One-Dimensional Model for Mud Flows

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    In this paper a transient, one-dimensional model for dynamic flood routing of mud flows is presented. The governing equations of mass and momentum conservation incorporate laminar flow resistance effects and utilize a power law expression to represent the cross-sectional geometry of the channel. The equations are solved by the method of characteristics on fixed time lines and program execution is performed on a microcomputer. Numerical results are compared with published experiment data for a laminar flow, dambreak problem of viscous oil.

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ABSTRACT

In this paper a transient, one-dimensional model for dynamic flood routing of mud flows is presented. The governing equations of mass and momentum conservation incorporate laminar flow resistance effects and utilize a power law expression to represent the cross-sectional geometry of the channel. The equations are solved by the method of characteristics on fixed time lines and program execution is performed on a micro-computer. Numerical results are compared with published experimental data for a laminar flow, dambreak problem of a viscous oil.

INTRODUCTION

During the spring of 1983, widespread landslides and debris flows caused an estimated 250 million dollars in damage in the state of Utah. Along a thirty-mile length of the Wasatch Front Mountains, over ninety significant landslides and debris flows sent torrents of mud, debris and water down steep canyons onto residential areas located on alluvial fans at the base of the mountains.

The ability to model these types of events is clearly needed and will be useful in preparing maps which delineate potential flood damage areas. The purpose of this paper is to present a one-dimensional mathematical model which can be used to route a mudflow down a confining channel. Equations of mass and momentum conservation are presented, with frictional resistance terms, which account for the laminar flow of a Bingham plastic fluid. The equations are solved by the method of characteristics on fixed time lines. To verify the model, comparison is made with experimental results of a laminar flow dambreak problem.

GOVERNING EQUATIONS

The flow is governed by the equations of mass and momentum conservation which are given respectively by [6]

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in which \( x \) = coordinate along the channel; \( t \) = time; \( A \) = cross-sectional area of flow; \( V \) = average velocity; \( B \) = channel top width; \( y \) = flow depth; \( A' \) = rate of change of area with \( x \) for a constant depth (nonprismatic term); \( g \) = gravitational constant; \( S_o \) = slope of the channel bottom; and \( S_f \) = resistance slope.

In most hydraulic applications, the flow is turbulent and \( S_f \) is generally given by Manning's equation. The flow of mud presents an entirely different situation. DeLeon and Jeppson [1] summarize the data from a number of debris flows, mud flows and pipe sludge flows and conclude that the flow is usually laminar. By fitting a line through a number of data points, these authors postulate a power law relation between the Chezy coefficient and the flow Reynolds number. Jeyapalan et al. [2], in their analysis of mine tailing dam failures, develop an expression for \( S_f \) by analyzing the laminar flow of materials with Bingham plastic fluid characteristics. Other researchers, [1,5] have noted a similar behavior for mud flows, which often exhibit plug like flow with a critical yield stress. In this work, the resistance term for a Bingham plastic fluid is adopted [2]. Mathematically,

\[
S_f = \frac{2\eta_p Vh^2}{\gamma y^2R^2} + \frac{\tau_y h}{\gamma yR}
\]  

in which \( \eta_p \) = plastic viscosity; \( \gamma \) = unit weight of the fluid; \( \tau_y \) = yield stress of the fluid; \( h \) = hydraulic depth; and \( R \) = hydraulic radius. The first term on the right hand side of Eq. 3 is similar in form to the expression postulated by DeLeon and Jeppson [1].

Equations 1 and 2 are hyperbolic in nature and have the property that, through linear combination, they can be reduced to equations involving differentiation in one less direction than the original equations [6]. This characteristic form for Eqs. 1 and 2 is given by

\[
\frac{dt}{dt} (V + \omega) = g(S_o - S_f) + \frac{c}{A} VA' + (V + c)L_a \frac{g}{c^2} \frac{c}{\delta x} d\tau
\]

\[
\frac{dx}{dt} = V + c
\]

in which \( c \) = celerity of an elementary gravity wave is given by

\[
c = \left(\frac{gA}{B}\right)^{\frac{1}{2}}
\]
and \( \omega \) = Escoffier stage variable is given by

\[
\omega = \int_{0}^{y} \frac{g}{c} \, d\eta
\]  

(7)

Eqs. 4 comprise a forward (+) and a backward (-) characteristic equation valid on the curves in the x-t plane defined by Eqs. 5, respectively.

A power law expression is used to represent the top width and area in Eqs. 4 and 5. Mathematically,

\[
B = (k_L + k_R) \, y^m
\]  

(8)

\[
A = \left( \frac{k_L + k_R}{m + 1} \right) \, y^{m+1}
\]  

(9)

Here \( k_L \) and \( k_R \) define the left and right width at any depth \( y \) and the exponent \( m \) defines the shape of the cross-section. The parameters \( k_L \), \( k_R \) and \( m \) can be specified functions of distance \( x \) to capture the nonprismatic nature of the channel. Using the definitions of Eqs. 8 and 9, Eqs. 6 and 7 reduce to

\[
c = \left( \frac{g}{m + 1} \right)^{1/2}
\]  

(10)

\[
\omega = 2[g(m + 1)y]^{1/2}
\]  

(11)

NUMERICAL SOLUTION

The numerical solution of Eqs. 4 and 5 is developed with reference to Fig. 1. At a sequence of points \( x_k \), \( k = 1,2,\ldots,n \), at some time \( t_i \), the solution is known. It is desired to find the solution for the points \( x_k \) on time line \( t_{i+1} \), an interval \( \Delta t \) later. The characteristic

Fig. 1 - Characteristics Computational Scheme.
curves in Fig. 1, i.e., L-P and R-P are approximated by parabolas in the x-t plane. With this approximation, the finite difference form of Eqs. 5 is given by

\[ \frac{x_p - x_L}{\delta t} = \lambda_L (V_L + c_L) + \lambda_p (V_p + c_p) \]  \tag{12}

\[ \frac{x_p - x_R}{\delta t} = \lambda_R (V_R - c_R) + \lambda_p (V_p - c_p) \]  \tag{13}

in which \( \lambda_L = \lambda_p = \lambda_R = \frac{1}{2} \). The forward and backward version of Eqs. 4 are also written in finite difference form. Mathematically,

\[ \frac{(V_p + \omega_p) - (V_L + \omega_L)}{\delta t} = \lambda_L F^+_L + \lambda_p F^+_p \]  \tag{14}

\[ \frac{(V_p - \omega_p) - (V_R - \omega_R)}{\delta t} = \lambda_R F^-_R + \lambda_p F^-_p \]  \tag{15}

in which

\[ F^\pm = g(S_0 - S_f) + \frac{c}{A} V_B \frac{V}{x} \pm (V \pm c) \int_0^Y \frac{g}{c^2} \frac{\partial c}{\partial x} \, \mathrm{d}n \]  \tag{16}

The set of four nonlinear equations, Eqs. 12-15, determines the locations of points L and R as well as \( V_p \) and \( y_p \). The variation of \( y_L \), \( V_L \), \( y_R \) and \( V_R \) is determined by parabolic interpolation along time line \( t_i \). A simple search procedure assures that the interpolation nodes \((x_{k-1}, x_k, x_{k+1})\) always straddle the points in question, so that extrapolation is avoided.

Eqs. 12-15 are solved iteratively by Newton's method \([4]\). A first guess to the solution is found by solving a linear version of Eqs. 12-15 in which \( \lambda_L = \lambda_R = 1 \) and \( \lambda_p = 0 \). The equations are solved at a number of points between \( x_1 \) and \( x_n \) to define the wave profile. At the boundaries of the flow domain, if only the velocity or depth is specified, the remaining unknown is determined by application of the appropriate backward or forward characteristic equation. For the case of advance on a dry bed, Whitham's assumption is used, i.e., \( V_B = V_{Bn-1} \).

During the early stages of flooding, the effects of boundary roughness and channel slope are small. A solution which ignores friction and slope is therefore used as the initial condition from which to start the numerical solution.
RESULTS

The model is compared with several dambreak experiments performed by Jeyapalan et al. [3]. In these experiments, oil is used to simulate a laminar flow of a viscous fluid. The experiments are conducted in a 6 foot long glass flume which has a constant width of 1 foot. The dam is located 4 feet from the downstream edge of the flume giving a reservoir length of 2 feet. Table I gives the parameters characterizing the examples presented herein. The test numbers listed in Table I correspond to several of the flood examples presented in [3]. In Table I, \( H_0 \) = depth of oil immediately behind the dam before failure; \( \beta \) = bottom slope of the flume; and for all cases \( \gamma = 0 \).

Results of the numerical simulation are presented in Figs. 2-3 and compared with the available experimental data. The agreement between theory and experiment is generally good. The numerical algorithm is programmed in Fortran and executed on an Apple Macintosh micro-computer. Computation times are on the order of 0.25-0.34 seconds per computational node.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>( H_0 ) (ft)</th>
<th>( \beta ) (degrees)</th>
<th>( \gamma ) (1b/ft(^3))</th>
<th>( \eta ) (1b sec/ft(^2))</th>
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<tr>
<td>2</td>
<td>0.50</td>
<td>0</td>
<td>56</td>
<td>0.078</td>
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<tr>
<td>7</td>
<td>0.50</td>
<td>0</td>
<td>56</td>
<td>0.156</td>
</tr>
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</table>

Fig. 2 - Wave Advance.
Fig. 3 - Wave Profile at t=1.95 sec., Test 2.

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REFERENCES


TP-70  Corps of Engineers Experience with Automatic Calibration of a Precipitation-Runoff Model
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