Opportunities and Recommendations to Improve the Implementation of Bulletin 17C

David Margo, P.E.[1] and Michael Bartles, P.E.[2]

[1] Lead Civil Engineer, U.S. Army Corps of Engineers, Institute for Water Resources, Risk Management Center, 1000 Liberty Ave, Pittsburg, PA 15222; 412-667-6629; David.A.Margo@usace.army.mil

[2] Hydraulic Engineer, U.S. Army Corps of Engineers, Institute for Water Resources, Hydrologic Engineering Center, 609 Second Street, Davis, CA 95616; 530-302-3706; Michael.D.Bartles@usace.army.mil.

ABSTRACT

The Expected Moments Algorithm (EMA) codified in Bulletin 17C and implemented in the U.S. Army Corps of Engineers Statistical Software Package (HEC-SSP) and the U.S. Geological Survey Flood Frequency Analysis Software (PeakFQ) significantly improves flow frequency analysis. Several years of experience within USACE has identified multiple opportunities to incrementally improve the implementation procedures. Under certain conditions, each of the changes presented in this paper can improve the accuracy and credibility of a flow-frequency analysis. First, the procedure in the current software calculates a new weighted skew value after each EMA iteration resulting in an incorrect value for the final weighted skew estimate. Second, the current software does not report the effective record length (ERL) value needed for a typical stochastic flood hazard analysis when historical, censored, or regional skew information is included. Third, the current software uses the geometric mean of the low and high flow values to calculate plotting positions for flow interval data resulting in a lack of transparency and an interdependency between plotting position and distribution fitting calculations. Fourth, the current software does not calculate or report an estimate of the expected probability curve which is needed for risk informed decision making. Fifth, recoding of potentially influential low floods (PILF) can result in unrealistic values for the mean square error (MSE) of the at-site skew, confidence intervals, and ERL. This paper provides examples and proposed solutions for each of these items.

INTRODUCTION

Bulletin 17B guided flow-frequency analyses within the United States from the early 1980s through the late 2010s. This guidance recommended the use of the Log Pearson Type III probability distribution for annual maximum flows on unregulated streams fit by the method of moments (Interagency Advisory Committee on Water Data, 1982). The Bulletin 17C guidance brought about several major improvements to the computation of peak flow-frequency within the United States. This guidance incorporated changes motivated by four of the items listed as future work within Bulletin 17B and more than 30 years of post-Bulletin 17B research on flood processes and statistical methods (England, et al., 2018). As part of the Bulletin 17C methodology, the parameters of the Log Pearson Type III distribution are estimated using the Expected Moments Algorithm (EMA). Like Bulletin 17B, the Bulletin 17C methodology uses the method of moments, but does so in a more generalized way that accommodates historical flow intervals and censored unobserved flows, rather than using a series of adjustment procedures (Cohn, Lane, & Baier, 1997). The use of Bulletin 17C procedures provides improved confidence intervals for the resulting frequency curve that incorporate diverse information appropriately, as historical data and censored values impact the uncertainty in the estimated frequency curve (Cohn, Lane, & Stedinger, 2001). Within the Bulletin 17C methodology, every annual peak flow in the analysis period, whether observed or not, is represented by a flow range. That range might simply be limited to an exact gaged value when one exists. However, it could also reflect an uncertain flow estimate.

PURPOSE

The goal of this paper is to motivate discussion and action directed toward making incremental improvements to the implementation procedures of Bulletin 17C and the supporting software. The items included in this paper were identified based on applying the methods and software in practice within the U.S. Army Corps of Engineers.

PROPOSED IMPROVEMENTS

Weighted Skew. The use of regional skew information within Federal flood-frequency guidelines dates to Bulletin 17, which included a procedure for weighting at-site skew and a regional skew (Water Resources Council, 1976). Tasker showed that the minimum variance skew estimator would be obtained by weighting at-site and regional skews by the inverse of their variances (Tasker, 1978). Bulletin 17B recommended the use of an inverse MSE weighting scheme to reflect estimator bias (Interagency Advisory Committee on Water Data, 1982). This weighting scheme is expressed as:

$$\tilde{G} = \frac{MSE_{\hat{\gamma}} * G + MSE_G * \hat{\gamma}}{MSE_{\hat{\gamma}} + MSE_G} \tag{1}$$

where:

$\hat{\gamma}$	at-site skew
G	regional skew
$MSE_{\hat{\gamma}}$	mean-square error of the at-site skew
MSE_G	mean-square error of the regional skew
$\tilde{G}$	weighted skew

Equation (1) has been shown to minimize the MSE of the skew estimator so long as G is unbiased and independent of the at-site skew estimator (Griffis, Stedinger, & Cohn, 2004). Equation (1) is simply a weighted average of two normally distributed independent random variables.

The current software implementation of EMA utilizes the following procedure from Appendix 7 in Bulletin 17C (2018) to fit the LPIII distribution given a regional skew estimate.

1. Estimate at-site skew
    a. Using at-site data, fit the LPIII distribution with EMA to estimate at-site skew ($\hat{\gamma}$)
    b. Estimate the at-site skew $MSE_{\hat{\gamma}}$ with EMA
2. Estimate weighted skew
    a. Estimate LPIII parameters using Equations 7-1, 7-2, and 7-3 from Bulletin 17C
    b. Estimate a weighted skew coefficient ($\tilde{G}$) using Equation (1)

c. Test for convergence; if not converged, return to step 2a.

The above procedure leads to erroneous results when 1) the at-site and regional skew estimates differ by an appreciable amount or 2) many EMA iterations are required for convergence. In general, more EMA iterations are needed when large amounts of historical and censored data are included. The error occurs because a new weighted skew estimate is calculated after each EMA iteration in Step 2b. The at-site skew estimate for the current EMA iteration is influenced by the weighting that took place in the previous EMA iteration. This means the at-site skew estimate for the 2nd and all subsequent EMA iterations is no longer an independent at-site skew value and should not be used in Equation (1). The current procedure results in an overweighting of the regional skew that compounds with each additional EMA iteration.

The issue was discovered during a recent flow-frequency analysis that utilized systematic, historical, paleoflood, and regional skew information. The evolution of the at-site and weighted skew estimates is shown in Figure 1 for analyses that included systematic only, systematic + historical, and systematic + historical + paleoflood information. As more at-site information was added to the analysis reflected by an increase in the ERL, the at-site skew $MSE_{\hat{\gamma}}$ should decrease and the weighted skew estimate (solid circles) should have trended toward the at-site skew estimate (open squares). Instead, the weighted skew estimate trended in the opposite direction toward the regional skew estimate (dashed line) due to the overweighting of the regional skew.

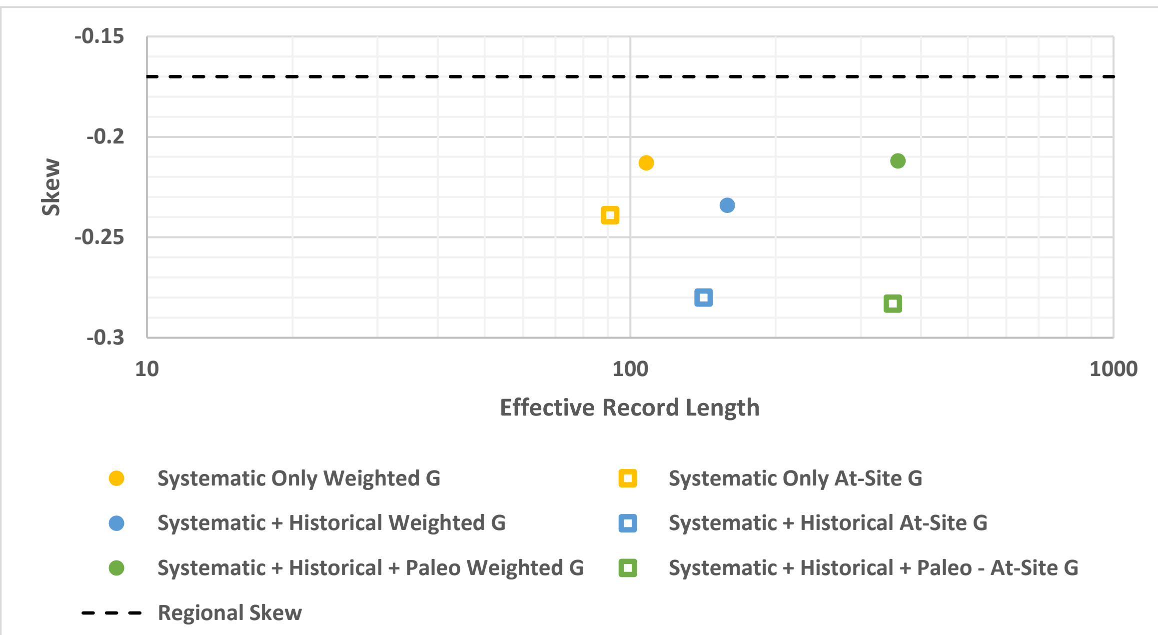

Figure 1. Real World Example of Erroneous Weighted Skew Results using Current Procedure

The issue can also be demonstrated by experiment using synthetic datasets having fixed parameters and varying amounts of historical and censored data. Four sample datasets were generated to have an LPIII distribution with a mean (of logs) of 4.0, standard deviation (of logs) of 0.3, and at-site skew (of logs) of 0.3. These datasets had ERLs of approximately 50, 110, 220, and 450 years. A regional skew value of 0.1 with an MSE_G of 0.078 was assumed for all four datasets. Figure 2 shows the evolution of the MSE and weighted skew estimate (solid circles) for each dataset.

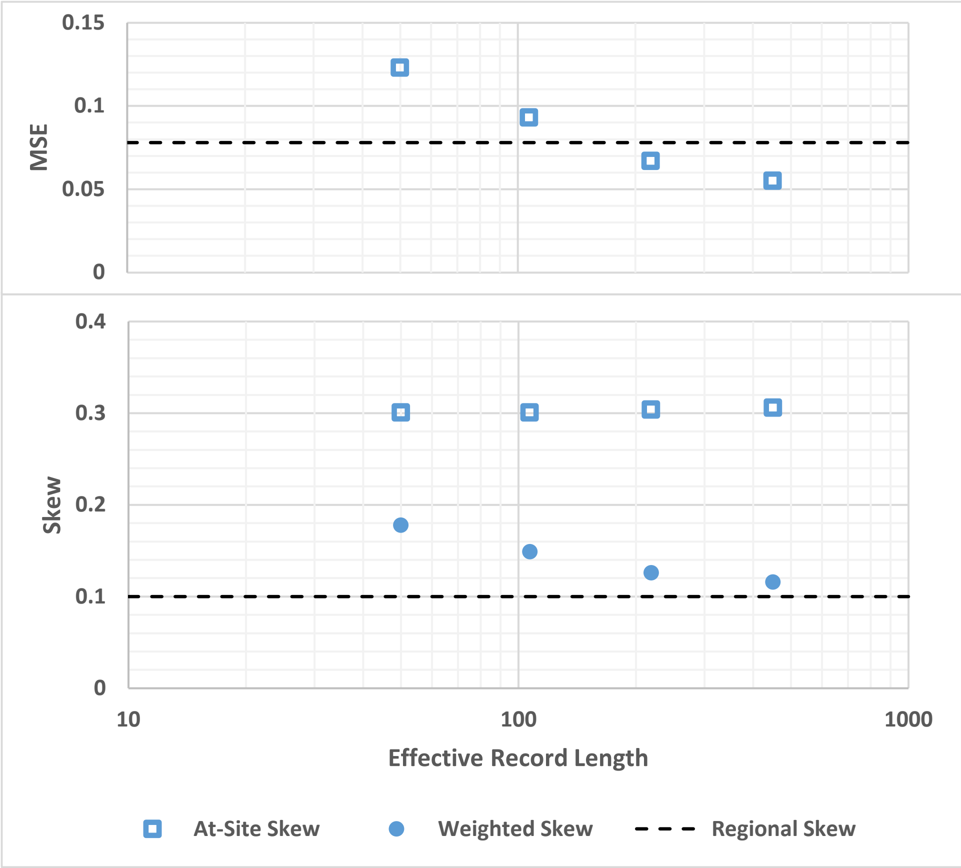

Figure 2. Fixed Parameter Example of Erroneous Weighted Skew Results using Current Procedure

The results shown in Figure 1 and Figure 2 demonstrate a similar behavior: the weighted skew coefficients obtained using the current implementation of EMA generally trend toward the regional skew coefficient as ERL increases. This result is counterintuitive and violates the first principles embodied in Equation (1). As ERL increases, the information content of the at-site skew estimator increases while the information content of the regional skew information remains the same. This means that the influence of regional skew on the weighted skew estimate should decrease with increasing at-site ERL. However, this behavior is not realized using the current procedure. The magnitude of the error is primarily a function of the number of EMA iterations which generally increases with increasing ERL when historical and censored data is included.

Griffis et al, improved the estimation of the third moment within EMA by directly incorporating regional skew information (Griffis, Stedinger, & Cohn, 2004). The improvement is presented below as Equation (2) which is equivalent to Equation 7-10 within Bulletin 17C:

$$\begin{aligned}\hat{\gamma}_{i+1} = \frac{1}{(n+n_G)\hat{\sigma}_{i+1}^3}\Big[c_3\Big\{\sum(X_s^{>}-\hat{\mu}_i)^3 + \sum(X_l^{>}-\hat{\mu}_i)^3 \\ +\sum(X_h-\hat{\mu}_i)^3\Big\} + n_l^{<}\mathrm{E}[(X_l^{<}-\hat{\mu}_i)^3] \\ +n_h^{\top}\mathrm{E}[(X_h^{<}-\hat{\mu}_i)^3] + n_G G\hat{\sigma}_{i+1}^3\Big]\end{aligned} \quad (2)$$

where:

n_G equivalent years of record assigned to the regional skew information

G regional skew value

The value of n_G can be estimated using an inverted form of Equation 6 from Bulletin 17B, an inverted form of Equation 27 from Griffis (2003), or Equation 46 from Griffis (2003). The examples presented in this paper use Equation 6 from Bulletin 17B.

Implementation of EMA software using Equation (2) ensures that the weighted skew corresponds to the adjusted mean and standard deviation fit to the data (Griffis, Stedinger, and Cohn, 2004), and eliminates the overweighting of the regional skew information that occurs with the current procedure. We propose modifying software that supports Bulletin 17C to use the following procedure:

1. Estimate at-site skew
   a. Using at-site data, fit the LPIII distribution with EMA to estimate at-site skew ($\hat{\gamma}$)
   b. Estimate the at-site skew $MSE_{\hat{\gamma}}$ with EMA
2. Estimate weighted skew
   a. Estimate n_G
   b. Estimate LPIII parameters using Equations 7-1, 7-2, and 7-10 from Bulletin 17C
   c. Test for convergence; if not converged, return to Step 2b.

The above procedure eliminates the use of Equation (1) and replaces the use of Equation 7-3 with Equation 7-10 from Bulletin 17C. The previously described examples were used to demonstrate the improvement with the proposed procedure. The results are shown in Figure 3 and Figure 4.

The weighted skew coefficients obtained using the proposed procedure exhibit the correct behavior trending toward the at-site skew coefficient as the ERL increases and the at-site skew MSE decreases.

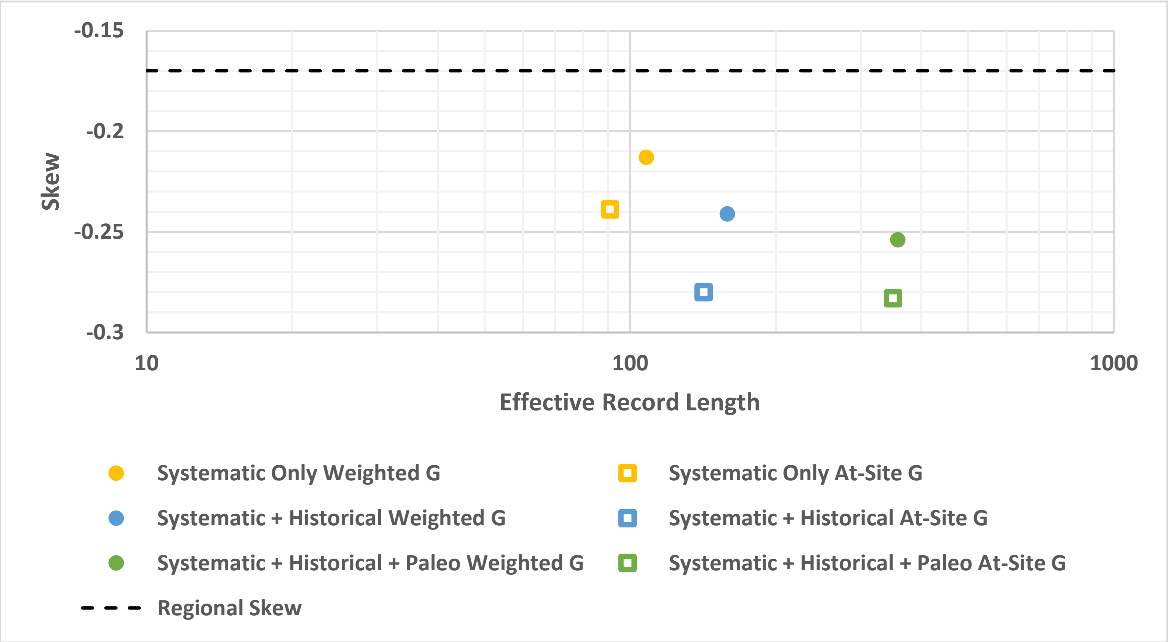

Figure 3. Real World Example Showing Improved Weighted Skew Estimates Using Proposed Procedure

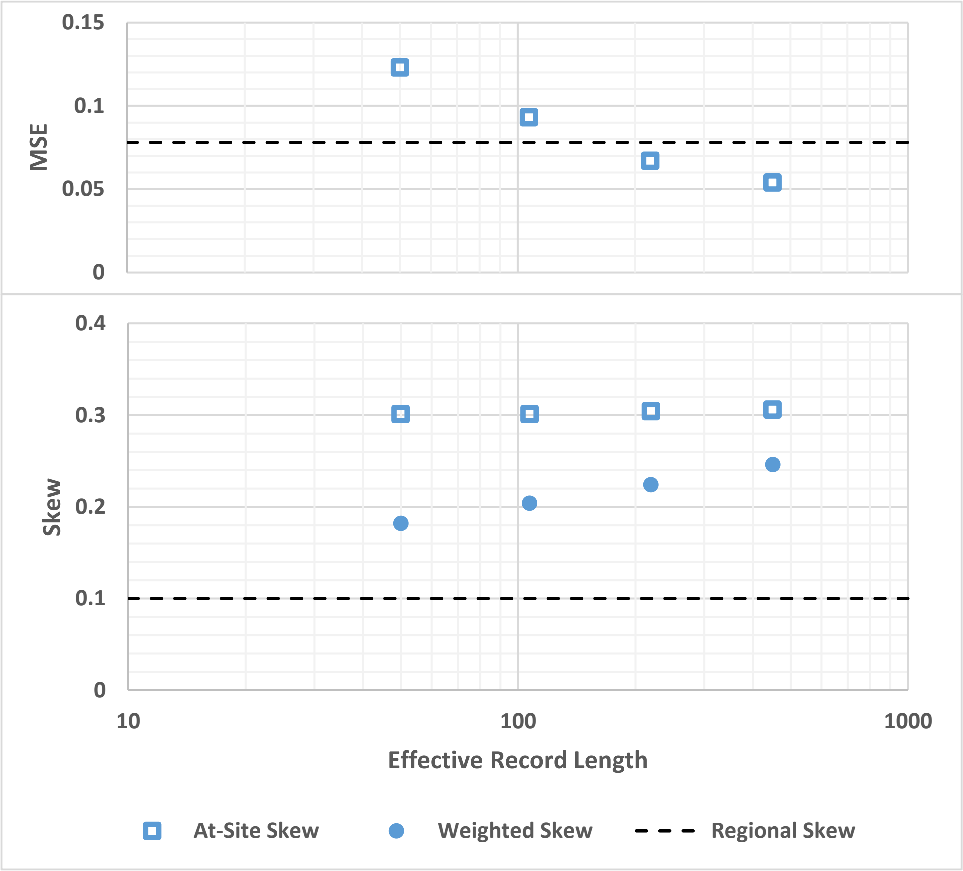

Figure 4. Fixed Parameter Example Showing Improved Weighted Skew Estimates Using Proposed Procedure

Effective Record Length. A variety of stochastic (monte-carlo) based methods exist for the purpose of modeling uncertainty in an analytical flow-frequency curve. These models are commonly used to support a variety of risk-informed decisions. Some examples include the Watershed Analysis Tool (HEC-WAT), Flood Damage Reduction Analysis (HEC-FDA), Reservoir Frequency Analysis (RMC-RFA), and Stochastic Event Flood Model (SEFM). Effective record length is commonly used as an input parameter to model the uncertainty in the flow-frequency curve using techniques such as the bootstrap (Efron, 1979) or parameter sampling distributions (USACE, 2016). Current Bulletin 17C procedures and software do not calculate or report the ERL needed for these types of analyses.

Effective record length can be defined as "the number of years of systematic data that would produce the same MSE [or quantile variance] as a given combination of historical and systematic data." (Stedinger and Cohn, 1986). When all the input data are systematic (exact), ERL is simply equal to the record length. When some input data consists of flow interval, censored, or regional skew information, ERL is unknown and must be estimated.

Cohn, Lane, and Baier (1997) proposed using Equation (3) to estimate effective record length after demonstrating that record length is asymptotically proportional to the inverse of quantile variance.

$$ERL_p = N_S \frac{Var[\hat{X}_p|N_S]}{Var[\hat{X}_p|N_T]} \tag{3}$$

where:

ERL_p	effective record length at the pth quantile
p	a quantile where AEP = 1 - p
N_S	number of systematic (exact) data
N_T	number of combined systematic, historical, and censored data
$Var[\hat{X}_p\|N_S]$	variance of the logarithm of flow at the pth quantile using only the systematic (exact) data
$Var[\hat{X}_p\|N_T]$	variance of the logarithm of flow at the pth quantile using the combined systematic, historical, censored, and regional skew information

We propose three improvements and clarifications to the ERL procedure for implementation in software that supports Bulletin 17C. The first improvement is to assume a linear relationship instead of a proportional relationship. This improves the accuracy of ERL estimates at record lengths on the order of 100 years or less. The second improvement is to define quantile variance conditional on the parameter set for the combined systematic, historical, censored, and regional data. The linear (and proportional) relationships are only valid when the parameter set (i.e., the aleatory uncertainty) is held constant. Strictly speaking, only the standard deviation (σ) and skew (γ) parameters need to be held constant. Figure 5 shows the concept of a linear relationship conditional on a fixed parameter set where each parameter set corresponds to a unique line. The third improvement is to clarify that an average effective record length should be estimated for a set of quantiles specified by the user. The selected set of quantiles should reflect the range of primary importance for the stochastic analysis. Effective record length will be different at each quantile, but typical stochastic flood models require a single representative value.

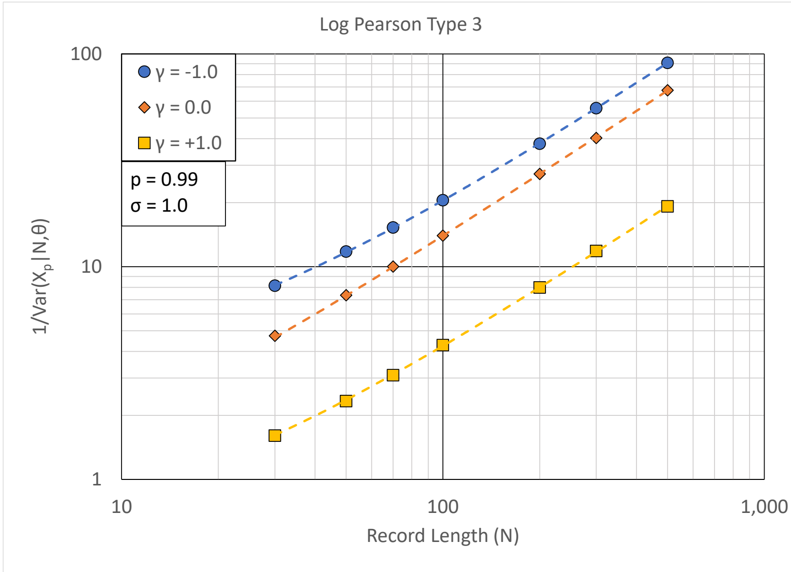

Figure 5. Example of Linear Relationship Between Record Length and Inverse of Quantile Variance Conditional on a Parameter Set

The recommended improvements can be implemented by writing a standard linear interpolation formula like Equation (4) and then solving for the unknown ERL_p. We propose using the resulting Equation (5) in software that supports Bulletin 17C to estimate and report ERL.

$$\frac{ERL_p - N_1}{Var[\hat{X}_p|N_T,\theta_T] - Var[\hat{X}_p|N_1,\theta_T]} = \frac{N_2 - N_1}{Var[\hat{X}_p|N_2,\theta_T] - Var[\hat{X}_p|N_1,\theta_T]} \quad (4)$$

$$ERL_p = N_1 + \left[\frac{Var[\hat{X}_p|N_2,\theta_T]}{Var[\hat{X}_p|N_T,\theta_T]}\right]\left[\frac{Var[\hat{X}_p|N_1,\theta_T] - Var[\hat{X}_p|N_T,\theta_T]}{Var[\hat{X}_p|N_1,\theta_T] - Var[\hat{X}_p|N_2,\theta_T]}\right][N_2 - N_1] \quad (5)$$

where:

$Var[\hat{X}_p|N_2,\theta_T]$	quantile variance for a hypothetical systematic dataset having a size of N_2 and a parameter set of Θ_T
$Var[\hat{X}_p|N_1,\theta_T]$	quantile variance for a hypothetical systematic dataset having a size of N_1 and a parameter set of Θ_T
θ_T	parameter set for the combined systematic, historical, and censored data

In theory, N_1 and N_2 can be assigned any arbitrary integer values so long as $N_1 \neq N_2$. In practice, values of $N_1=N_S$ and $N_2=N_T$ are a reasonable choice. An upper bound on the order of $N_2<500$ is another logical choice when $N_T>500$ due to a long paleoflood record.

Figure 6 and Equation (6) summarize an example estimate of ERL for Example 4 (Arkansas River at Pueblo, CO) presented in Appendix 10 of Bulletin 17C (England, et al., 2018). The total record length of 840 years yields an $ERL_{0.99}$ of 167 years. Notice that the quantile variance for the combined dataset ($Var[X_{0.99}| N_T, \Theta_T] = 0.00717$) shown in the top left panel of Figure 6 is practically the same as the quantile variance for the equivalent systematic dataset ($Var[X_{0.99}| ERL_{0.99}, \Theta_T] = 0.00712$) shown in the top right panel of Figure 6 satisfying the definition of ERL. For demonstration purposes, quantile variance for the two hypothetical systematic datasets shown in the bottom left and right panels of Figure 6 and the equivalent systematic dataset were estimated using HEC-SSP version 2.2 by generating synthetic systematic datasets having the specified record length and parameter set. Notice that the parameter sets (μ, σ, γ) for all four datasets in Figure 6 are practically the same. For software implementation, the quantile variance can be calculated directly and more accurately based on the specified record lengths and parameter set without needing actual systematic data values. A slightly more precise estimate of ERL could also be obtained by implementing an optimization routine to find the value of ERL_p that minimizes $\left|Var\left[\hat{X}_p \middle| N_T, \theta_T\right] - Var\left[\hat{X}_p \middle| ERL_p, \theta_T\right]\right|$. A solver would be trivial to implement, and the computation time would likely be negligible.

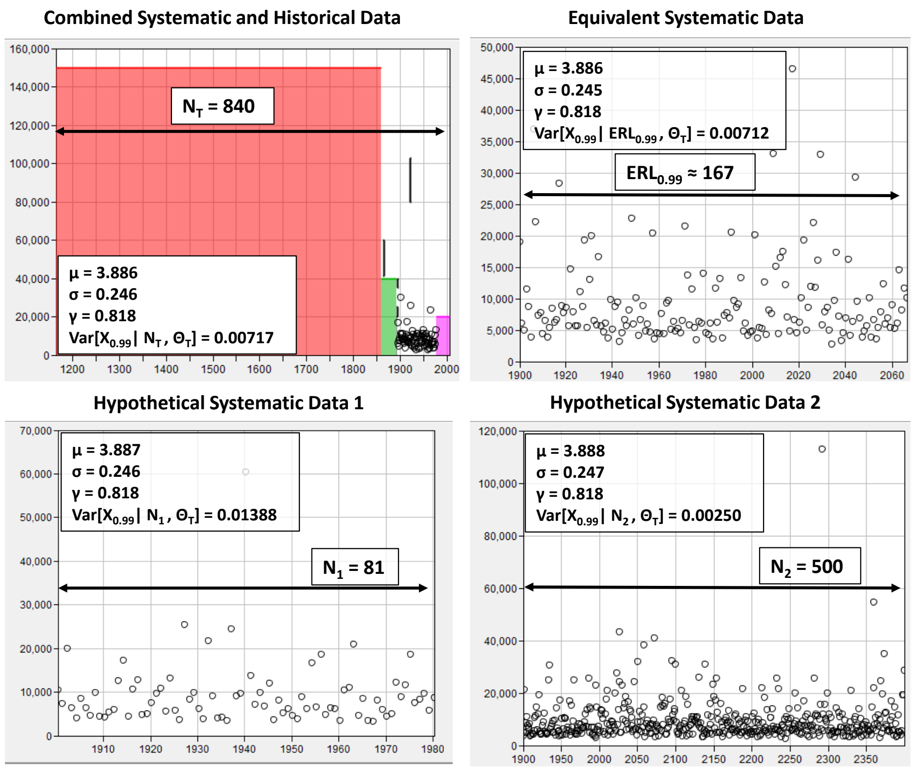

Figure 6. Example Effective Record Length Estimate for Example 4 (Arkansas River at Pueblo, CO) in Appendix 10 of Bulletin 17C

$$ERL_p = 81 + \left[\frac{0.00250}{0.00717}\right]\left[\frac{0.01388 - 0.00717}{0.01388 - 0.00250}\right][500 - 81] = 167 \; years \tag{6}$$

Plotting Positions for Interval Data. Empirical exceedance probability estimates for observed annual maxima are made using plotting positions. A generic plotting position formula for systematic (exact) observations can be expressed as (U.S. Army Corps of Engineers, 2021):

$$P_i = \frac{i - A}{n + 1 - 2A} \tag{7}$$

where:

P_i	exceedance probability of the ith observed annual maxima
i	rank of observed annual maxima from largest (i = 1) to smallest (i = n)
n	number of observed annual maxima
A	constant

An empirical probability distribution results from the application of Equation (7). Empirical distributions can be used to visually assess the fit of a parameterized analytical distribution such as Log Pearson Type III. When historical and censored data are included, an alternative plotting position methodology is needed. Hirsch and Stedinger (1987) developed a method to compute plotting positions for censored data. They emphasized the correct interpretation of the information conveyed by historical flood data, the recognition of the limited precision of estimates of the exceedance probabilities of historical floods, and showed that any estimator will be somewhat imprecise. (Hirsch & Stedinger, 1987).

Bulletin 17C recommends use of the Hirsch-Stedinger threshold exceedance plotting position procedure (England, et al., 2018). The formula for floods exceeding a single perception threshold is expressed as:

$$P_i = \frac{k}{n}\left(\frac{i - A}{n + 1 - 2A}\right) \tag{8}$$

where:

P_i	exceedance probability of the ith above threshold observed annual maxima
I	rank of the above threshold floods from largest (i = 1) to smallest (i = k)
k	number of floods above the threshold
n	total record length including the threshold

Current software uses a more generalized set of equations like Equation (8) to calculate plotting positions for above and below threshold floods given one or more thresholds. For historical flow interval data, the geometric mean (or log-average) of the user specified low and high flow values is used to determine if a flood is above or below a given threshold and to determine the rank of a flood. The geometric mean, $\bar{Q}_i$, for a flow interval can be computed as:

$$\bar{Q}_i = \sqrt{(Q_{i,l})(Q_{i,u})} \tag{9}$$

where:

$Q_{i,l}$	the low value of the flow interval
$Q_{i,u}$	the high value of the flow interval

Use of the geometric mean lacks transparency and creates an interdependency between plotting position and distribution fitting that can force a user to make a tradeoff. Tradeoffs are commonly encountered when using paleoflood data, which can include evidence of a pre-historic flood called a paleostage indicator (PSI) or evidence of an absence of flooding called a non-exceedance bound (NEB). Flow intervals are typically used to model a PSI with a corresponding perception threshold. Perception thresholds are typically used to model a NEB. The interdependency occurs because both the parameter and plotting position estimates are a function of the low and high flow values. In certain cases, the user must manipulate the low and high values or the corresponding perception threshold value to obtain the desired plotting position which could have an adverse impact on the parameter estimates.

Consider the example shown in Table 1 where there is geologic evidence of a PSI. Both the age and discharge needed to define the PSI and corresponding perception threshold are uncertain due to various geologic and hydraulic uncertainties.

Table 1. Paleostage Indicator Example

Feature	**Age (years before present)**		**Discharge (cfs)**	
PSI	Young	1100	Low	275,000
	Best Estimate	1500	Best Estimate	375,000
	Old	1800	High	425,000

The PSI was included within a flow-frequency analysis as a flow interval observation spanning 275,000 to 425,000 cfs. A best estimate perception threshold was included as a perception range spanning 375,000 cfs to infinity. The perception threshold was applied over a historical period corresponding to the best estimate age of the PSI. A chronology plot of this information is shown within Figure 7.

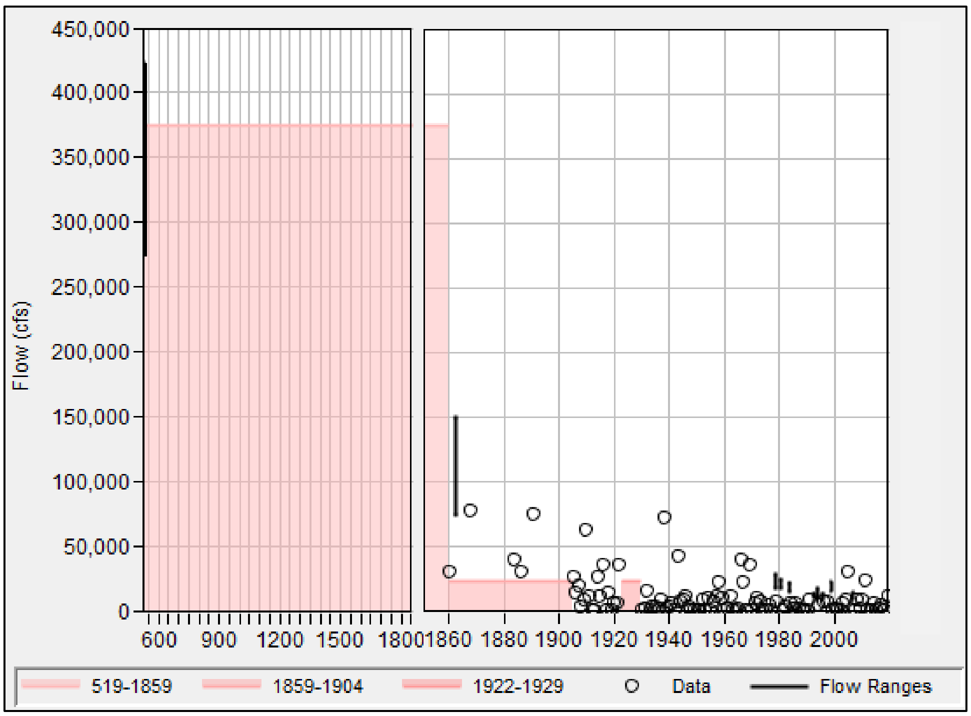

Figure 7. Chronology Plot for Plotting Position Example

Computing the flow-frequency analysis using the current procedure resulted in the information contained within Figure 8. Of particular importance is the computed plotting position for the PSI, which has an AEP on the order of 0.005 (200-year return period). This result occurs because the geometric mean of the PSI flow interval (342,000 cfs) is less than the perception threshold (375,000 cfs). The plotting position conflicts with the knowledge that the PSI exceeded all thresholds over the 1500-year period of analysis. To obtain the correct plotting position, the user must modify the perception threshold value or the PSI flow interval values so that the geometric mean of the PSI interval is greater than the threshold. However, these modifications would conflict with the best estimates and would change the parameter estimates.

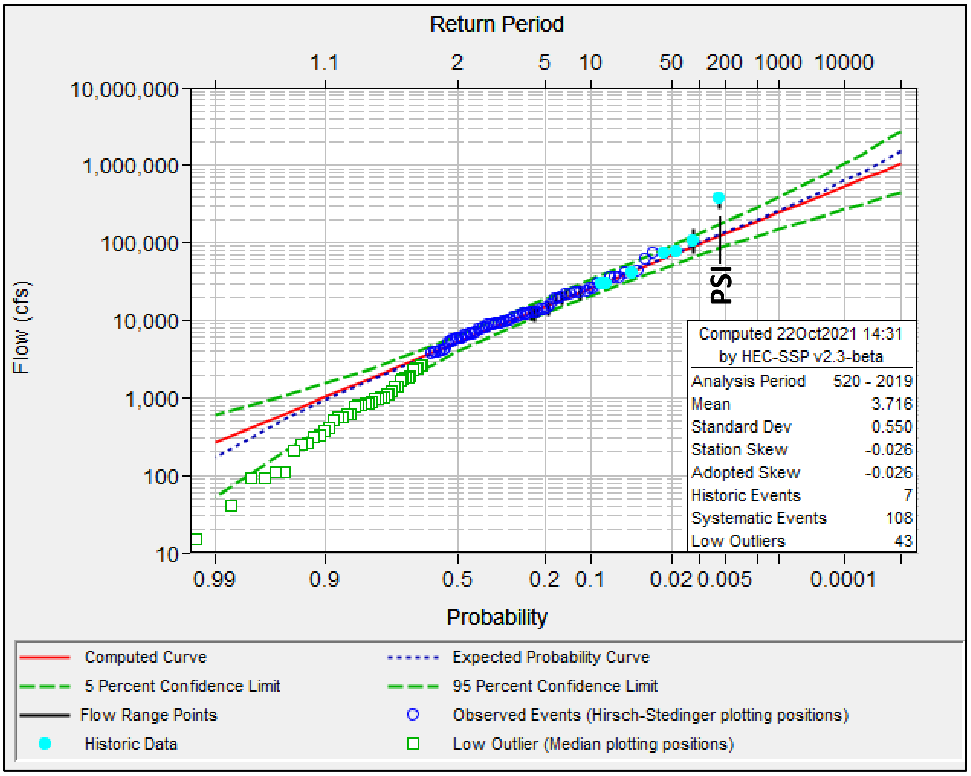

Figure 8. Example of Erroneous Plotting Positions Using Current Procedure

We propose modifying the procedure in the software by using a user-specified flow magnitude to determine if a flood represented by a flow interval is above or below a threshold and to rank the flood. Recomputing the flow-frequency analysis with a specified flow value of 375,000 cfs for the PSI results in the information contained within Figure 9. The resultant plotting position of the PSI now has an AEP on the order of 0.0003 (1 in 3000-year return period) consistent with the available information. The parameterized LPIII distribution, confidence limits, and expected probability curves are unaffected by this change. Only the plotting positions are affected. This gives the user more transparency and control to obtain the desired plotting positions without needing to manipulate inputs that affect the parameter estimates.

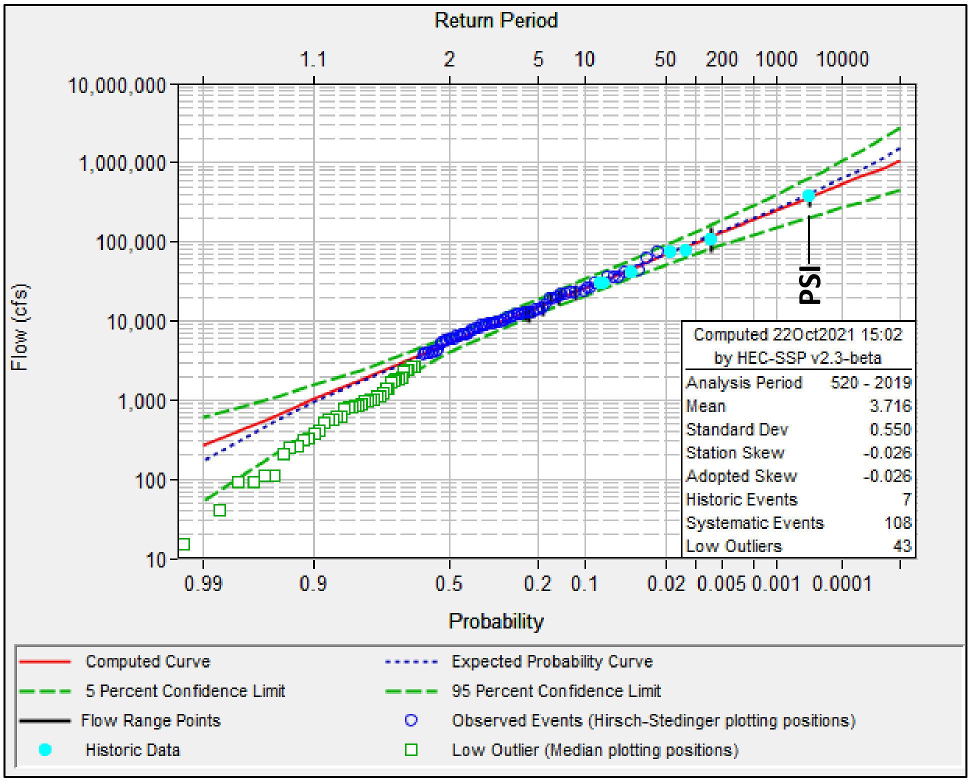

Figure 9. Example of Correct Plotting Position Using Proposed Procedure

Expected Probability. Bulletin 17C procedures and supporting software calculate and report a computed frequency curve and user-specified confidence interval. The expected flow-frequency curve is not calculated or reported. An expected probability adjustment was provided in Bulletin 17B (Interagency Advisory Committee on Water Data, 1982). However, the Bulletin 17B adjustment should not be used because it does not account for the uncertainty in the skew parameter.

The expected flow-frequency curve "can serve as a basis for computing the expected return on an investment" (Beard, 1960). A risk informed approach is necessary because it is not possible to know the future outcome of an investment decision with certainty. Given a decision and a conditional probability distribution of possible outcomes, the expected utility of the decision can be expressed by Equation (10). The integral is calculated over a complete set of possible outcomes (DeGroot, 2004).

$$E[U_D(d)] = \int f(o|d)\, U(o)\, do \tag{10}$$

where:

$E[U_D(d)]$	expected utility (U) of a decision (d)
$f(o\|d)$	probability density of possible outcomes (o) given the decision (d)
$U(o)$	utility (U) of an outcome (o)

The optimal (best) decision is the one that maximizes the expected utility. In flood hydrology, this concept applies to examples such as maximizing net economic benefits, maximizing cost effectiveness of life safety investments, or providing a desired level of protection. The expected flow-frequency curve is the optimal estimator in this context and should be used to inform decisions in flood hydrology. This can be accomplished by explicitly modeling the uncertainty in a stochastic model or by applying the expected flow-frequency curve as a single best estimate in a deterministic calculation (USACE, 1994).

Conceptually, the expected flow-frequency curve is optimal because it includes both the natural variability described by the parameters and the knowledge uncertainty described by the ERL whereas a computed flow-frequency curve only includes the natural variability component of uncertainty (USACE, 2017). Greater knowledge uncertainty due to a shorter ERL results in an asymmetrical uncertainty distribution in both the more frequent lower tail and less frequent upper tail of a flow-frequency curve. This asymmetry is properly accounted for with the expected flow-frequency curve (USACE, 1994).

The expected flow-frequency curve can be defined by the expected value of the AEP at a given value of flow. In Bayesian statistics, the expected flow-frequency curve is referred to as the posterior predictive flow-frequency curve. “Bayesian and classical [or frequentist] statistical approaches can be used to develop design flood values which, given available hydrologic information, will (on average) be exceeded with the specified 1% design probability.” (Stedinger, 1983). There are some important philosophical differences between the Bayesian and frequentist interpretations of probability. However, these differences are mostly inconsequential for the practicing hydrologist.

Given a probability distribution of annual exceedance probability conditional on a given flow, the expected flow-frequency curve can be estimated using Equation (11).

$$E[AEP|Q_i] = \int F(AEP|Q_i)\, dAEP \tag{11}$$

where:

$E[AEP|Q]$ expected value of the annual exceedance probability (AEP) at a given flow (Q_i)

$F(AEP|Q)$ cumulative distribution function of the annual exceedance probability (AEP) at a given flow (Q_i)

The cumulative distribution function of the annual exceedance probability at a given flow can be estimated directly from the confidence intervals of a flow-frequency curve. This method ensures that the expected flow-frequency curve is internally consistent with the reported confidence intervals. The following procedure is proposed for software implementation.

1. Compute flows for a range of confidence limits between 0 and 100% and a range of AEPs between 0 and 1 to obtain a two-dimensional table of flow as a function of AEP and confidence limit.
2. Invert the table from step 1 using interpolation to obtain a two-dimensional table of AEP as a function of flow and confidence limit.
3. Estimate the integral in Equation (10) using a midpoint Riemann sum for each flow value in the table from Step 2 to obtain a one-dimensional table of expected AEP as a function of flow.
4. Invert the table from step 3 by interpolation to obtain a one-dimensional table of flow as a function of expected AEP at user specified AEP values.

5. Report the resulting set of AEP vs flow values from Step 4 as the expected flow-frequency curve.

Figure 10 and Figure 11 show an example of the general concept and proposed procedure to estimate the expected flow-frequency curve at a flow of 100,000 cfs for Example 4 (Arkansas River at Pueblo, CO) presented in Appendix 10 of Bulletin 17C (England, et al., 2018). The expected value of AEP defined by Equation (11) is simply the area under the cumulative distribution function shown in Figure 11. Conceptually, the cumulative distribution function can be defined by taking a horizontal slice through the confidence limits at a given flow. The cumulative probability for a confidence limit corresponds to the value of AEP where the horizontal slice intersects the confidence limit. For this example, the horizonal slice at 100,000 cfs intersects the upper 95% confidence limit at an AEP of about 0.003 which means that $F(0.003|100{,}000) = 0.95$.

The computed AEP at 100,000 cfs is about 0.00063 (1 in 1600) and the expected AEP is about 0.001 (1 in 1000). Due to the nature of the uncertainty asymmetry, the annual exceedance probability for the expected flow-frequency curve will always be more frequent than (i.e., to the left of) the computed curve for annual exceedance probabilities less than 0.5. As the ERL increases, the expected flow-frequency curve will trend toward the computed curve.

We propose to implement the procedure described above to calculate and report the expected flow-frequency curve in software that supports Bulletin 17C. This provides the user with more information and options to apply the results of a flow-frequency analysis in a risk-informed context.

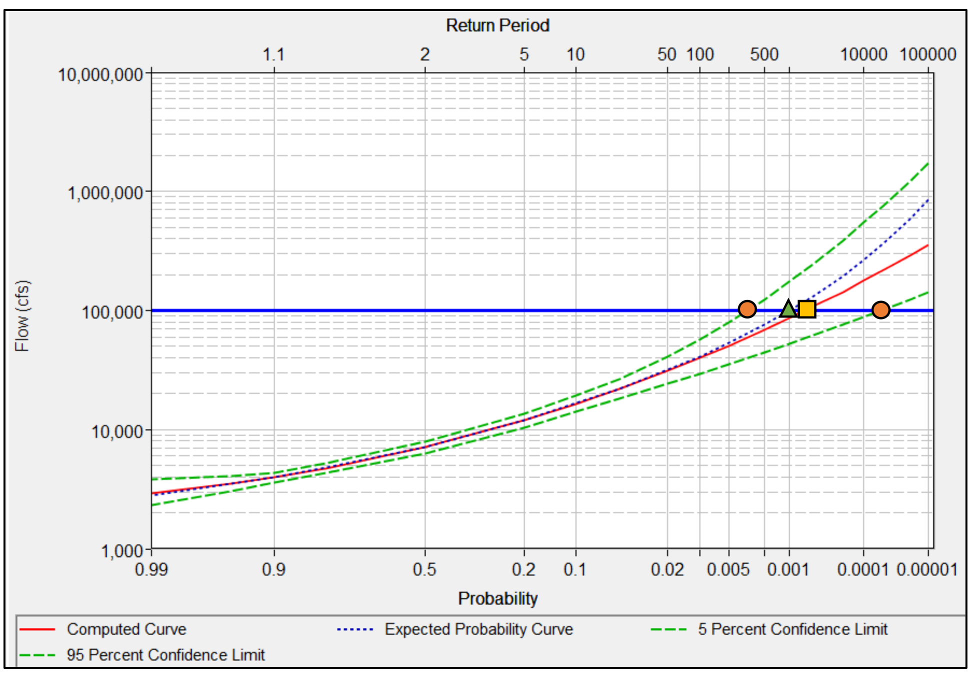

Figure 10. Example of Concept Used to Develop the Cumulative Distribution Function of Annual Exceedance Probability at a Given Flow from the Confidence Limits

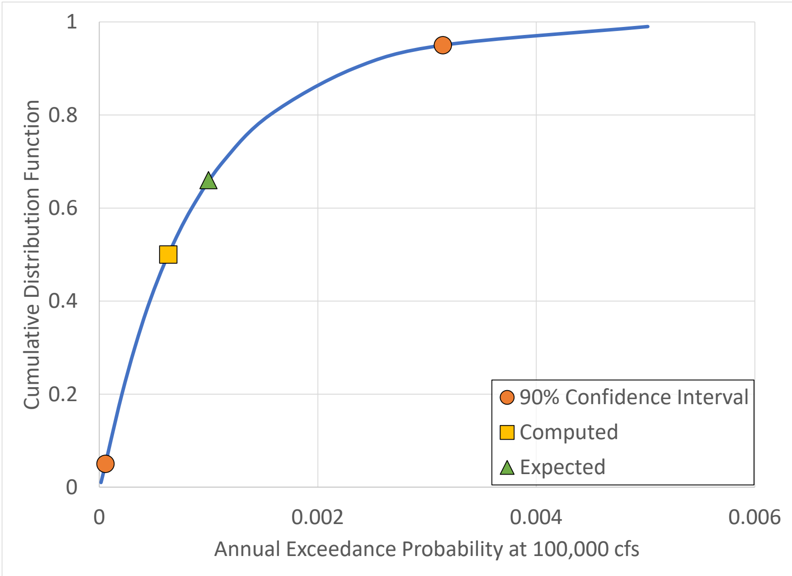

Figure 11. Example of Concept Used to Estimate the Expected Flow-Frequency Curve from the Cumulative Distribution Function of Annual Exceedance Probability at a Given Flow

Potentially Influential Low Floods. Bulletin 17C procedures and supporting software use the Multiple Grubbs-Beck Test (MGBT) to screen for PILFs. As the name implies, PILFs are small floods that act as low-outliers whose influence may be detrimental to the fit of the upper tail (i.e. the infrequent region) of a flow-frequency curve. The goals of the current procedure are to reduce the leverage of these low floods and to accommodate zero flows in a flow-frequency analysis.

By default, the current software will identify PILFs and automatically recode them using a perception range. The perception range has a low threshold equal to the Grubbs-Beck critical value and a high threshold equal to infinity. A perception threshold is then applied to every year in the period of analysis where an existing low threshold value is less than the Grubbs-Beck critical value. Identified PILFs are modified and recoded as censored flow intervals with a low value of zero and a high value equal to the Grubbs-Beck critical value. Figure 12 shows an example of the current automatic recoding procedure for Example 2 (Orestimba Creek near Newman, CA) presented in Appendix 10 of Bulletin 17C (England, et al., 2018). The flow intervals in the chronology plot on the right are shown to emphasize the recoding of the thirty identified PILFs as censored flow intervals based on the Grubbs-Beck critical value of 782 cfs.

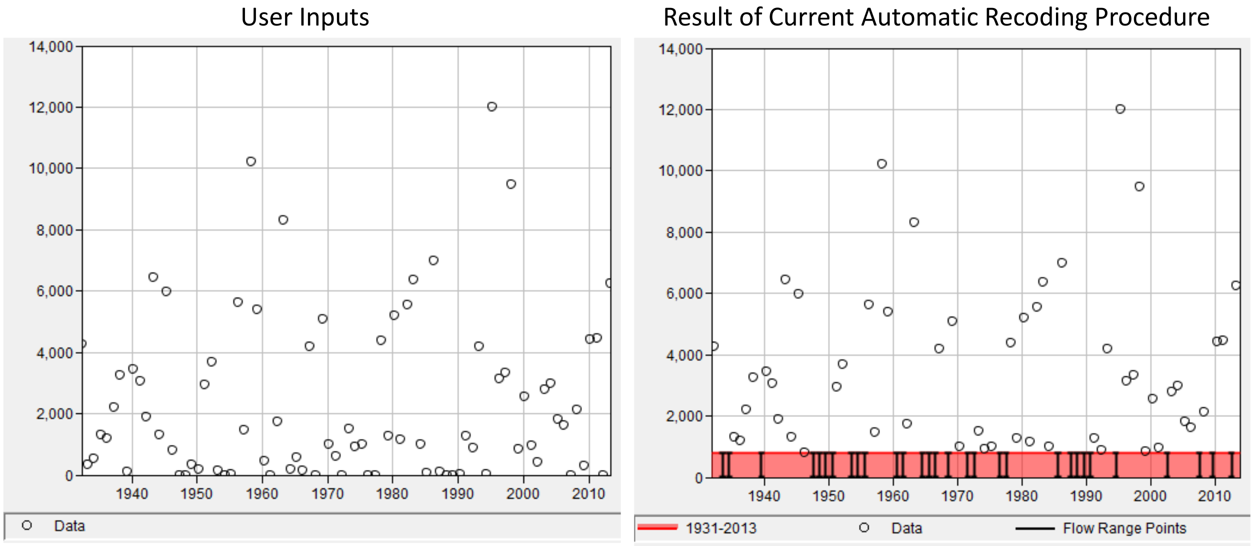

Figure 12. Depiction of Current Automatic Recoding Procedure for PILFs for Example 2 in Bulletin 17C

The current recoding procedure results in unreasonable values for the EMA computed at-site skew MSE, the confidence intervals, and the ERL. To resolve this issue, the current software calculates and reports the at-site skew MSE using Equation 6 from Bulletin 17B with the value of N set equal to the total number of years in the analysis. An example calculation of the at-site skew MSE for Example 2 in Bulletin 17C is shown as Equation (12). This procedure provides a more reasonable estimate for the at-site skew MSE. However, the procedure does not resolve the corresponding issue with the confidence intervals and ERL. Also, the at-site skew MSE will be underestimated to some degree because the assumed value for N is the upper bound based on the available information. The actual value of N will always be less than the assumed value when the analysis includes censored data. When there is a significant amount of censored data, the actual value of N could be significantly less than the assumed value resulting in a significant underestimation of the at-site skew MSE.

$$MSE_G = 10^{\left[(-0.52+0.30|-0.929|)-(0.94-0.26|-0.929|)\log_{10}\frac{82}{10}\right]} = 0.132 \qquad (12)$$

When a user overrides the default option in the current software by manually entering a user specified low-outlier threshold equal to the computed Grubbs-Beck critical value, the results should be the same. Instead, the current procedure computes and reports the at-site skew MSE using EMA. For example, the at-site skew MSE computed using EMA for Example 2 in Bulletin 17C would be 0.048 if the user manually enters a threshold value of 782 cfs. This is much less than the at-site skew MSE of 0.132 that is computed using the default procedure. This EMA estimate of MSE corresponds to an effective record length for the at-site skew on the order of 350 years which is obviously impossible given only 82 years of total record length.

More reasonable values for the at-site skew MSE using EMA along with more reasonable confidence intervals and ERL estimates can be obtained by only adding perception thresholds for years with an identified PILF. We propose modifying the automatic recoding procedure to obtain a result like Figure 13 for Example 2 in Bulletin 17C where only the thirty identified PILFs are recoded using a perception range with a corresponding censored flow interval.

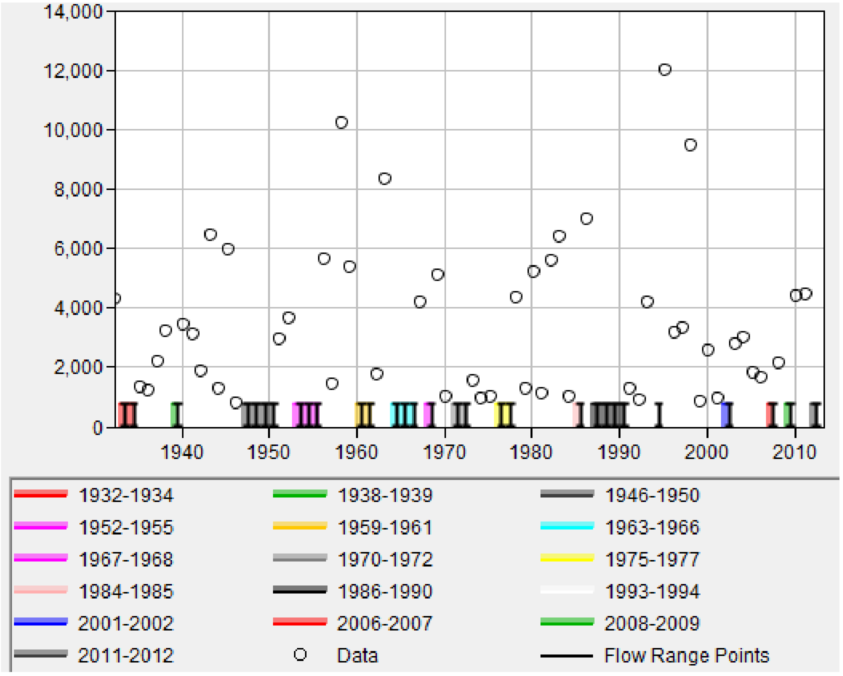

Figure 13. Depiction of Proposed Automatic Recoding Procedure for PILFs for Example 2 in Bulletin 17C

Table 2 shows a comparison of the results obtained for three different options. The first option is the current procedure that adds perception threshold(s) over the entire period of analysis and uses Equation 6 from Bulletin 17B with $N_G=N_T$ to calculate the at-site skew MSE. The second option is the current procedure with a manually entered threshold equal to the Grubbs-Beck critical value which uses EMA to calculate the at-site skew MSE. The third option is the proposed procedure that only recodes identified PILFs with a perception range and calculates at-site skew MSE using EMA.

Table 2. Comparison of PILF Recoding Options for Example 2 in Bulletin 17C

Parameter	*Option 1* *Current Procedure*	*Option 2* *Current Override Procedure*	*Option 3* *Proposed Procedure*
N_S	52	52	52
N_T	82	82	82
MSE_G	0.132	0.048	0.136
N_G	82	350	79
$Var[\hat{X}_{0.99}]$	0.00879	0.00879	0.0227
$ERL_{0.99}$	220	220	71

Results for this example suggest that Option 3 should be the preferred option because it is the only option that obtains realistic values for the at-site skew MSE and the ERL given the available information. The value of the at-site skew MSE and the corresponding value of N_G for Option 1 are reasonable but not as reasonable as Option 3. Some information is lost when the PILFs are censored which should result in $N<N_T$ and a larger at-site skew MSE. The value of at-site skew MSE and the corresponding value of N_G for Option 2 are not realistic given the available information. An effective record length for the at-site skew on the order of 350 years is not possible given only 82 years of total record length. For the same reason, the value of $ERL_{0.99}$ for Option 1 and 2 is not realistic. Only Option 3 provides a complete set of plausible values that are consistent with the available information. Similar results and conclusions can be obtained with Examples 3, 5, and 6 in Bulletin 17C.

CONCLUSION

Research and documentation related to the findings presented in this paper is ongoing. Some of the proposals have already been implemented within the USACE HEC-SSP software while others are still being evaluated and tested. Discussions with USGS are underway to collaborate on this research with the goal of reaching consensus on future revisions to both the HEC-SSP and PeakFQ software.

The five proposed changes to the Bulletin 17C implementation procedures are summarized below.

1. Use Equation 7-10 in Bulletin 17C to estimate the expected moment for weighted skew. This replaces the use of Equation 7-3 and the weighted skew equation.
2. Calculate and report an average effective record length for a range of user specified AEPs for use in stochastic modeling. The method for estimating ERL assumes a practically linear relationship between record length and quantile variance.
3. Use a user specified flow value for plotting positions associated with a flow interval to provide more user control and transparency. This replaces the use of the geometric mean of the low and high flow values.
4. Calculate and report the expected flow-frequency curve by integrating over the computed confidence intervals. This provides more information and options for the user.
5. Change the automatic recoding of PILFs to only apply a perception threshold in years having an identified PILF. This provides a more reasonable estimate of the at-site skew MSE, confidence intervals, and ERL.

Our goal is to motivate collaboration with other interested developers, practitioners, and organizations to continue making incremental improvements to the Bulletin 17C procedures and associated software.

ACKNOWLEDGEMENTS

Multiple USACE colleagues contributed to this research effort. The authors recognize Dr. John England, Dr. Beth Faber, Haden Smith, and Greg Karlovits for their insights, advice, and expertise.

REFERENCES

Beard, L. R. (1960). Probability Estimates Based on Small Normal-Distribution Samples. *Journal of Geophysical Research, V. 65, No. 7*, 2143-2148.

Cohn, T. A., Lane, W. M., & Baier, W. G. (1997). An Algorithm for Computing Moments-Based Flood Quantile Estimates When Historical Flood Information is Available. *Water Resources Research*, 2089-2096.

Cohn, T. A., Lane, W. M., & Stedinger, J. R. (2001). Confidence Intervals for Expected Moments Algorithm Flood Quantile Estimates. *Water Resources Research*, 1695-1706.

England, J. F., Cohn, T. A., Faber, B. A., Stedinger, J. R., Thomas, W. O., Veilleux, A. G., . . . Mason, R. R. (2018). *Guildelines for Determining Flood Flow Frequency, Bulletin 17C.* Washington, D.C.: U.S. Department of the Interior.

Griffis, V. W. (2003). *Evaluation of Log-Pearson Type 3 Flood Frequency Analysis Methods Addressing Regional Skew and Low Outliers.* Ithaca, NY: Cornell University.

Griffis, V. W., Stedinger, J. R., & Cohn, T. A. (2004). Log Pearson Type 3 Quantile Estimators with Regional Skew Information and Low Outlier Adjustments. *Water Resources Research, Volume 40, Number 7*.

Hirsch, R. M., & Stedinger, J. R. (1987). Plotting Positions for Historical Floods and Their Precision. *Water Resources Research, Volume 23, Number 4*, 715-727.

Interagency Advisory Committee on Water Data. (1982). *Guidelines for Determining Flood Flow Frequency, Bulletin 17B.* Reston, VA: U.S. Geological Survey.

Tasker, G. D. (1978). Flood Frequency Analysis with a Generalized Skew. *Water Resources Research, Volume 14, Number 2*, 373-376.

U.S. Army Corps of Engineers. (2021). *HEC-SSP Statistical Software Package User's Manual, Version 2.3.* Davis, CA: Hydrologic Engineering Center.

Water Resources Council. (1976). *Guidelines for Determining Flood Flow Frequency, Bulletin 17.* Washington, D.C.: Hydrology Committee.