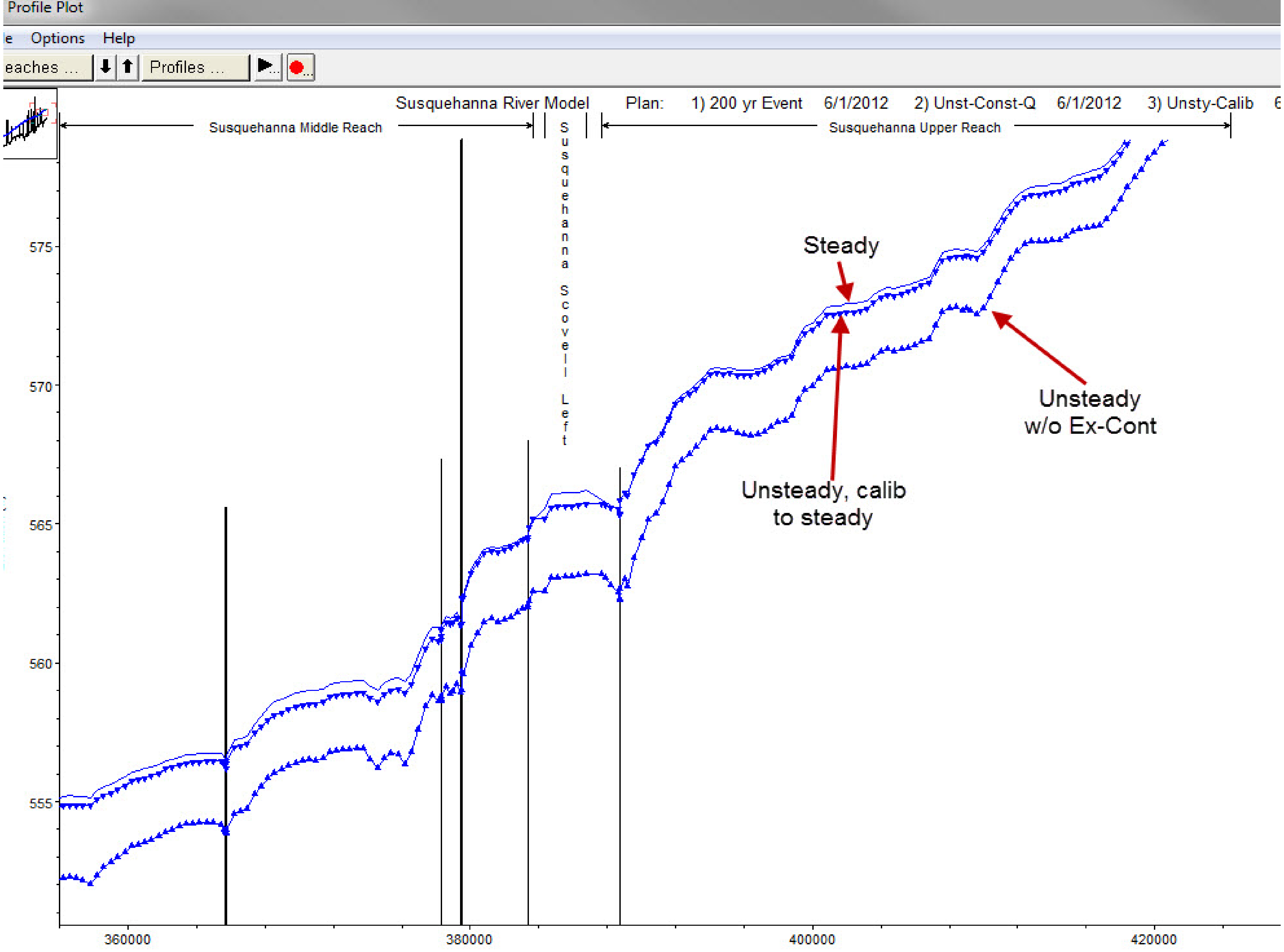
The reach length used for the floodplain area is computed by taking the arithmetic average of the left and right overbank reach lengths $(L_L + L_R)/2 = L_F$.

Expansion/Contraction Coefficients

Steady - One by default with values of 0.1set for contraction and 0.3 for expansion

Unsteady - Not used by default, coefficients set to zero. However, they can play an important role in unsteady flow simulations thru added forces for turbulence not reflected in cross section shape change.


Geometric Data - Existing For Unsteady Flow
File Edit Options View Tables Tools GIS Tools Help
Tools
Editors
River Reach
Storage Area
Junct.
Cross Section
Brdg/Culv
Inline Structure
Lateral Structure
Storage Area
2D Flow Area
SA/2D Area Conn
Pump Station
HTab Param.
View Picture
Manning's n or k values (Horizontally varied) ...
Reach Lengths ...
Contraction\Expansion Coefficients (Steady Flow) ...
Contraction\Expansion Coefficients (Unsteady Flow) ...
Minor Losses ...
Bank Stations ...
Levees ...
Ice Cover ...
Names
Picture File Associations ...
Ineffective Flow Areas ...
Bridge Width Table ...
Weir and Gate Coeff Table ...
HTab Internal Boundaries Table ...
Linear Routing Coefficients ...
Priessmann's Slot on Lidded XS's ...
Plot WS extents for Profile:
(none)
Edit Contraction/Expansion Coefficients (Unsteady Flow)
River: Susquehanna
Reach: Upper Reach
Edit Interpolated XS's
Selected Area Edit Options
Add Constant ...
Multiply Factor ...
Set Values ...
Replace ...
River Station
Contraction
Expansion
1 205.877 0.1 0.3
2 205.797 0.1 0.3
3 205.719 0.1 0.3
4 205.640 0.1 0.3
5 205.562 0.1 0.3
6 205.486 0.1 0.3
7 205.409 0.1 0.3
8 205.335 0.1 0.3
9 205.256 0.1 0.3
10 205.180 0.1 0.3
11 205.102 0.1 0.3
12 205.025 0.1 0.3
13 204.948 0.1 0.3
14 204.872 0.1 0.3
15 204.796 0.1 0.3
16 204.718 0.1 0.3
17 204.642 0.1 0.3
18 204.563 0.1 0.3
OK
Cancel
Help
2472540.28, 465085.20


Profile Plot
le Options Help
eaches ... Profiles ...
Susquehanna River Model Plan: 1) 200 yr Event 6/1/2012 2) Unst-Const-Q 6/1/2012 3) Unsty-Calib 6
Susquehanna Middle Reach
Susquehanna Scovell Left
Susquehanna Upper Reach
575
570
565
560
555
360000
380000
400000
420000
Steady
Unsteady w/o Ex-Cont
Unsteady, calib to steady

Application Strategy

1. **Check the model performance with a range of steady flows**

   Rough **stage** calibration.

   Possible **supercritical** flow locations.

   Modeling of hydraulic **structures**.

Application Strategy

2. **Prepare hydrographs (boundary conditions)**

    Upstream flows

    Tributary (local flows)

    Ungaged/unmodeled flows

    Downstream Boundary conditions

Application Strategy

3. Calibration

Manning's *n* affects both stage and timing.

Storage areas/ineffective flow areas can be very important.

Fine tuning of flood wave attenuation via **conveyance** adjustment.

Structures such as bridges, diversions and pumps can be useful sources of data.

Questions?

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