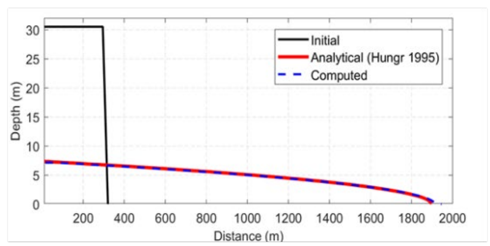
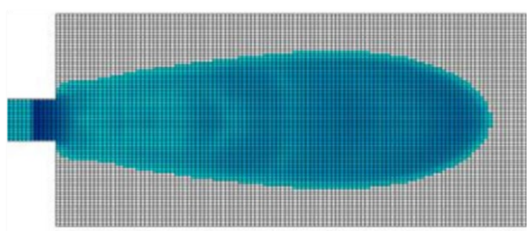
US Army Corps
of Engineers
Hydrologic Engineering Center

HEC-RAS
Mud and Debris Flow Manual





Version 6.0
September 2020

Approved for Public Release. Distribution Unlimited.

CPD-xx

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YYYY)*	**2. REPORT TYPE**	**3. DATES COVERED** *(From - To)*
September 2020	Computer Program Documentation	

4. TITLE AND SUBTITLE
HEC-RAS
Mud and Debris Flow Manual
Version 6.0

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)
Stanford Gibson PhD, CEIWR-HEC
Alejandro Sánchez PhD, CEIWR-HEC

5d. PROJECT NUMBER

5e. TASK NUMBER

5F. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
US Army Corps of Engineers
Institute for Water Resources
Hydrologic Engineering Center (HEC)
609 Second Street
Davis, CA 95616-4687

8. PERFORMING ORGANIZATION REPORT NUMBER
CPD-68

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/ MONITOR'S ACRONYM(S)

11. SPONSOR/ MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT
HEC-RAS version 6.0 includes mud and debris flow capabilities in 1D and 2D unsteady flow. These capacities use the single-phase, non-Newtonian equations and algorithms in DebrisLib, a USACE library developed jointly by HEC and the ERDC-Coastal and Hydraulics Laboratory. Users can define material properties and select a rheological model for their geophysical flows and HEC-RAS will compute an internal loss term in the momentum equation that accounts for particle interactions. This document describes the user inputs, parameters, model selection criteria and algorithms available to compute mud and debris flows in HEC-RAS.

15. SUBJECT TERMS
Mud flow, Debris flow, non-Newtonian, post wildfire, mine tailing dam breach

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON:** Stanford Gibson
a. REPORT	**b. ABSTRACT**	**c. THIS PAGE**			
U	U	U	UU	962	**19b. TELEPHONE NUMBER**

Standard Form 298 (Rev. 8/98)
Prescribed by ANSI Std. Z39-18

HEC-RAS
Mud and Debris Flow
Manual

September 2020

US Army Corps of Engineers
Institute for Water Resources
Hydrologic Engineering Center
609 Second Street
Davis, CA 95616

(530) 756-1104
(530) 756-8250 FAX
www.hec.usace.army.mil

CPD-68?

River Analysis System, HEC-RAS, User's Manual

HEC cannot provide technical support for this software to non-Corps users. See our software vendors list (on our web page) to locate organizations that provide the program, documentation, and support services for a fee. However, we will respond to all documented instances of program errors. Documented errors are bugs in the software due to programming mistakes not model problems due to user-entered data.

Terms and Conditions of Use:

Use of the software described by this document is controlled by certain terms and conditions. The user must acknowledge and agree to be bound by the terms and conditions of usage before the software can be installed or used. The software described by this document can be downloaded for free from our internet site (www.hec.usace.army.mil).

The United States Government, US Army Corps of Engineers, Hydrologic Engineering Center ("HEC") grants to the user the rights to install the HEC River Analysis System (HEC-RAS) "the Software" (either from a disk copy obtained from HEC, a distributor or another user or by downloading it from a network) and to use, copy and/or distribute copies of the Software to other users, subject to the following Terms and Conditions for Use:

All copies of the Software received or reproduced by or for user pursuant to the authority of this Terms and Conditions for Use will be and remain the property of HEC.

User may reproduce and distribute the Software provided that the recipient agrees to the Terms and Conditions for Use noted herein.

HEC is solely responsible for the content of the Software. The Software may not be modified, abridged, decompiled, disassembled, unobfuscated or reverse engineered. The user is solely responsible for the content, interactions, and effects of any and all amendments, if present, whether they be extension modules, language resource bundles, scripts or any other amendment.

The name "HEC-RAS" must not be used to endorse or promote products derived from the Software. Products derived from the Software may not be called "HEC- RAS" nor may any part of the "HEC- RAS " name appear within the name of derived products.

No part of this Terms and Conditions for Use may be modified, deleted or obliterated from the Software.

No part of the Software may be exported or re-exported in contravention of U.S. export laws or regulations.

Waiver of Warranty:

THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS PROVIDE HEC-WAT \"AS IS,\" WITHOUT ANY WARRANTY OR CONDITION, EXPRESS, IMPLIED OR STATUTORY, AND SPECIFICALLY DISCLAIM ANY IMPLIED WARRANTIES OF TITLE, MERCHANTABILITY, FITNESS FOR A PARTICULAR PURPOSE AND NON-INFRINGEMENT. Depending on state law, the foregoing disclaimer may not apply to you, and you may also have other legal rights that vary from state to state.

Limitation of Liability:

IN NO EVENT SHALL THE UNITED STATES GOVERNMENT AND ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, BE LIABLE FOR LOST PROFITS OR ANY SPECIAL, INCIDENTAL OR CONSEQUENTIAL DAMAGES ARISING OUT OF OR IN CONNECTION WITH USE OF HEC-WAT REGARDLESS OF CAUSE, INCLUDING NEGLIGENCE.

THE UNITED STATES GOVERNMENT'S LIABILITY, AND THE LIABILITY OF ITS AGENCIES, OFFICIALS, REPRESENTATIVES, AND EMPLOYEES, INCLUDING ITS CONTRACTORS AND SUPPLIERS, TO YOU OR ANY THIRD PARTIES IN ANY CIRCUMSTANCE IS LIMITED TO THE REPLACEMENT OF CERTIFIED COPIES OF HEC-RAS WITH IDENTIFIED ERRORS CORRECTED. Depending on state law, the above limitation or exclusion may not apply to you.

Indemnity:

As a voluntary user of HEC- RAS you agree to indemnify and hold the United States Government, and its agencies, officials, representatives, and employees, including its contractors and suppliers, harmless from any claim or demand, including reasonable attorneys' fees, made by any third party due to or arising out of your use of HEC- RAS or breach of this Agreement or your violation of any law or the rights of a third party.

Assent:

By using this program you voluntarily accept these terms and conditions. If you do not agree to these terms and conditions, uninstall the program and return any program materials to HEC (if you downloaded the program and do not have disk media, please delete all copies, and cease using the program.)

Cover sketch adapted from: Flood Plain Management Program, Handbook for Public Officials, State of California, Department of Water Resources, August 1986

Table of Contents

CHAPTER 1 **NON-NEWTONIAN- USER MANUAL**

Introduction

At very high solid concentrations fluids begin to depart from some of the basic hydraulic assumptions in HEC-RAS. In particular, post-wildfire storm events and mine-tailing dam breaches tend to carry enough sediment and other solids to change the flow physics and make the "Newtonian" fluid assumptions used throughout HEC-RAS inappropriate. The Mud and Debris module in HEC-RAS uses DebrisLib (Floyd et al., 2019) to account for internal losses that affect these high-concentration flows, and applies non-Newtonian models in HEC-RAS.

There are a range of approaches to simulating non-Newtonian fluids including single-phase and multi-phase approaches. The current capabilities in HEC-RAS use single phase approaches, which model fluid behavior with rheological models (i.e. stress-strain relationships).

Incorporating Non-Newtonian Effects into the Hydraulic Equations

The Technical Reference Manual includes a detailed description of how the non-Newtonian terms fit into the unsteady flow equations in HEC-RAS. But the basic idea is that HEC-RAS adds an additional dimensionless "loss slope" to the friction slope that calculates friction losses in the Newtonian momentum equation in HEC-RAS.

The momentum equation is:

$$\frac{\partial Q}{\partial t} + \frac{\partial QV}{\partial x} + gA\left(\frac{\partial z}{\partial x} + S_f\right) = 0$$

(where S_f stands in for all of the dimensionless loss "slopes" in Newtonian simulations including expansion and contraction and wind). The single-phase approach to mud and debris flow, simply adds another dimensionless loss slope, a mud and debris slope (S_{MD}):

$$\frac{\partial Q}{\partial t} + \frac{\partial QV}{\partial x} + gA\left(\frac{\partial z}{\partial x} + S_f + S_{MD}\right) = 0$$

Casting the non-Newtonian effects as a "friction" slope is the mathematical move that allows us to import rheological[1] theory into the momentum equation, because we can connect this term to the expected stress-strain behavior of different materials.

[1] "Rheology" is simply the study of how materials deform under stress. Here it is just short hand for "theoretical stress-strain relationships" like those in the right column of Figure 1-4.

The equation for shear stress is:

$$\tau = \gamma R S_f$$

Where γ is the unit weight of the fluid, R is the hydraulic radius and S_f is the friction slope.

Therefore, the friction slope can be expressed as a function of the shear and two known or specified variables:

$$S_f = \frac{\tau}{\gamma R}$$

We can compute the Mud and Debris slope the same way, so it is proportional to an internal shear stress (by the ratio of variables known or specified in the model):

$$S_{MD} = \frac{\tau_{internal\ fluid}}{\gamma R}$$

If we can express the internal losses of the fluid in terms of an internal shear stress, we can incorporate those effects into the momentum equation in HEC-RAS. These shear stresses come from Rheological models.

Rheology (Stress-Strain Relationships) of Non-Newtonian Fluids

Rheology is the study of how materials deform under stress. So "rheological models" are often expresses as simple relationships between stress and strain. Standard hydraulic models already assume a rheological model for hydrodynamic simulations. They assume that water begins to "deform" (movement or strain) under any stress (zero intercept on the stress-strain relationship), the strain increases linearly with the stress, and the water viscocity is the ratio between stress and strain (Figure 1-1a – left). These are the assumptions of "Newtonian" flow.

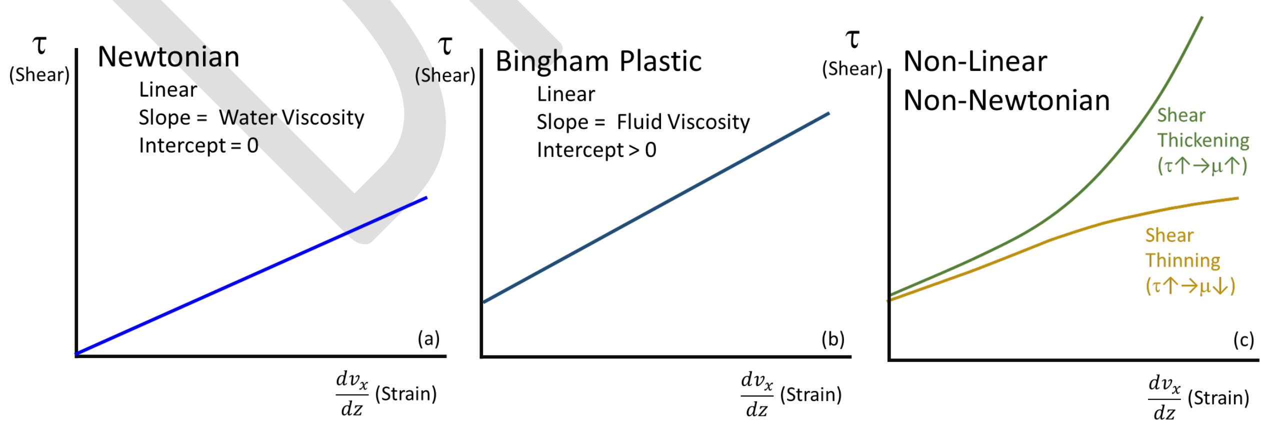


Figure 1-1: Rheological models used to simulate (a) clear water and (b,c) mud and debris flows.

When fluids diverge from these assumptions including a non-zero stress-strain intercept (Figure 1-1b – center) or a non-linear stress-strain relationship, or both (Figure 1-1b – right).

Taxonomy of Mud and Debris Flows

These high-concentration flows do not all depart from the Newtonian assumptions in the same way. As concentration increases, and particle interactions become more important to the fluid energy losses. But the size of the solids also affects the rheological properties of the fluid. Because of this complexity, the categories and taxonomy of natural and anthropogenic non-Newtonian flows can be confusing. The interacting effects of concentration and grain size are both captured in the taxonomies in Different practitioners and agencies might use the terms "debris flow," "mud flow," Hyperconcentrated flow," "land slide" and "avalanche" to refer to different and overlapping processes.

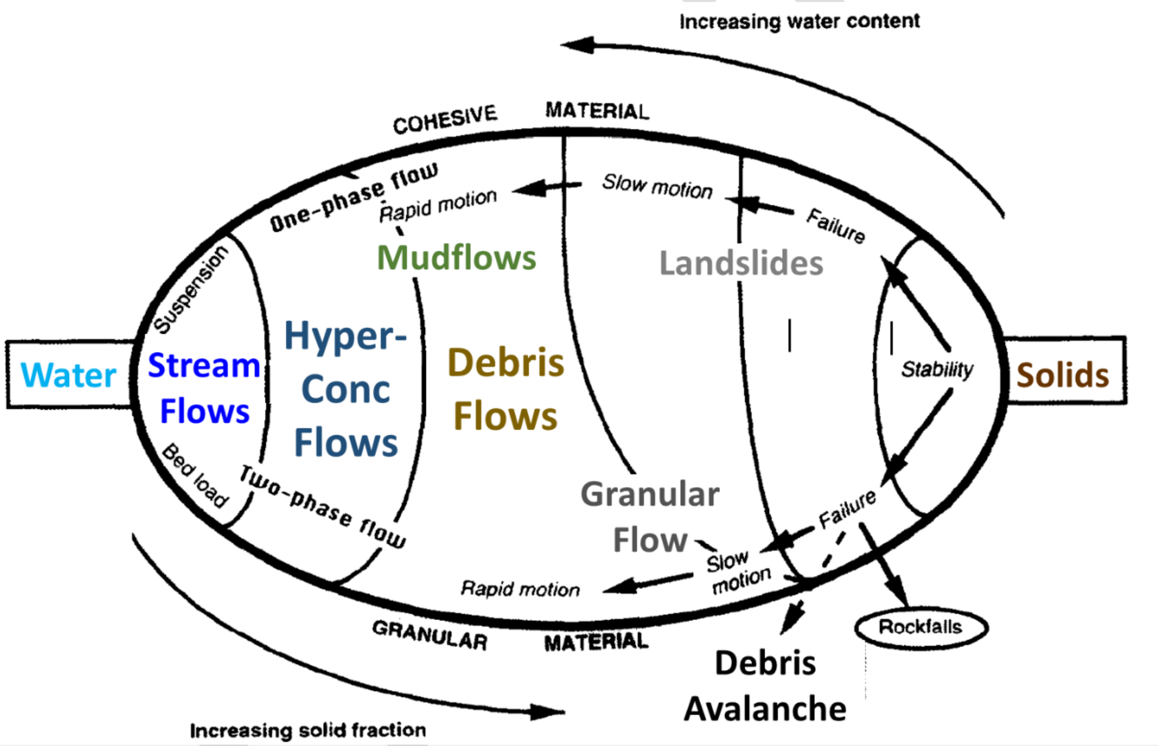


Figure 1-2: Coussot and Meunier's (1986) taxonomy of Geologic flows.

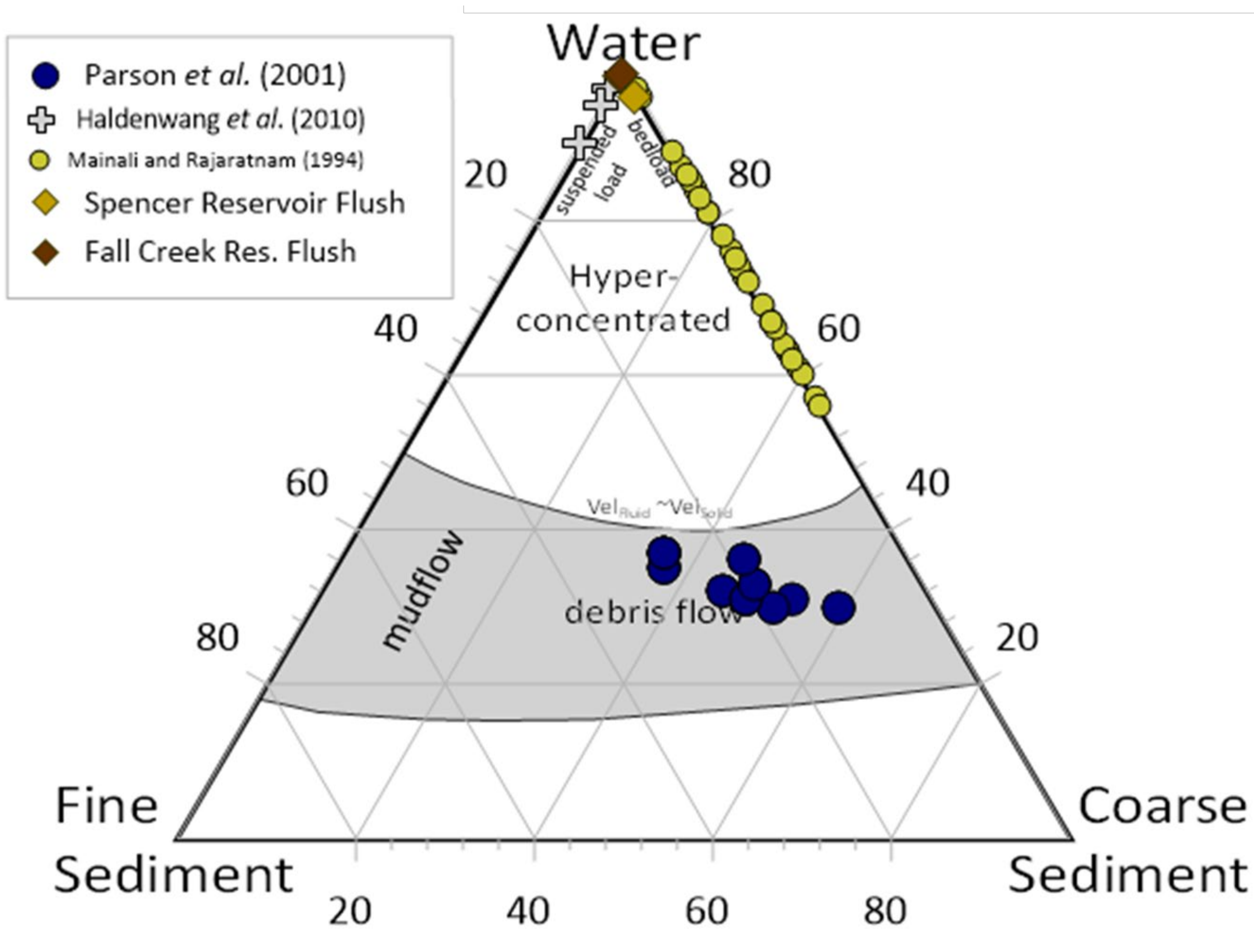
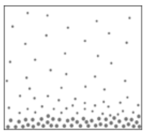
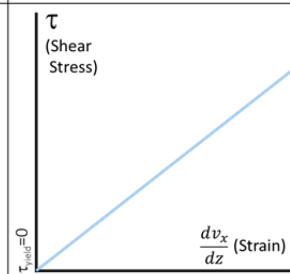
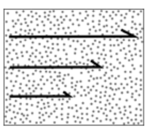
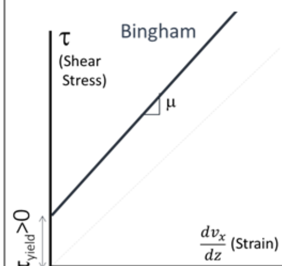
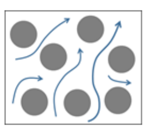
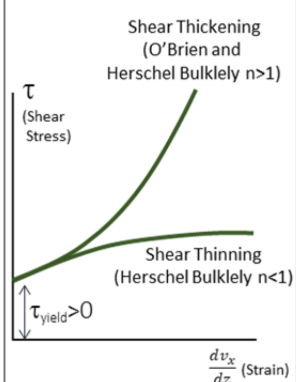
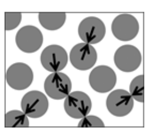
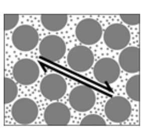
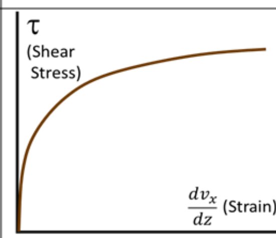


Figure 1-3: Data sets used to test the HEC-RAS non-Newtonian model plotted on a modified verison of the Philip and Davies (1991) taxonomy (from Gibson et al., 2020 in revision).

Because the geophysical flows we would like to model with HEC-RAS have different rheological properties and classifications, HEC-RAS follows the taxonomy in DebrisLib[2] which is laid out in Figure 1-4 (also see Figure 2-1). As the sediment load increases and gets coarser, the flow transitions from Newtonian, to hperconcetrated, mud, debris, and finally clastic "flows". The rheological models also progress from Newtonian, to Bingham (linear with a Yield Stress) to various non-linear models, and finally as the dominant internal loss process transitions from inter-particle collisions to inter-particle friction, DebrisLib includes geotechnical approaches to account for those processes.

[2] DebrisLib is a non-Newtonian, mud and debris flow library that HEC developed jointly with ERDC-CHL. Most USACE models including HEC-RAS, HEC-HMS, ADH,

Flow Classification	Process	Threshold	O'brien *et al.* (1993) Shear Component	Hershel Bulkeley Shear Component	Rheological/ Geotechnical Model
Sediment Transport	Bedload and Suspended Load 	C_v<5%	None	None	
Hyper-concentrated Flow	Yield Stress & Viscous Losses 	C_v>5%	Bingham $\tau_y + \mu(\dot{\gamma})$	Linear $\tau_y + K(\dot{\gamma})^{n=1}$	
Mudflow	Inter-Particle Turbulence 		$\rho_m l_m^2 (\dot{\gamma})^2$	$K(\dot{\gamma})^{n\neq1}$	
Debris Flow	Grain Collision 	Bagnold # N_{BAG}>40	$\frac{0.01\rho_s d_s^2 (\dot{\gamma})^2}{\left(\left(\frac{0.615}{C_v}\right)^{1/3}-1\right)^2}$		
Clastic Flow	Matrix Strength 	Friction # N_{fri}>100*	Replaces the yield stress (τ_y) in either model with a Coulomb (geotechnical) model: $\tau_y = \tau_c + \sigma \tan\varphi$		

Lower Concentration → Higher Concentration

Finer Sediment → Coarser Sediment

Figure 1-4: Non-Newtonian flow taxonomy, with the rheological models and equations used to model them (from Gibson et al., 2020 in revision).

Non-Newtonian Transport Editor

HEC-RAS can incorporate non-Newtonian effects in any unsteady flow simulation including 1D or 2D. But the current version of HEC-RAS does not include non-Newtonian effects in steady or quasi-unsteady flow.

Therefore, the non-Newtonian editor is an Option in the Unsteady flow editor.

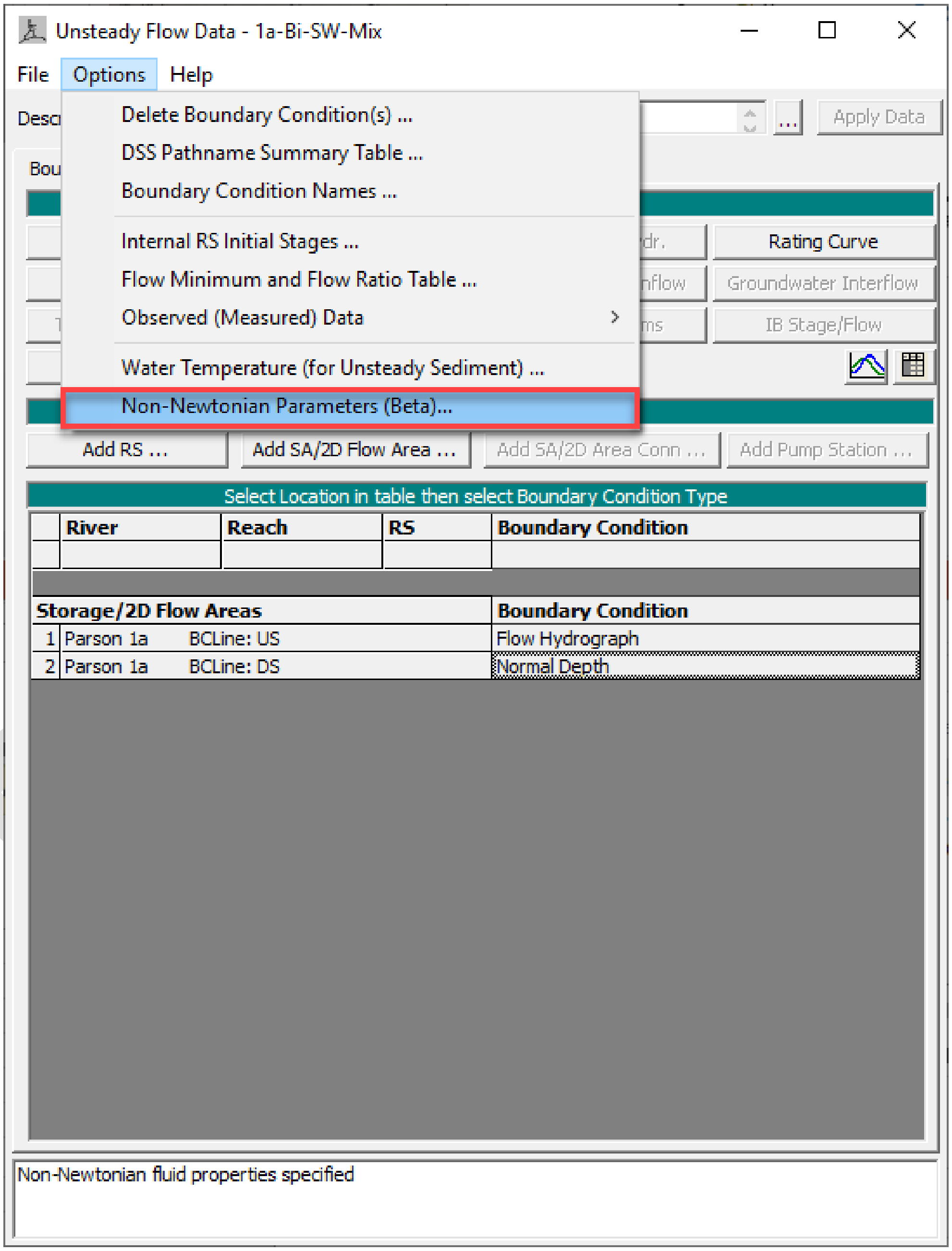


Figure 1-5: Select the Non-Newtonian Editor from the Unsteady Flow Options Menu.

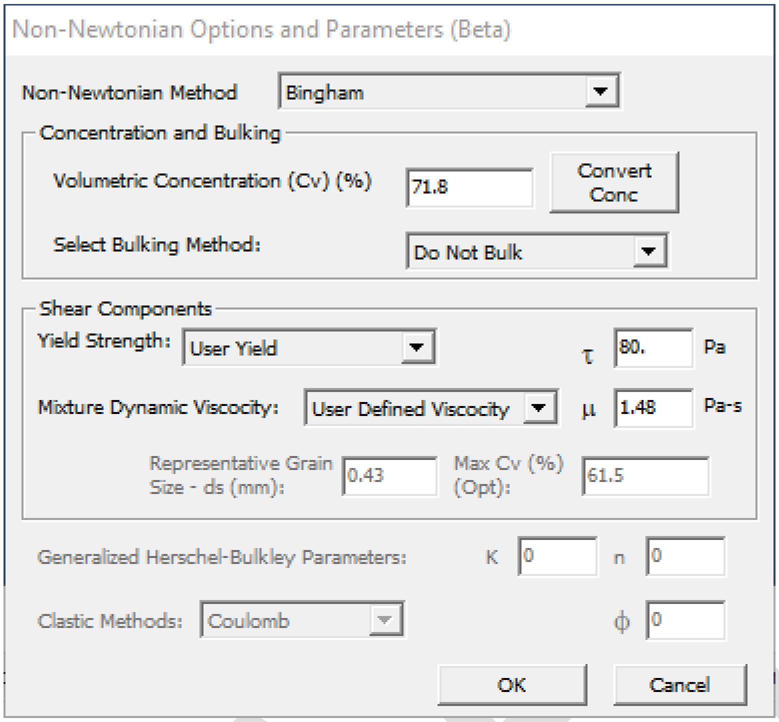


Figure 1-6: HEC-RAS Non-Newtonian editor.

Non-Newtonian Methods

The terms, variables, and parameters on the non-Newtonian editor change based on the method selected. Select the **Non-Newtonian Method** first to activate the appropriate fields and methods associated with that option. The default method is **Newtonian Assumptions** which means that HEC-RAS uses the standard "clear water" equations and does not apply any Non-Newtonian methods.

The current version of HEC-RAS includes five additional methods:

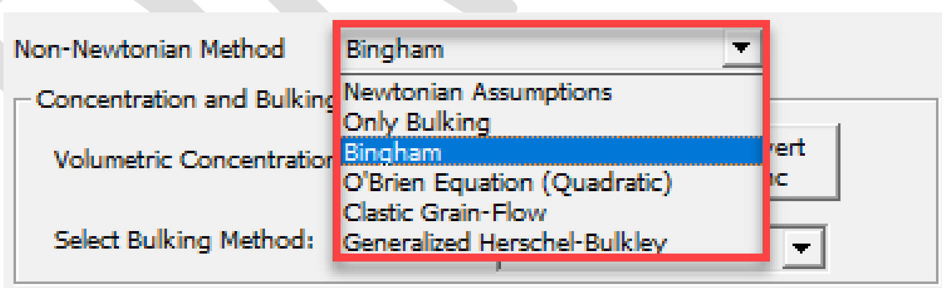


Figure 1-7: Non-Newtonian methods available in

The first mud and debris method (**Bulking Only**) is not – actually -a Non-Newtonian method, because it only changes the volume of the fluid. The **Bulking Only** method does not depart from the Newtonian model or compute internal losses. The other five mud and debris flow methods are non-Newtonian approaches, computing internal losses from stress-strain models that do not have a zero intercept and/or are not linear.

The models and math behind these methods are described in detail in the [Technical Reference Manual]. This section will describe use and parameterization.

Bingham User Parameters

The Bingham equation is often applied to Hyperconcentrated flows and mud flows. In theory, these lower concentration flows fit the linear model better. However, its relatively simple formulation makes it easier to calibrate. Fewer free parameters make it less vulnerable to equifinality issues. So it has been applied successfully to higher-concentration debris flows in laboratory and field applications.

The Bingham model only requires two user inputs: the yield strength (the intercept of the stress-strain relationship) and the sediment laden viscosity (the slope of the stress-strain relationship). The options for these are described in the Yield and viscosity sections below.

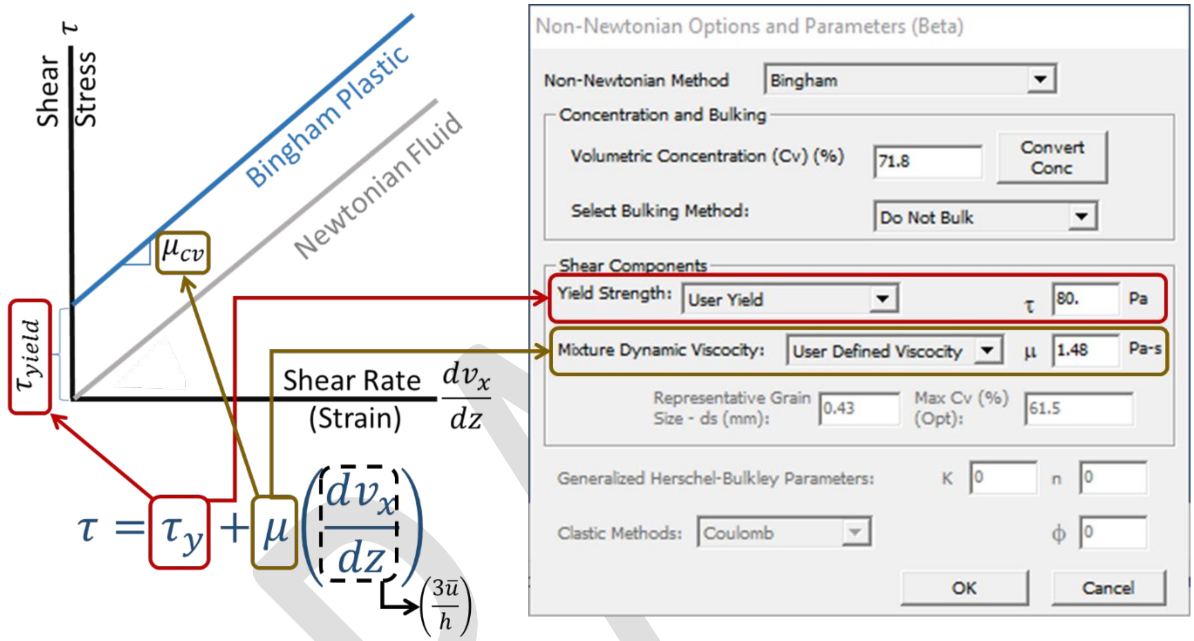


Figure 1-8: The Bingham model has two parameters in the Non-Newtonian interface.

O'Brien Equation (Quadratic) User Parameters

The O'Brien equation uses a quadratic model to add non-linear impacts of particle collision and turbulence to the linear yield and viscosity terms in the Bingham model. It is not as flexible as Herschel-Bulkley. The non-linear effects are always a function of the square of strain, so they are always strong shear-thickening effects. But the O'Brien model is easier to parameterize than Herschel-Bulkley. The O'Brien equation uses physical values to develop theoretical quadratic effects. The liability of this approach is that if the theoretical formulation does not reflect the processes in the geophysical flow, it will introduce errors. But the benefit of this physical-theoretical approach is that all of the inputs in the non-linear terms are physical parameters that are either default or relatively intuitive for the user to specify.

In addition to the yield stress[3] and sediment laden viscosity that are required for the Bingham model, the O'Brien model only requires the volumetric concentration (which is

---

[3] Gibson et al. (2020 in revision) demonstrated that lower yield and viscosity values are often appropriate for the O'Brien approach when compared to Bingham because the O'Brien

already required for bulking and for some yeild and viscosity estimates) and a representative grain size. HEC-RAS has also exposed the default maximum volumetric concentration in O'Brien's Bagnold term (0.615 or 61.5%). This term is ok for lower concentration flows (Cv<50%). But as concentration approaches or exceeds this theoretical maximum (see discussion associated with this input below) users should increase it to make it larger than the volumetric concentration.

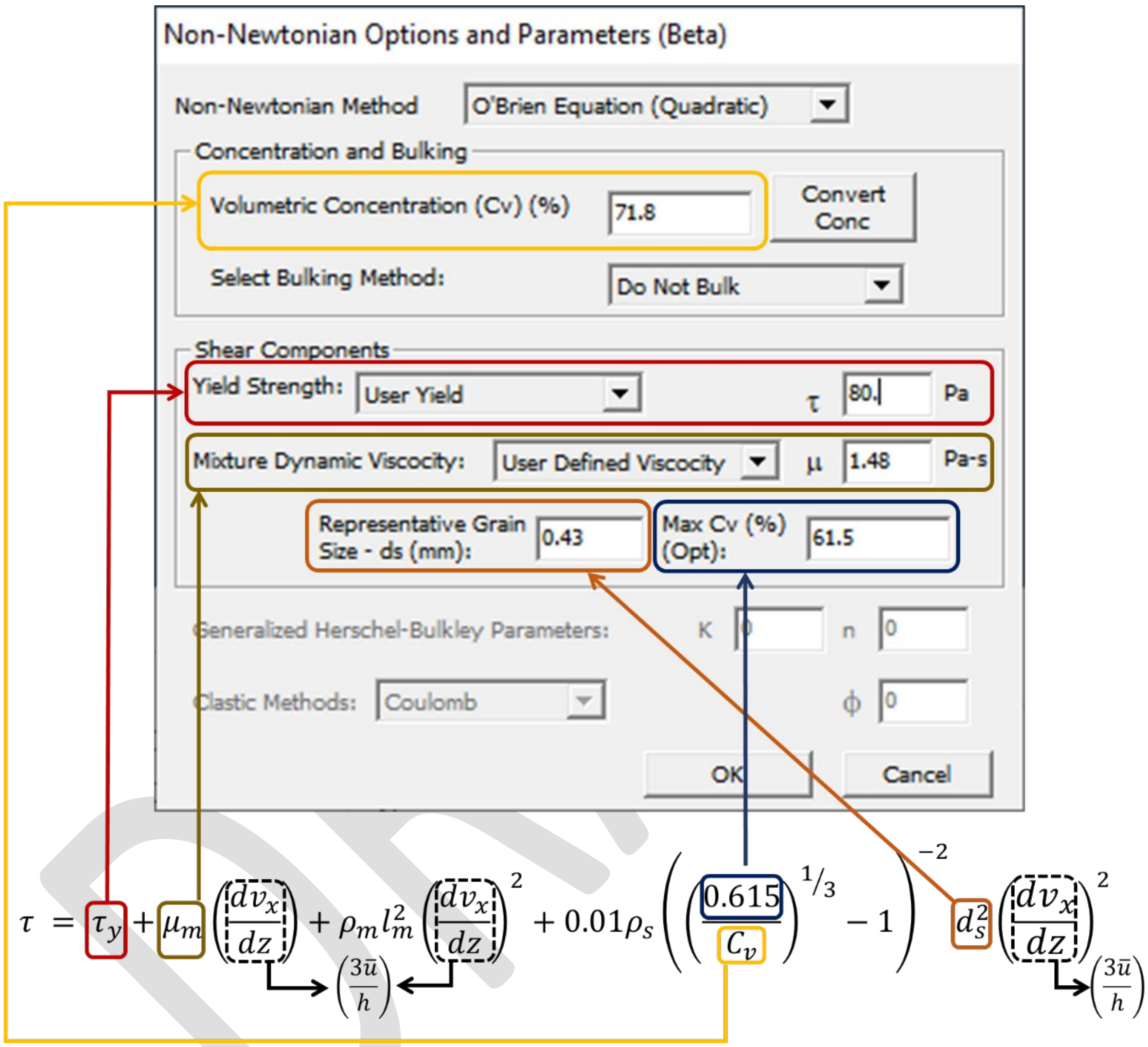


Figure 1-9: User defined parameters of the quadratic O'Brien equation, and their location in the Non-Newtonian interface.

---

equation is explicitly accounting for processes in the quadratic term that Bingham is lumping into the linear parameters.

Herschel-Bulkley User Parameters

Hershel-Bulkley is a flexible, relatively simple, but highly empirical approach. It allows a wide range of non-linear rheological approaches with a fairly simple formulation. However, estimating these terms can be very difficult outside of a laboratory (see discussion of these terms below).

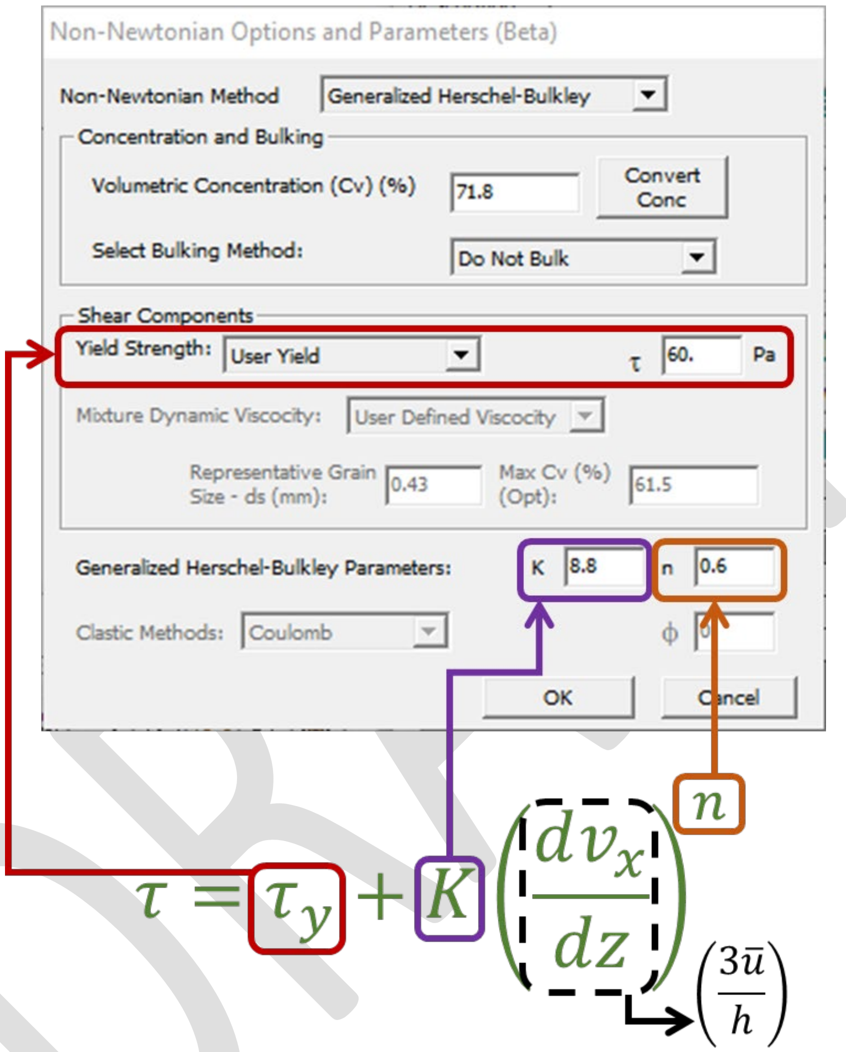


Figure 1-10: User parameters of the Herschel-Bulkley in the Non-Newtonian interface.

Like Bingham and the O'Brien quadratic models, Hershel Bulkley has a yield stress. This yield stress has the same units and methods as the other models so it is specified in the same **Shear Components** location as the other models. However, the coefficient in front of the strain is no longer a simple **Mixture Dynamic Viscosity**. Because Herschel-Bulkley raises the strain to a power, the units of the coefficient diverge from the simple viscosity units for any power other than 1. Therefore, if $n \neq 1$, K is not a physical viscosity, but just an empirical coefficient of the power function. It is not appropriate to use the viscosity equations for this coefficient. Therefore if users select the **Generalized Herschel-Bulkley** method the **Mixture Dynamic Viscosity** options become unavailable, and the **Herschel-Bulkely** terms become active.

User Inputs and Model Parameters

Concentration

Volumetric concentration is the first variable the user must estimate. Most of the non-Newtonian models are very sensitive to the volumetric concentration. Some of the other parameters can even be estimated with empirical equations with concentration in the exponent, making results even more sensitive to this variable. Enter the *volumetric* concentration *in percent* at the top of the Non-Newtonian editor. The current version of HEC-RAS uses one volumetric concentration for all time and space.[4] Select the single concentration that is most appropriate for the modeling objectives.

The following sections describe how to compute the volumetric concentration from other concentration measurements and a few methods to estimate this variable.

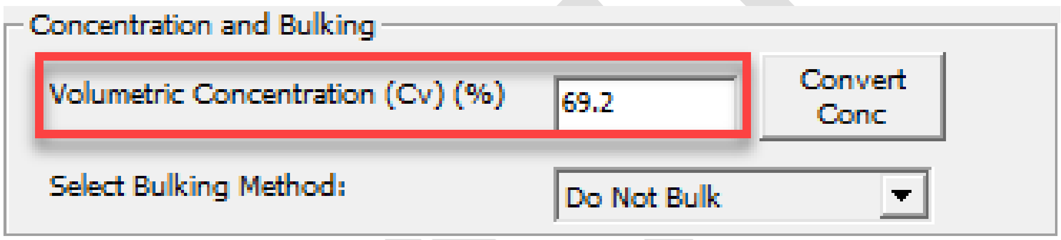


Figure 1-11: User defined volumetric concentration of solids.

Warning: One of the most common errors in this editor is defining volumetric concentration as a decimal instead of a percent. For example, in Figure 1-11, defining concentration as 0.692 would register as less than 1% solids, which would produce almost no debris flow effects.

Converting Concentration

There are several different ways to report concentration. At low concentrations, like those encountered in almost all natural, fluvial, sediment transport conditions, the different concentration conventions are close enough that practitioners use them interchangeably and often without distinction. But as the solid fraction of geophysical and hyperconcentrated flows increases, the difference between the specific gravity of sediment and water makes concentration by mass, volumetric concentration, and parts per-million (ppm) diverge.

Concentration by weight (C_w) is greater than volumetric concentration (C_w) because soil is denser than water. Table 1-1 (after a table in Julian 2010) demonstrates how the four primary concentration conventions interact (with the total mixture density) as the solid content increases.

[4] We are working on a time-series of concentration and, eventually, more sophisticated methods for routing concentration through the model.

Table 1-1: Equivalent concentrations and fluid densities (after Julian 2010).

Volumetric Concetration C_v (%)	Concetraion by Weight C_w (%)	Parts per Million C_{ppm} (ppm)	Concentration mg/L $C_{mg/L}$ (mg/L)	Density of Mixture $\rho_{mixture}$ (kg/m^3)
0.01%	0.03%	265	265	1,000
0.05%	0.13%	1,324	1,325	1,001
0.10%	0.26%	2,645	2,650	1,002
0.25%	0.66%	6,598	6,625	1,004
0.50%	1.31%	13,141	13,250	1,008
0.75%	2.0%	19,632	19,875	1,012
1.00%	2.6%	26,069	26,500	1,017
2.5%	6.4%	63,625	66,250	1,041
Hyperconcentration				
5.0%	12.2%	122,401	132,500	1,083
7.5%	17.7%	176,863	198,750	1,124
10.0%	22.7%	227,467	265,000	1,165
25.0%	46.9%	469,027	662,500	1,413
50.0%	72.6%	726,027	1,325,000	1,825
75.0%	88.8%	888,268	1,987,500	2,238
100.0%	100.0%	1,000,000	2,650,000	2,650

At low concentrations (<5%) the differences yields trivial divergence between ppm and mg/L. For example, at C_v=5%, C_w is 12.2%, but the concentration in ppm and mg/L units only differ by about 8%. However, when the half of the volume of the mixture is solid, 73% of the weight is solid, and the $C_{mg/L}$ is almost twice the ppm.

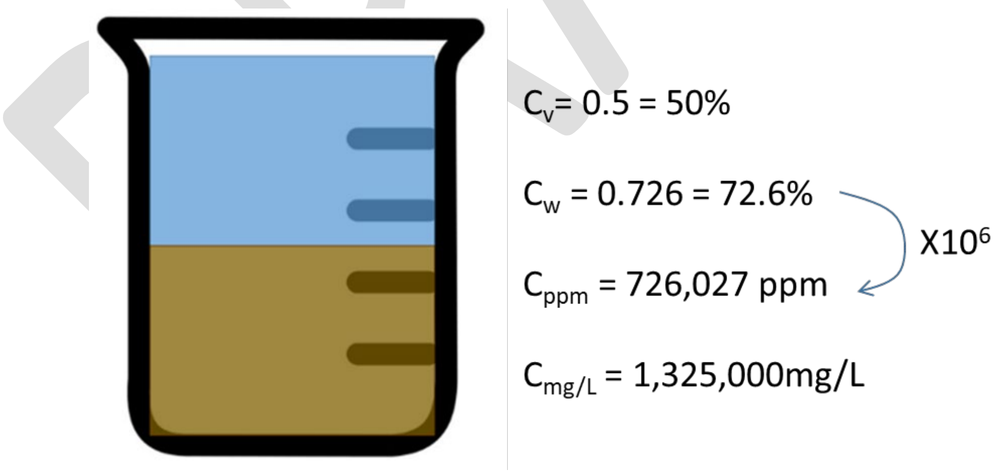


Figure 1-12: Four different concentration conventions for a mixture that is half solids by volume (e.g. it is >70% solids by weight because of the higher density of solid particles).

The conversion between concentration by weight and parts-per million is the simplest conceptually. C_{ppm} is six orders of magnitude greater than C_w (or C_{ppm}=10^6C_w).

Equations for converting (Julian, 2010) between Cv and Cw are:

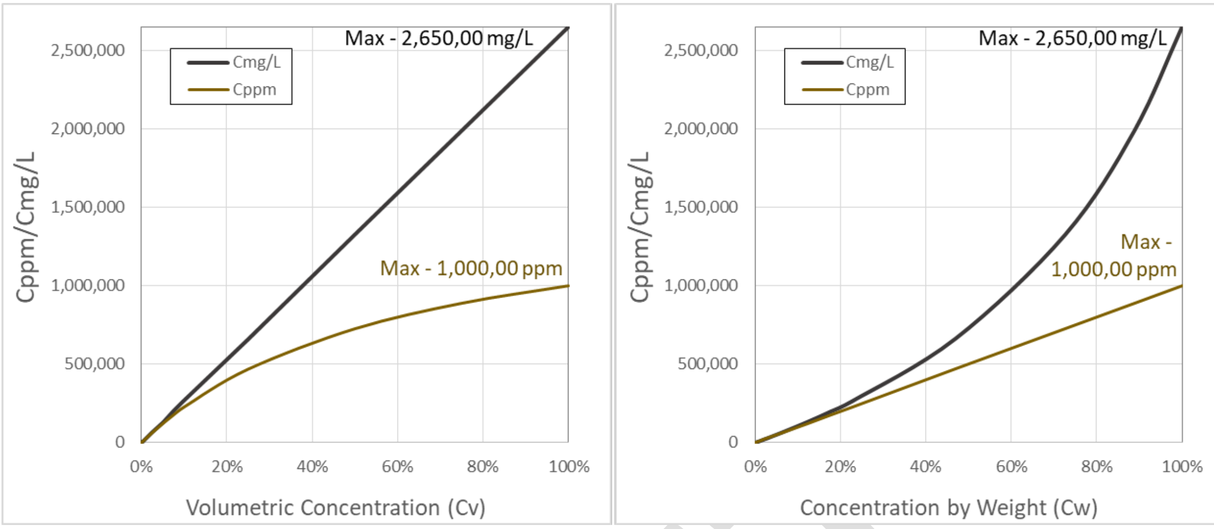


Figure 1-13: Relationship between volumetric concentration (C_v - left) and concentration by weight (C_w - right) and the corresponding concentrations in mg/L and ppm.

$$C_{volume} = \frac{C_{weight}}{sg-(sg-1)C_{weight}} \quad C_{weight} = \frac{sg \cdot C_{volume}}{1+(sg-1)C_{volume}}$$

where *sg* is the specific gravity of the solid (2.65 assumed). Therefore, the concentration by weight for a mixture that is half solid by volume is:

$$\boldsymbol{C_{weight}} = \frac{\mathbf{2.65 * 50\%}}{\mathbf{1 + (2.65 - 1)50\%}} = \mathbf{73\%}$$

A mixture cannot have more than one-million parts per million, but the maximum mg/L concentration is 2,650,000 mg/L. Both of these end cases are solid rock.

Concentration Calculator

Because there are at least four concentration conventions,[5] these concentrations vary dramatically for non-Newtonian mixtures, and the non-Newtonian equations are very sensitive to concentrations, it is critical that users identify or compute the *Volumetric Concentration* to input into HEC-RAS (Figure 1-11). To help users and project teams navigate the concentration options, HEC-RAS includes a **Concentration Conversion Calculator**. Press the button labeled **Convert Conc** to launch the **Concentration Conversion Calculator**.

[5] There are actually more, because "concentration" is often embedded in various density conventions (e.g. density of the solids or density of the mixture), or water content.

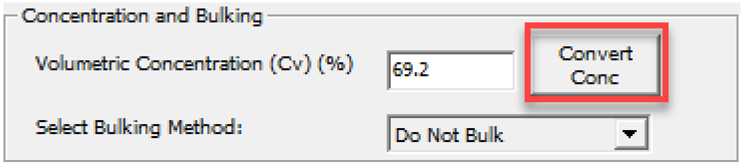


Figure 1-14: Launch the concentration calculator by pushing the Convert Conc button.

The **Concentration Conversion Calculator** requires three inputs. The Concentration the input concentration convention (users can choose from five options) and the specific gravity of the solids (default = 2.65). Specify these three inputs and press **Compute**. The calculator will generate the concentrations in C_v, C_w, w[6], *mg/L* and *ppm*.

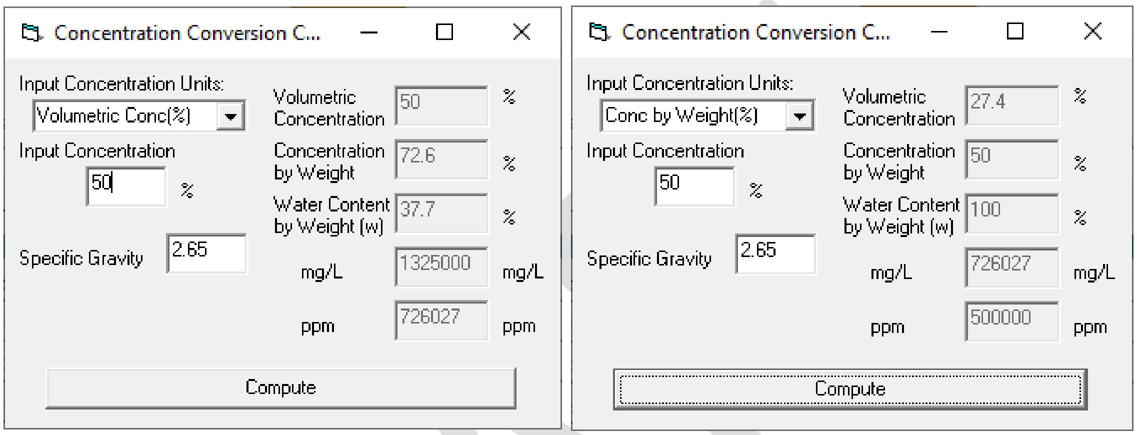


Figure 1-15: Concentration calculator results, computing the other concentration conventions for mixtures that are half solids by volume (left) and half solids by weight (right).

Estimating Concentration

Estimating the volumetric concentration of a mud or debris flow is difficult, but there are a couple major approaches. For forensic analysis, estimating the total deposits and pass through load yields the total mass transported, which can be distributed over a hydrograph to compute a concentration. The total deposits can be calculated by comparing pre-event and post-event LiDAR or by inferring the mass of the deposits from maintenance records.

For predictive models, there are several regression equations that estimate post-wildfire debris yields and estimating the volume of solids in a mine tailings impoundment and making credible assumptions about the volume of those solids that would be mobilized, the flow that would mobilize them, and how the solids would be distributed over the hydrograph, will help modelers estimate an approximate C_v.

Modeling Note: The post-wildfire debris load equations are included in the recent release version of HEC-HMS.

[6] Water content by weight. Note, the equation for water content by weight from Julian (2010) ($w=(1-C_v)/sg\ C_v$) is only useful for significantly hyperconcentrated mixtures. It reports water contents above 100% for lower concentrations, and the calculator caps these at 100%.

Bulking Options

At high concentrations the solid component has a significant effect on the volume of the mixture. This can confound flow conventions if users and modeling teams are not careful. There are two main ways of incorporating solid volume into mud and debris models, and users should select the appropriate approach under **Select Bulking Method** to reflect that decision.

Incorporate Volume of Solids in Flow Data (Do Not Bulk)

One way to account for the volume of the solids is to include it in the flow. With this approach, the flow (Volume/Time) is the flux of the mixture. This will be a common approach for measured flows, because field measurements (or estimates) will not separate the fluid and solid components. If the flows include the total volume of the mixture, the mud and debris calculations should not bulk them. Increasing the volume based on the concentration would double count the influence of the solids.

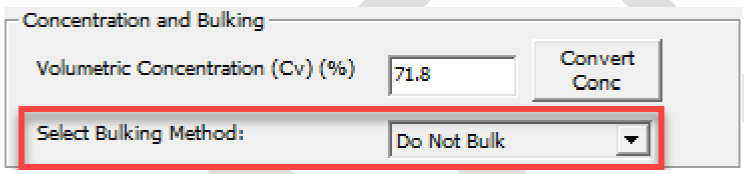


Figure 1-16: If the volume of the solids is included in the flow data, the debris model should not use the concentration to bulk the flow.

The model still requires a concentration for the non-Newtonian equations. But if the flow includes the volume of the solids, select **Do Not Bulk** under **Select Bulking Methods.**

Modeling Note: Volumetric Concentration is often a calibration parameter (because it is often uncertain and sensitive). However, the result will be less sensitive to Cv if the volume is incorporated in the flow, rather than computed from Cv and a base water flow.

Add Solid Volume to Water Flow Data (Bulk Fluid Volume)

In the second approach users define only the water flow in the unsteady flow file, and then HEC-RAS adds the volume of the solids during the non-Newtonian simulation. This approach is common if the flows come from a hydrologic model (or a runoff model like HEC-HMS that computes separate hydrographs and sedigraphs), if Cv is a calibration parameter, or if the modeling team wants to quantify the effect of the mud or debris (by comparing the result to a clear water flow with the same flow file).

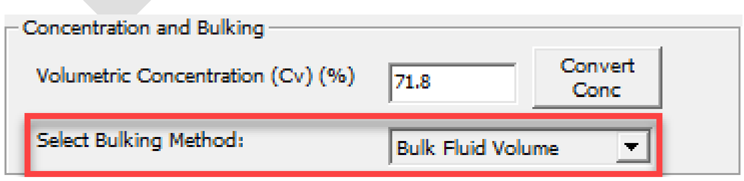


Figure 1-17: If the HEC-RAS flow file only includes the volume of the water, the mud and debris calculation should increase the volume of the flow to account for the volume of the solids.

If the flow data only account for the water, select **Bulk Fluid Volume** under **Select Bulking Method**. HEC-RAS will increase the volume of the boundary flows to account for the solid components based on the user-specified volumetric concentration using the relationship describe in the Bulking section of the Technical Reference Manual.

Yield Stress

All of the linear and non-linear rheological models require a yield stress. Mathematically, the yield stress is the y-axis intercept of the stress-strain relationship. Conceptually, it is the range of stresses over which the mixture does not move.

Bingham:

$$\tau = \boxed{\tau_y} + \mu_m \left(\frac{3\bar{u}}{h}\right)$$

O'Brien Quadratic:

$$\tau = \boxed{\tau_y} + \mu_m \left(\frac{3\bar{u}}{h}\right) + \rho_m l_m^2 \left(\frac{3\bar{u}}{h}\right)^2 + 0.01\rho_s \left(\left(\frac{0.615}{C_v}\right)^{1/3} - 1\right)^{-2} d_s^2 \left(\frac{3\bar{u}}{h}\right)^2$$

Herschel-Bulkley:

$$\tau = \boxed{\tau_y} + K \left(\frac{3\bar{u}}{h}\right)^n$$

Figure 1-18: All of the non-geotechnical rheological models have a yield strength (τ_y) term.

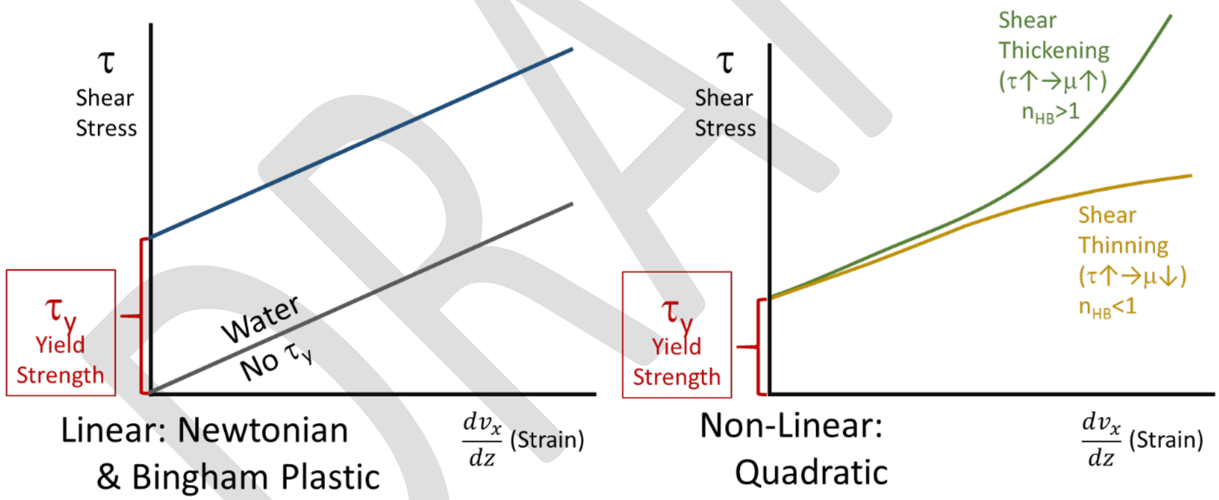


Figure 1-19: Yield stress is the intercept (ty) of the stress-strain relationships. Newtonian fluids like water do not have internal strength so they do not have a yield strength (i.e. there is no, non-zero stress at which they are at rest or do not deform).

This is one of the important differences between Newtonian and Non-Newtonian fluids. Newtonian fluids have a zero stress-strain intercept, which just means that they deform (move) at under the slightest stress. Water has no internal strength, so very small stresses move it. It is only at rest under no-stress conditions.

Non-Newtonian mixtures often have internal strength, however. They resist motion under a range of stresses. The driving forces have to exceed this internal strength before the material moves (deforms or strains). The rheological models account for this with a Yield Strength. This y-intercept in the stress-strain relationship is a motion threshold. As long as $\tau<\tau_y$ the fluid is at rest.

This yield strength drives one of the most important processes in mud and debris flows that Newtonian models cannot simulate: run out. Water will flow downslope indefinitely. Even if flow attenuates and slope decreases, as long as the flow has some slope or momentum it will stay in motion. Mud and debris flows can come to rest, even on a relatively steep slope. As driving forces decrease, the strength of the particle interactions can exceed the stress of the slope and momentum of the fluid, causing it to stop or "run out." The Yield Strength drives this process.

Yield stress is difficult to measure. Laboratory measurements like tilt tests can estimate yield strength when solid particles are small enough to sample and if the fluid can be sampled or reconstituted. But sampling mud and debris flows is difficult and modelers have to make some assumptions in predictive models. Therefore, HEC-RAS includes three approaches to Yield Strength.

Exponential

Because direct measurements of yield strength are rare, the **Exponential** empirical method is the default approach. Several researchers have found that yield strength is an exponential function of the volumetric concentration. Therefore, the **Exponential** method incorporates two empirical parameters into an exponential function of the volumetric concentration:

$$\tau_y = ae^{bC_v}$$

where a and b are calibration coefficients, and C_v volumetric concentration between 0 and 1.

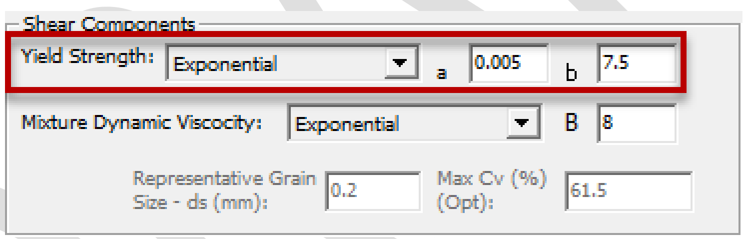


Figure 1-20: The exponential equation for yield strength embed two empirical coefficients in an exponential function of volumetric concentration.

O'Brien and Julian published values for these empirical parameters. These coefficients vary widely, so they are often calibration parameters. But these values can serve as a starting point for a calibration.

Table 1-2: Yield stress parameters for the O'Brian equation from Julian (1995)

Material	a (Pa)	b
"Typical soil"	0.005	7.5
Kaolinite	0.05	9
Sensitive Clays	0.03	10
Bentonite	0.002	100

User Yield

The user specified Yield Strength is the most direct way to input the yield strength. Just select **User Yield** and define the Yield Strength (in Pa – the initial release of the Non-Newtonian editor uses SI units but is compatible with SI and US customary simulations).

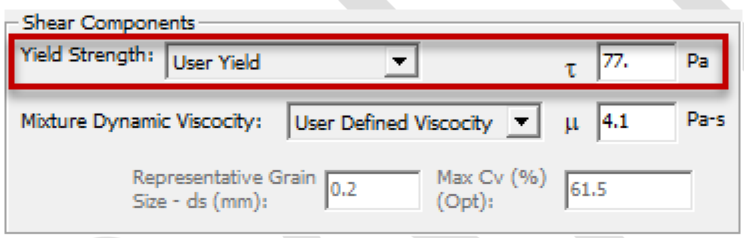


Figure 1-21: Define the yield strength directly with the User Yield method.

Use Coulomb

In the rheological models the yeild stress is – conceptually - an internal property of the single-phase fluid mixture. However, there is another way to think about Yield and runout. As the concentration increases (or as the mixture dewaters) the particle interactions transition from collision to inter-particle friction (Figure 1-4 and Figure 2-4). In this transition from collision to friction, the dominant processes transition from fluid mechanics to geotechnical processes. The third approach to yield strength takes this approach. Selecting **Coulomb** under **Yield Strength** activates the **Coulomb** model under **Clastic Methods** even if a Rheological model (i.e. Bingham, O'Brien, HB) is selected for the **Non-Newtonian Method**. In this mode, HEC-RAS will use geotechnical **Coulomb** theory to compute a Yield Strength (τ_y) in the rheological model. With this approach, the Yield Strength will be the stress required to initiate motion along the friction plane.

This approach differs from selecting **Clastic Methods** and **Coulomb** because applying the Coulomb approach as a clastic method only considers the geotechnical threshold stress. Selecting **Coulomb** as a **Yield Strength** method in conjunction with the Rheological Non-Newtonian methods (i.e. Bingham, O'Brien, HB) uses the Coulomb method to compute the threshold of motion by using it for yield stress (τ_y in each equation in Figure 1-18) but then adds the viscous and/or non-linear components.

The Coulomb method requires a friction angel to compute the threshold of motion.

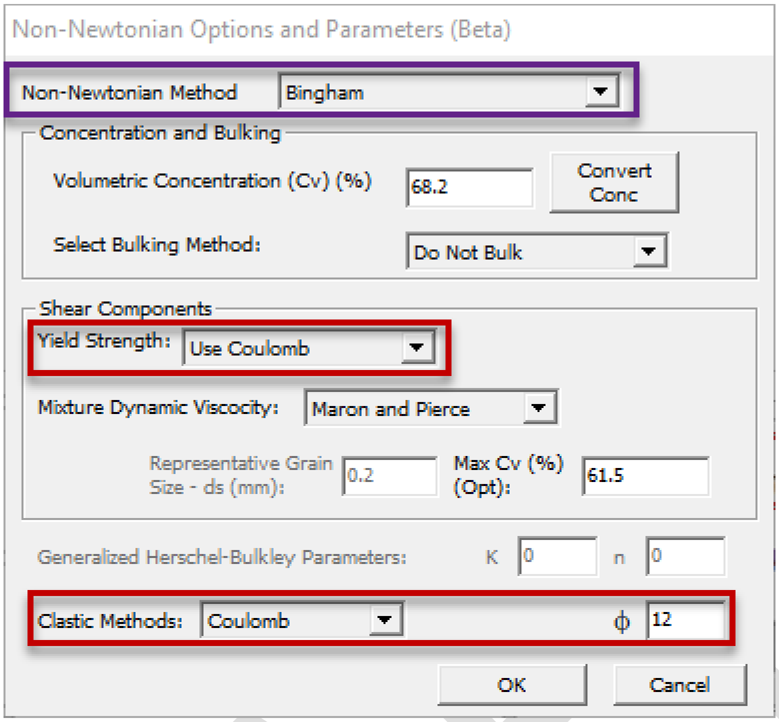


Figure 1-22: Selecting the Coulomb method for Yield Strength makes the Coulomb approach available under Clastic Methods, but uses this geotechnical threshold of motion as the Yield Strength of the Rheological method selected.

Mixture Dynamic Viscosity

The **Mixture Dynamic Viscosity** (sometimes called the "Sediment Laden Viscosity") is the viscosity of the mixture. HEC-RAS and DebrisLib use "single phase" models to simulate mud and debris flows. Single phase models do not compute separate fluid and solid mechanics. They assume that the mixture is a homogeneous fluid. Single phase models account for the impacts of the solid fraction by changing the properties of the fluid, including the viscosity.

Two of the rheological models incorporate the impact of the solids, in part, by using a **Mixture Dynamic Viscosity**, which is the apparent viscosity of the mixture, including the influence of the solid phase on the liquid phase. Mud and debris flows are more viscous than water. The **Mixture Dynamic Viscosity** includes the impacts of the solid phase on the stress-strain relationship of the mixture. In the Bingham model (Figure 1-8) this mixture viscosity is the slope of the stress-strain relationship and the O'Brien methods adds quadratic terms for other processes, but still uses a **Mixture Dynamic Viscosity** to compute linear, internal, viscous losses.

HEC-RAS includes four methods to compute the **Mixture Dynamic Viscosity**. These are described in detail in the [Technical Reference Manual]. The four methods include:

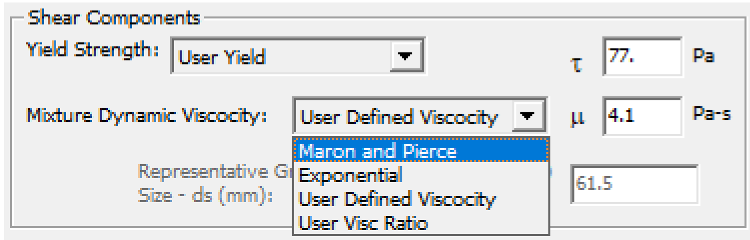


Figure 1-23: Methods available to compute sediment laden viscosity.

Maron and Pierce

Maron and Pierce is the default method. This method is popular because it does not require user input or parameters. However, it does compute the **Mixture Dynamic Viscosity** based on the ratio of the **Volumetric Concentration** (C_v) to a maximum possible concentration (***Max*** $\boldsymbol{C_v}$). ***Max*** $\boldsymbol{C_v}$ is not a fixed value. It can vary with the gradation (particle size distribution) of the material. HEC-RAS uses the default ***Max*** $\boldsymbol{C_v}$ from the Bagnold packing assumption in the O'Brien quadratic (see more discussion of this parameter in the ***Max*** $\boldsymbol{C_v}$ section below). However, if you select **Maron and Pierce** the interface will make this parameter editable whether the O'Brien method is selected or not. Make sure that ***Max*** $\boldsymbol{C_v}$ is always greater than *Cv*.

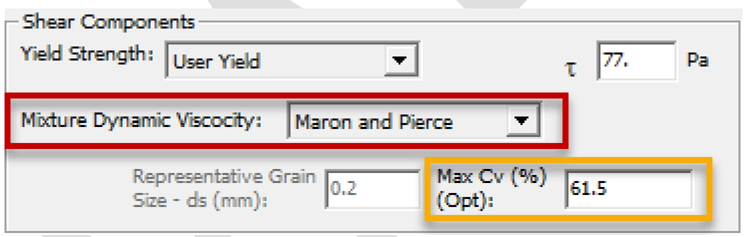


Figure 1-24: Maron and Pierce is the default Dynamic Viscosity which does not require a user input but does make the Max Cv field editable and will use this value in the analysis.

Exponential

Like Yield Strength, multiple investigations have found that Mixture Dynamic Viscosity has an exponential relationship with volumetric concentration.

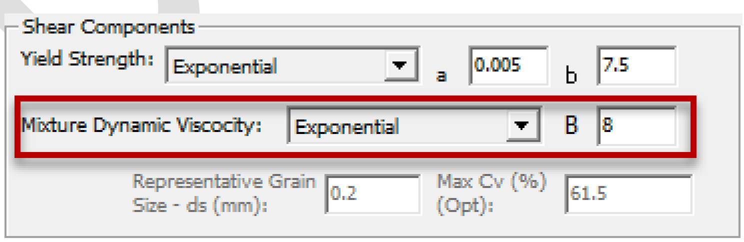


Figure 1-25: Exponential viscosity method. B is a multiplier of Cv in the exponent.

The form used by O'Brien (1998) and documented in Julian (1995) has an empirical "multiplier" in the exponent like the Yield Stress Equation but no coefficient in front of the exponential function. Therefore, the exponential Viscosity equation only has one parameter (compared to the exponential Yield equation that has two):

$$\mu_m = \mu_w e^{BC_v}$$

Common values for *B* are include in Table 1-3. However, because this parameter is in the exponent, the computed viscosity is very sensitive to this value, which has a broad observed range. This is often a calibration parameter.

Table 1-3: Coefficients for the viscosity exponential multiplier for different soil types from Julian (1995).

Material	**B**
"Typical soil"	8
Kaolinite	8
Sensitive Clays	5
Bentonite	100

Modeling Note: Because the exponential equations (for both Yield Strength and Viscosity) include an empirical parameter *and* the volumetric concentration in the exponent, they can introduce equifinality issues (e.g. compensating errors, increasing one to compensate for setting the other too low) if modelers vary both of them during calibration. When a model has multiple sensitive parameters like this, making the calibration process a non-unique, multi-vitiate solution space, it is often good practice to fix the parameter with the lowest uncertainty and vary the less certain parameter.

User Defined Mixture Dynamic Viscosity

Like the Yield Strength, users can enter the **Mixture Dynamic Viscosity** directly (in SI viscosity units Pascal-seconds). Also like Yield Strength, there are laboratory methods to measure the mixture viscosity, but these tend to be difficult to transfer to the field (and *in situ*) measurements of debris flows are rare because the events tend to be unexpected and dangerous. But laboratory experiments often have defined **Mixture Dynamic Viscosity** and some users may find that calibrating the viscosity directly is more intuitive that calibrating an empirical power multiplier in the exponential approach.

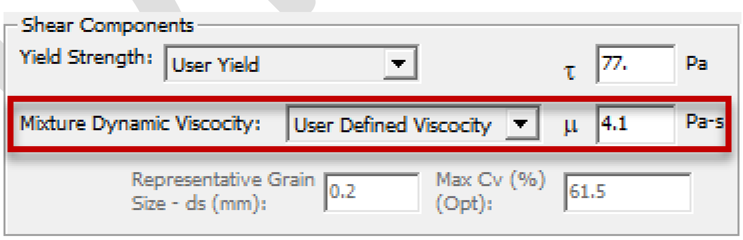


Figure 1-26: User Defined Mixture Dynamic Viscosity.

Modeling Note: Because the current version of HEC-RAS has uses a single, users specified, Volumetric Concentration (C_v) for all time and space, both the **User Defined** and **Exponential** methods compute constant viscosities throughout the simulation. However, in future versions, which will vary C_v in time (and eventually in space) the **Exponential Method** will become dynamic, adjusting with C_v, while the **User Defined** method will remain static.

User Visc Ratio

It may be more intuitive for modelers to think about the relative viscosity of the mixture, compared to water, instead of the absolute mixture viscosity. The **User Visc Ratio** computes the **Mixture Dynamic Viscosity** with a simple, user specified, multiplier of the dynamic viscosity of the water. This approach also incorporates dynamic effects of temperature on the

Dynamic Temperature

Some of the mixture dynamic viscosity methods compute the viscosity of the mixture relative to the water viscosity, which is a function of temperature. The mud and debris equations will vary both the water and mixture viscosity with temperatures if these methods are selected and if the user defines a temperature time series. By default, HEC-RAS assumes a temperature of 50° F (10° C). To define a temperature time series, select the **Options** menu in the **Unsteady Flow** editor. Select **Water Temperature** (just above the Non-Newtonian option).

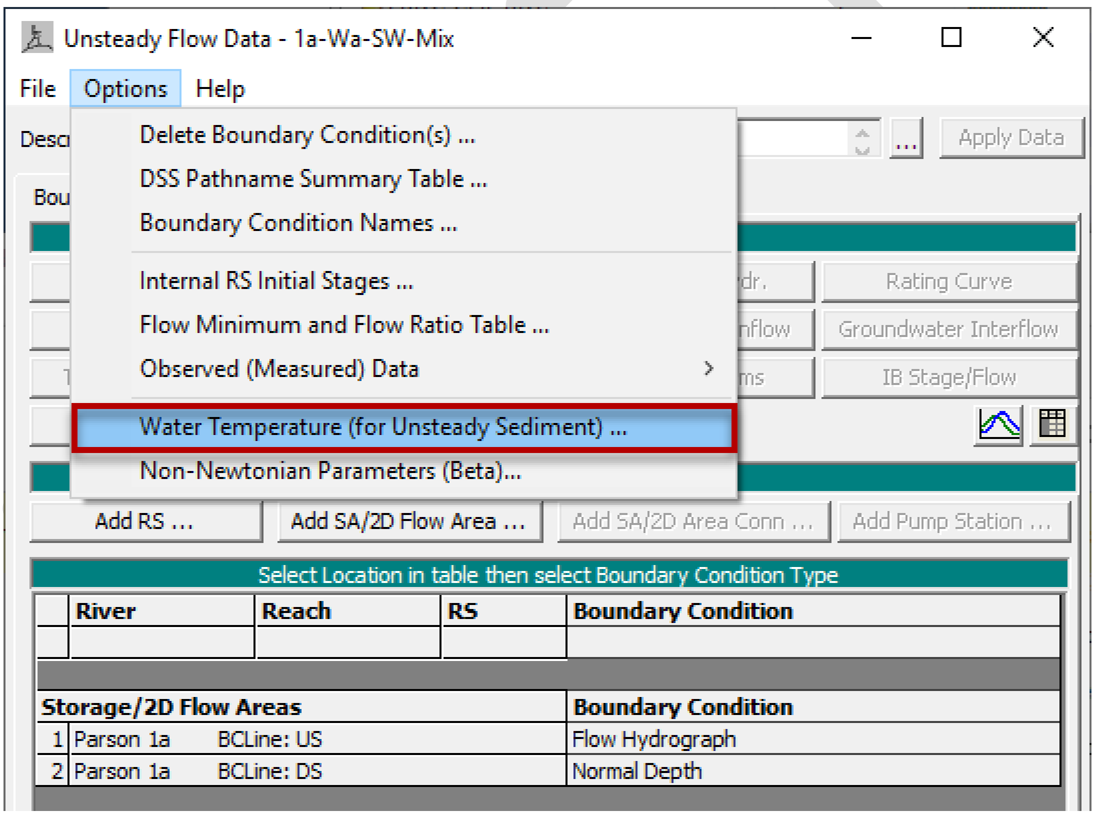


Figure 1-27: Choose the Water Temperature Option to define temperature time series.

Mud and debris flows are usually rapid events, so seasonal temperature changes (like the one in the do not tend to affect the simulations. However, users can define a new, non-default, constant temperature time series editor or use more detailed mixture temperature data if available.

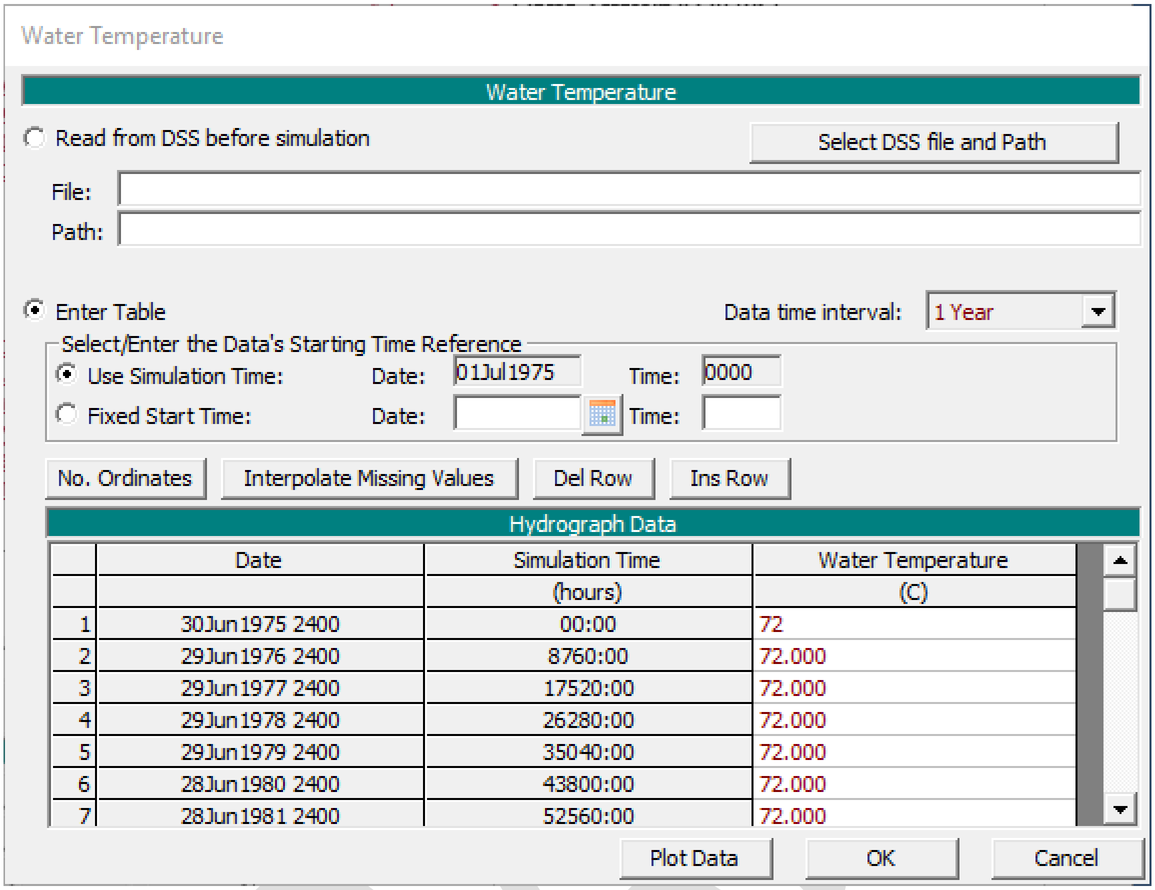


Figure 1-28: Overriding the default water temperature with a new "constant" temperature in the temperature time series editor.

Representative Particle Size

The representative grain size is only used in the dispersive term of the O'Brien equation, and only becomes active if that method is selected.

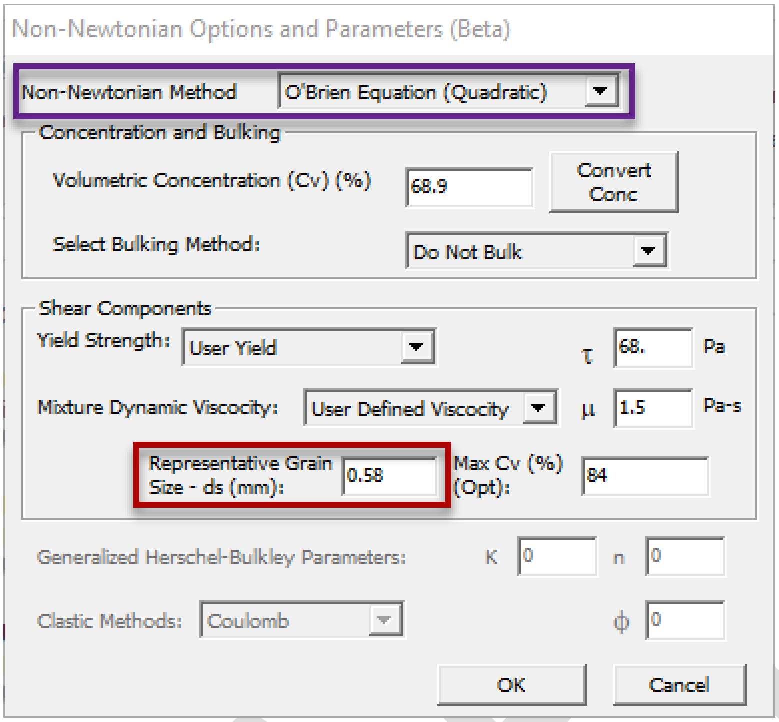


Figure 1-29: Representative grain size - Only used in the O'Brien Equation.

However, the value is squared in one of the quadratic terms of the equation, so results can be sensitive to this user decision.

$$\tau = \tau_y + \mu_m\left(\frac{3\bar{u}}{h}\right) + \rho_m l_m^2\left(\frac{3\bar{u}}{h}\right)^2 + 0.01\rho_s\left(\left(\frac{0.615}{C_v}\right)^{1/3} - 1\right)^{-2} \boxed{d_s^2}\left(\frac{3\bar{u}}{h}\right)^2$$

Various sediment transport analyses that collapse the particle size effects of a particle-size distribution into a representative grain size use the median grain size (d_{50}) or geometric mean of the distribution. However, it can be difficult to determine the median particle size of an event with substantial clay components that also moves car-size boulders. It is also an open question whether the median particle size is appropriate for the dispersive process Bagnold was describing (in a single particle-size theoretical model) under transport conditions that include substantial grain class fractions between 0.004 mm and 4,000 mm. Evaluate the uncertainty and sensitivity of this value in the simulation and determine if it should be a fixed parameter or a calibration parameter.

Max C_v

The Bagnold term in the O'Brien quadratic (the same term that includes the representative particle size) estimates losses from particle collisions. The term approximates the relative frequency of these collisions, in part, from the density of particles in the fluid relative to the maximum packing density.

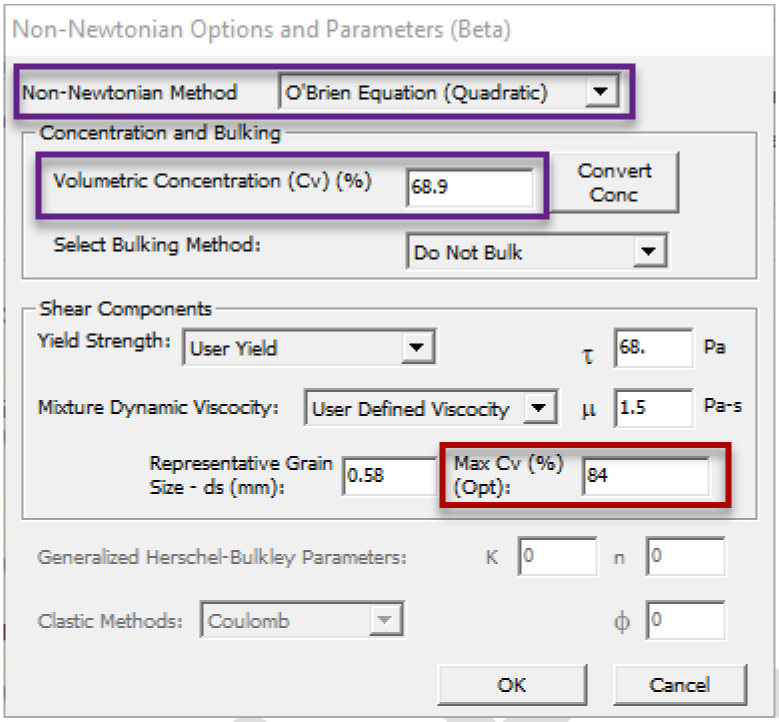


Figure 1-30: Max Cv option becomes available when the O'Brien Equation is selected. In this case the Cv is greater than the default max (61.5%). Therefore, the Max Cv was increased to a max debris flow concentration from a literature review.

If the solids were not in motion and/or dry, they would still have a volumetric concentration less than 1 (<100%). The solids are a porous media, so even at rest they have porosity. The maximum volumetric concentration of the solids is the inverse of the porosity of the solids in their highest density packing configuration. Porosity (void volume divided by total volume) of natural materials roughly vary between 0.35 and 0.45. Larger particles tend to have larger porosities but well graded (poorly sorted[7]) tend to have smaller porosities as the finer particles fill the interstitial space between the larger particles.

O'Brien Quadratic

$$\tau = \tau_y + \mu_m\left(\frac{3\bar{u}}{h}\right) + \rho_m l_m^2\left(\frac{3\bar{u}}{h}\right)^2 + 0.01\rho_s\left(\left(\frac{\boxed{0.615}}{C_v}\right)^{1/3} - 1\right)^{-2} d_s^2\left(\frac{3\bar{u}}{h}\right)^2$$

Maron and Pierce Mixture Viscosity Ratio

$$\mu_r = \frac{\mu_m}{\mu_w} = \left(1 - \frac{C_v}{\boxed{C_{max}}}\right)^{-2}$$

The O'Brien equation uses the maximum volumetric concentration from the Bagnold equation – which is the theoretical packing maximum for uniform spheres: 0.615 or 61.5% which corresponds to 39.5% porosity. Debris flows can have higher volumetric

[7] Engineers and geologists (unhelpfully) use opposite conventions to describe particle size distributions. Uniform distributions with very little particle-size diversity are poorly graded (in geotechnical terminology) and well sorted (in geologic terminology). Debris flows tend to be extreme examples of the opposite phenomenon, porous media that include a wide range of particle sizes. Soil and sediment that include significant components of a wide variety of grain classes are well graded or poorly sorted.

concentrations than this default while in motion. Users must be careful that the volumetric concentration is never higher than the maximum packing concentration. Therefore, if modelers select either of the methods (O'Brien or Maron and Pierce) that use Max C_v, the *Max* C_v field will become active and editable. It will populate with the Bagnold default (61.5% - like C_v, this variable is always in percent) and will use this value or a user specified value for both methods if both are selected.

Hershel-Bulkley Parameters

The Herschel-Bulkley method is a two-term non-linear approach to mud and debris rheology. This method raises the strain to a user-selected power, which can be greater or less than 1. Unlike the Bingham method that raises strain to the power of 1 or O'Brien that uses a quadratic (raising strain to the powers of 1 and 2) Herschel-Bulkley can raise strain to non-integer powers greater or less than 1.

Shear Thickening

$$\tau = \tau_y + K\left(\frac{dv_x}{dz}\right)^{n>1}$$

Bingham

$$\tau = \tau_y + K\left(\frac{dv_x}{dz}\right)^{n=1}$$

Shear Thinning

$$\tau = \tau_y + K\left(\frac{dv_x}{dz}\right)^{n<1}$$

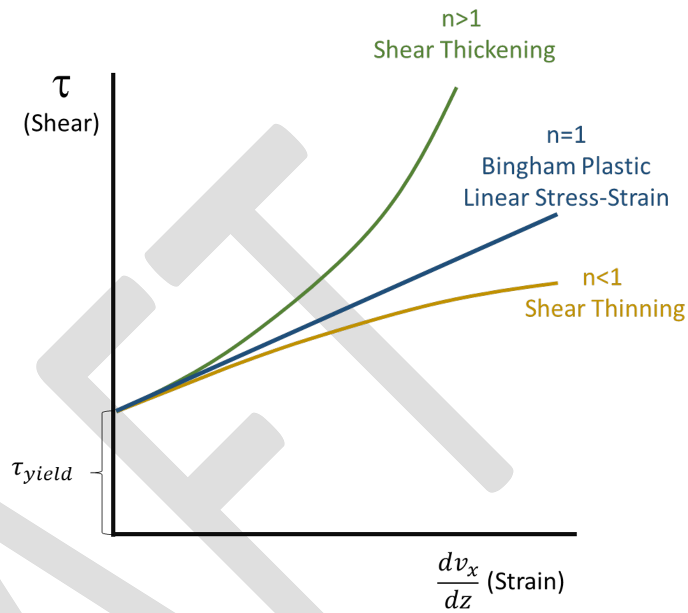


Figure 1-31: Changing the power in the Herschel-Bulckley model to simulate shear thickening (n>1), a Bingham Plastic (n=1), and shear thinning (n<1)

This flexibility allows users to define a range of non-linear stress-strain relationships including shear-thickening and shear-thinning rheologies. A shear-thinning mixture becomes easier to deform under higher stresses. A shear-thinning viscosity decreases as stress increases. As shear stress increases, the rate of strain increases non-linearly, so each increment of additional stress causes more strain than the previous increment.[8] The "shear-thinning" terminology illustrates this relationship. As shear increases, the material "thins" or becomes easier to strain. The Herschel-Bulkley model simulates shear thinning relationships by raising strain to a power less than one (n<1).

8 Increased viscosity at higher shear stresses essentially means that the slope of the stress-strain relationship increases with stress, which can be confusing with plots like Figure 1-31 (or most of the rheological plots in this document with strain on the x-axis). Because depth and velocity are model results, and DebrisLib uses them to compute an internal stress, the numerical model considers strain the independent variable and stress the dependent variable. But stress is the independent variable in physical deformation, so shear thinning and thickening responses are inverted in these plots (e.g. the slope of the strain-stress curves decrease at higher strains for shear thinning).

Shear-thickening materials get more viscous under higher stresses. Stress has a negative feedback on strain, making the material more difficult to deform. The Herschel-Bulkely model simulates shear-thickening by raising strain to a power greater than 1 ($n>1$).[9]

Setting the power of Herschel-Bulkley to one ($n=1$) collapses the model to the Bingham approach, because a linear stress-strain relationship with a yield stress is the definition of a Bingham Plastic.

The Herschel-Bulkely model requires three parameters:

The Yield Stress in Herschel-Bulkley is the same as the previous methods, and can be computed with the same options. But the linear parameter in front of the Strain term is loses its viscosity units if strain is raised to a power other than 1. Therefore, K is no-longer viscosity when Herschel-Bulkley diverges from the Bingham model ($n \neq 1$). Both K and n are empirical user parameters.

Figure 1-32 includes screen shots of shear-thickening and shear-thinning simulations (Gibson *et al.*, in revision) of Parsons *et al*'s (2000) experiments that displayed these processes.

[9] The O'Brien Quadratic is a de-facto shear thickening model because it includes squared strain terms ($n=2 \rightarrow n>1$).

Figure 1-32: Herschel-Bulkley simulations (from Gibson et al. in revision) of shear-thickening and shear-thinning laboratory experiments from Parsons et al. (2000).

CHAPTER 2 MUD AND DEBRIS FLOWS: NON-NEWTONIAN-TECHNICAL REFERENCE

Introduction

In Newtonian fluids the relationship between shear rate and shear stress is linear and passes though the origin. Non-Newtonian fluids have a shear rate vs shear stress relationship which can be nonlinear and/or does not pass though the origin. A wide range natural flows present non-Newtonian properties including mudflows, debris flows, lahars, and snow avalanches. The USACE has well established hydraulic hydrologic tools for simulating Newtonian flows but the tools available for non-Newtonian flows are quite limited. Hyperconcentated flows present physical present properties between clear-water and solid mass movements which complicate their computational modeling. Hyperconcentrations range from approximately 5-60%.

Most hydraulic and sediment transport simulations assume that the transporting fluid has "Newtonian" properties.

A Newtonian Fluid has two properties,

1) a linear stress-strain relationship and

2) a zero stress-strain intercept.

This assumption appropriate for most fluids, including sediment laden fluids with volumetric concentrations up to 30%. However, as sediment concentrations increase, they begin to affect the fluid properties, which alter the stress-strain relationship. There are many constitutive equations describing the shear-strain relationship in literature which have had some degree of success for different situations. However, due to the complex nature of the fluid-solid mixtures, these equations and their parameters have a large degree of uncertainty.

The mathematical models used to simulate non-Newtonian flows may be classified as single- and two-phase models. Single-phase models describe the properties of the mixture and solve conservation equations for the mixture (e.g. Hergarten and Robl 2015; Hunger and McDougall 2009). Two-phase models consider the fluid and solid phases of the mixture and solve conservation equations for both the mixture and each phase (e.g. Bozhinskiy and Nazarov 200; Iverson and Denlinger 2001). The mathematical approaches developed in HEC-RAS follow a single-phase approach.

Bulking Factor

Historically, the available and practical use of non-Newtonian modeling tools has been limited in engineering practice. The way in which hyperconcentrations have been accounted for in engineer design of for example detention basins is by increasing or bulking the flow hydrograph. The Bulking Factor (BF) is computed as

$$BF = \frac{1}{1 - C_v}$$

where C_v is the sediment concentration by volume. Therefore assuming an average volume concentration of 50% leads to a BF of 2. The advantage of the Bulking Factor is its simplicity. When utilizing the Bulking Factor solely for design it is also good practice to increase the friction energy loses by increasing the bottom roughness to account for additional internal friction and increasing the turbulent eddy viscosity to account for the increased horizontal transfer of momentum.

Non-Newtonian Flow Equations

1D Saint-Venant Equations

Most clear water hydraulic models compute the boundary friction force with a quasi-empirical formula that accounts for channel roughness, like the Manning's equation (SI units):

$$Q = \frac{AR^{2/3}}{n} {S_f}^{1/2}$$

The momentum equation incorporates this force by incorporating the dimensionless friction slope (Sf):

$$\frac{\partial Q}{\partial t} + \frac{\partial QV}{\partial x} + gA\left(\frac{\partial z}{\partial x} + S_f\right) = 0$$

Where Sf comes from the Manning equation:

$$S_f = \frac{Q^2 n^2}{R^{4/3} A^2}$$

Representing empirical resisting forces as additive, dimensionless slopes allows developers to include additional forces that can collapse to one of these representative slopes. So, HEC-RAS includes unsteady contraction-expansion losses (S_{CE}) and wind forces (S_W) by including them as additive slopes in the momentum equation. Likewise, the Debris Library computes internal fluid forces in mud and debris flows as a new slope term (S_{MD}) that HEC-RAS, AdH, and GESSHA can incorporate into their momentum equation solutions:

$$\frac{\partial Q}{\partial t} + \frac{\partial (QV)}{\partial x} + gA\left(\frac{\partial z}{\partial x} + S_f + S_{CE} + S_W + S_{MD}\right) = 0$$

While the bed exerts a force on the fluid, the fluid also exerts a force on the bed. The bed shear stress is another way of describing the momentum exchange at the fluid boundary. Likewise, the internal forces can also be expressed as stresses. Thinking of these forces as stresses is useful because mud and debris flows depart from the relatively trivial stress-strain assumptions embedded in the clear water flow equations. Depending on the concentration and grain size, the Debris Library will assign a stress-strain model to the fluid and will compute internal shear stresses for the different internal resisting forces.

The library will then convert these internal shears into the mud and debris slope (S_{MD}) to integrate these resisting process in the momentum equation by back calculating the slope from the shear:

$$S_{MD} = \frac{\tau_{MD}}{\gamma_m R}$$

Therefore, the mud and debris algorithms will identify the appropriate internal forces in the fluid, identify the appropriate stress-strain model for the fluid, compute an internal shear associated with these processes (τ_{MD}), and return as single mud and debris slope (S_{MD}) that can integrate these forces into the momentum equation.

2D Shallow Water Equations

The depth-averaged Shallow Water Equations (SWE) model solves volume and momentum conservation equations and includes temporal and spatial accelerations as well as horizontal mixing while the DWE model ignores these processes but is therefore simpler and more computationally efficient. The 2D volume conservation of the water-solid mixture is given by:

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (hV) = q$$

where η is the flow surface elevation, t is time, h is the water depth, V is the velocity vector, and q is a source or sink term, to account for external and internal fluxes. The depth-averaged momentum conservation equations may be written as (Hergarten and Robl, 2015):

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V = -g\cos^2\varphi \nabla \eta + \frac{1}{h}\nabla \cdot (v_t h \nabla V) - \frac{\tau}{\rho_m R}\frac{\cos\psi}{\cos\varphi}\frac{V}{|V|}$$

in which g is the gravitational acceleration, v_t is a turbulent eddy viscosity, τ is the total basal stress, ρ_m is the water-solid mixture density, R is the hydraulic radius, $|V|$ is the magnitude of the velocity vector, φ is the water surface slope, and ψ is the inclination angle of the current velocity direction. In the above equations, the second term on the right-hand-side represents the horizontal mixing due to turbulence and also in the case of a debris flow, horizontal mixing due to particle collisions. Utilizing the conservative form of the mixing terms is essential for accurate momentum conservation. The bottom friction coefficient is computed utilizing the Manning's roughness coefficient as

$$\tau = \tau_t + \tau_{MD}$$

where τ_t is the turbulent stress and τ_{MD} is the mud and debris stress which includes all non-Newtonian stresses. The turbulence bottom shear stress is computed as a function of the Manning's roughness coefficient

$$\tau_t = \rho_m C_d |V|^2$$

$$C_d = \frac{gn^2}{R^{1/3}}$$

where ρ_m is the density water-particle mixture and n is the Manning's roughness coefficient. The mud and debris stress is described in detail in the section "Rheological Models".

When the non-Newtonian stress is equal to zero and the cosine functions (slope corrections) are removed, the above 2D SWE equations reduce to the clear-water equations utilized in HEC-RAS.

When simulating hyperconcentratedted flows, the longitudinal and transverse components of the turbulent eddy viscosity are computed with the shear velocity from total shear stress (i.e. $u_* = \sqrt{\tau/\rho_m}$). There is no existing research on the appropriate values for the turbulence coefficients for hyperconcentratedted flows. However, testing has shown that using similar values to those for clear-water produce reasonable results. This is a subject which requires further research. The current guidance is to start with "clear-water" values for the turbulence coefficients and calibrate them as best as possible with measurements.

2D Diffusion Wave Equation

HEC-RAS also includes a simplified, unsteady, hydrodynamic model, which replaces momentum with the Diffusive-Wave Equation (DWE):

$$\frac{\partial \eta}{\partial t} + \nabla \cdot (\beta \nabla \eta) = q$$

where β is a non-linear "diffusion" coefficient which is a function of the bottom friction and non-Newtonian stress

$$\beta = \cos^{1/2}\psi \cos\varphi \frac{K}{A} \frac{h}{|\nabla \eta|^{1/2}}$$

in which

$$\frac{K}{A} = \left[\frac{n^2}{(Rcos\varphi)^{4/3}} + \frac{\tau_{MD}}{\gamma_m Rcos\varphi |V|^2} \right]^{-1/2}$$

In the above equations, K is the conveyance, and A is the vertical area. The diffusion equation has been modified for steep slopes following an approach similar to that of Hergarten and Robl (2015). Again, it is noted that when the non-Newtonian stress is equal to zero and the cosine functions (slope corrections) are removed, the above DWE equation reduce to the clear-water equations utilized in HEC-RAS.

For many of the types of non-Newtonian flows the DWE may not be applicable and in fact most 2D non-Newtonian models are not based on the DWE. However, there are some types of applications where the DWE model is useful and there are some examples in literature such as Lin et al. (2011).

Classification of Non-Newtonian Flows

Non-Newtonian flows include several regimes, depending on the solid concentration of the fluid and, for higher concentration mixtures, the grain size of the solids. It is helpful to think of this classification as a hierarchy. In general, as concentration increases (and the solid component coarsens) the fluid passes through five classifications:

1. Hyperconcentrated Flow
2. Mud and debris flow
3. Clastic Flow

Dividing a continuum into a classification imposes artificial boundaries and mathematical discontinuities. Non-Newtonian flows are complicated because they do not form a continuum on a single axis. These classifications are somewhat arbitrary and the terminology in the non-Newtonian literature

The four classes of non-Newtonian flows in the Debris library, the criteria used to separate them, and the model used to simulate them are summarized in Table 2-1 and Figure 2-1.

Table 2-1: Non-Newtonian flow classifications, thresholds, and the model used to simulate them.

Classification	Model	Condition
Hyperconcentrated	Bingham	Cv>30%
Mud and Debris Flow	Turbulent- Quadratic Herschel-Bulkley	Cv>60%
Snow Avalanche	Voellmy	
Clastic	Mohr-Coulomb	Ns>0.1

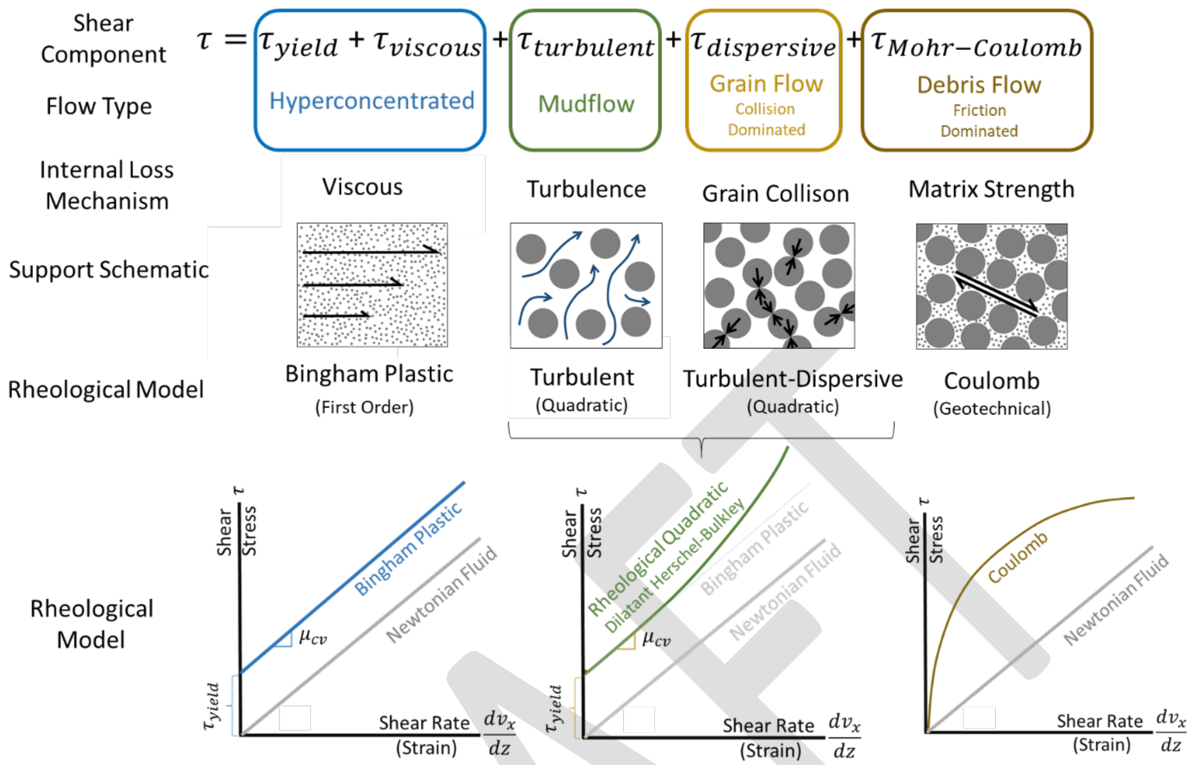


Figure 2-1: Classification, processes, conceptual model, and rheological model of the four non-Newtonian flow types in the Debris Library.

Hyperconcentrated Flows

When fine sediment concentrations (by volume) rise above about 30%, (Rickenmann, 1992) the viscosity of the mixture increases enough that the viscosity of water is no longer an appropriate approximation. The Debris library models Hyperconcentrated flows with a Bingham Plastic model.

The Bingham Plastic model has a linear stress-strain relationship like the Newtonian model, but it diverges from Newtonian assumptions in two ways. First, the Bingham model includes a yield stress. The yield stress (τ_y) introduces a non-zero intercept in the stress-strain relationship. In other words, there is a range of stress that does not deform the fluid (a range of stresses that do not induce strain). Second, while the Bingham stress-strain relationship is linear it does not have the same slope as the Newtonian fluid. The viscosity of the mixture (μ_m, which is higher than the viscosity of the fluid alone) dictates the slope of the Hyperconcentrated stress-strain relationship.

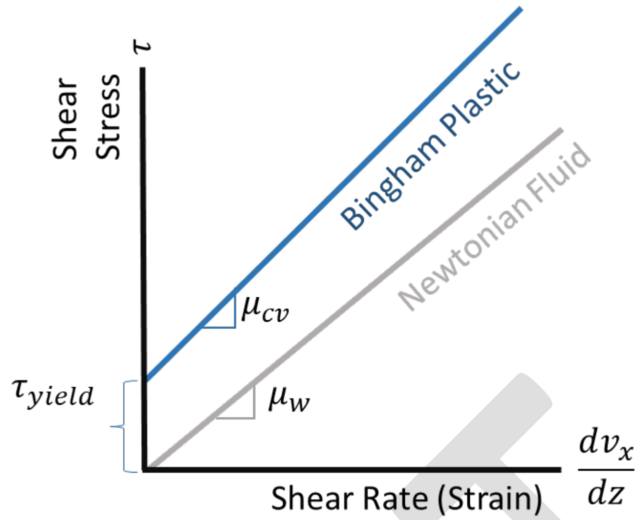


Figure 2-2: Comparison of the Bingham Plastic model with the standard Newtonian Assumptions.

Mud and Debris Flows

As concentration increases (Cv>60%), stress-strain relationship starts to depart from the linear, Bingham approximations. Non-Linear stress-strain relationships can be "dilatant" (stress rises faster than strain) or "pseudoplastic" (where strain increases faster than stress). Both mudflows and grain flows are dilatant. The Debris Library models both mudflows and grain flow stress as second order relationships with strain ($\tau = f(\dot{\gamma})^2$)), making these models "quadratic-dilatant" (Figure 2-3).

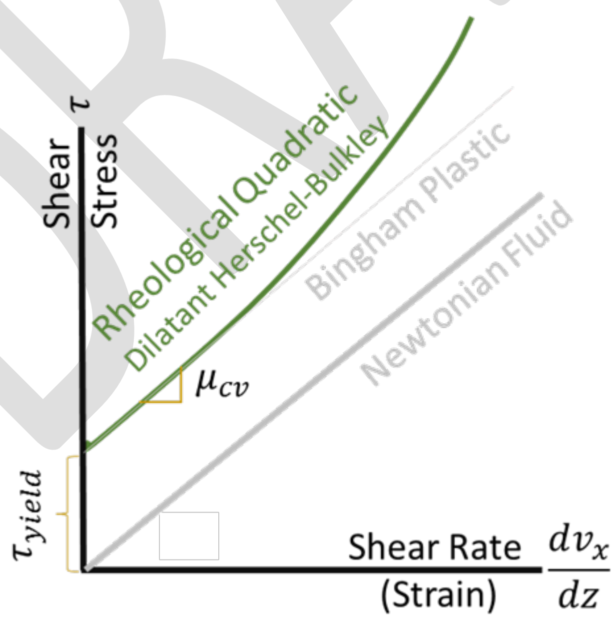


Figure 2-3: Quadratic dilatant stress-strain model applied to mudflows and grain flows (i.e. volumetric concentrations greater than 60%).

Both mudflows and grain flows have volumetric concentrations greater than 60%. At these concentrations, particle interactions become important (though they are more important for coarser particles) but the fluid is still the dominant phase, transporting the solids. These high concentration flows are distinguished primarily by the grain size of the transported materials and the grain interactions during the transport process. Both used the second-order (quadratic), dilatant, rheological model (Figure 2-3), but they include different second order terms in this relationship.

Mudflows (or turbidity currents) transport high concentrations of fine grain material. The influences of grain-to-grain collisions are not as important with these finer materials. However, at very high concentrations (C_v>60%), inter-particle turbulence introduces non-linearity into the stress-strain relationship. So, in addition to the yield and (linear) viscous shears from the hyperconentrated flows, mudflows add a non-linear turbulent shear.

Grain flows occur at the same volumetric concentrations as mudflows (Cv>60%), but transport coarser sediment. Therefore, the stress-strain relationship has to account for particle collisions, in addition to the viscous and turbulent processes. Therefore, mudflows have the same quadratic rheological behavior as Mudflows (Figure 2-3) but add an additional second order term, a dispersive stress, to the turbulent stress used for mudflows.

Debris flows have such high concentrations and, usually, large particles, that the particles are in persistent contact. The particles are no longer primarily suspended by the fluid and periodically collide. The coarse particle concentration is high enough that the fluid pushes the sediment and other large "debris" (e.g. trees and infrastructure) over other particles. Denlinger (2001) illustrates the main distinction between grain flows and debris flows in Figure 2-4. Grain flow particles are still largely suspended, making them "collision dominated" (Figure 2-4-left) while debris flows particles mostly maintain contact with each other, making them "friction dominated" (Figure 2-4 – right). Persistent, inter-particle friction dominates debris flow requires a geotechnical friction model. The Debris Library uses a Mohr-Coulomb model (Figure 2-5) to simulate these friction dominated processes.

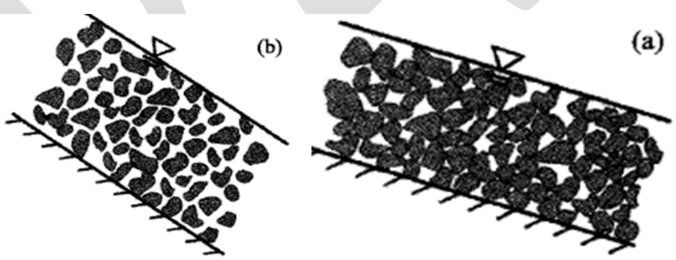


Figure 2-4: Conceptual model of the collision dominated grain flow (left) and the friction dominated debris flow (right) from Iverson and Denlinger (2001).

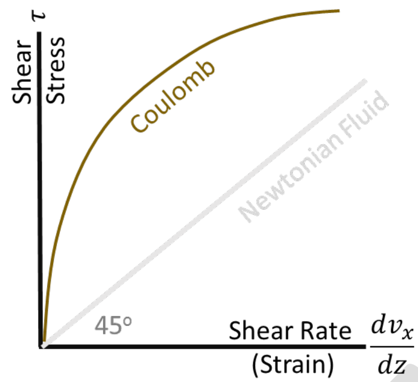


Figure 2-5: Mohr-Coulomb model used to simulate friction dominated, debris flows.

Landslides cap the upper end of the debris flow continuum. At low enough water contents and high enough volumetric solid concentrations, flow models are no longer appropriate. These events are gravity dominated, occur more rapidly, and require geotechnical failure models.

Rheological Models

Rheology is the study of mechanical properties and flow of matter, specifically non-Newtonian fluids, mixtures, and plastic solids.

Bingham

The Bingham (Bingham 1922) model is one of the simplest of the rheological models. It

$$\tau_{MD} = \tau_y + \tau_v$$

$$\tau_v = \mu_m \dot{\gamma}$$

where τ_y is the yield stress, τ_v is the viscous stress, μ_m is the mixture dynamic viscosity, and $\dot{\gamma}$ is the shear rate. This model has a linear stress-strain relationship, with a non-zero intercept. Therefore, τ_y and τ_v represent the intercept and the slope respectively of the stress-strain relationship. For stresses less than the yield stress the fluid behaves as a solid. The Bingham model is useful for simulating mudflows under low shear rates in which the yield and viscous stresses depend on the cohesion of fine sediments (Govier and Aziz 1982; Julien 1995; Julien and Leon 2000). However, the Bingham model has also been a practical model for use in simulating debris flows (Huang and Dai, 2014; Dai et al., 2014).

Quadratic

The so called Quadratic model was proposed by O'Brien and Julien (1985) and combines stresses due to: (1) cohesion, (2) internal friction between sediment and fluid, (3) turbulence, and (4) inertial impact between particles. The quadratic model may be written as

$$\tau_{MD} = \tau_y + \tau_v + \tau_d$$

$$\tau_v = \mu_m \dot{\gamma}$$

$$\tau_d = c_{Bd} \rho_s \lambda^2 d_s^2 \dot{\gamma}^2$$

where τ_d is the dispersive stress, c_{Bd} is an empirical coefficient, ρ_s is the sediment particle density, d_s is a representative particle diameter, and λ is the linear sediment concentration. The dispersive stress was originally proposed by Bagnold (1954). Bagnold (1954) and Takahashi (1980) proposed c_{Bd} = 0.01. The linear sediment concentration λ is defined by (Bagnold 1954)

$$\frac{1}{\lambda} = \left(\frac{C_{max}}{C_v}\right)^{1/3} - 1$$

in which C_v is the sediment concentration by volume and C_{max} is the maximum sediment concentration. An example of the dispersive stress as a function of concentration and shear rate is shown in the figure below. The formulation shows a sharp increase as the concentration approaches the maximum concentration.

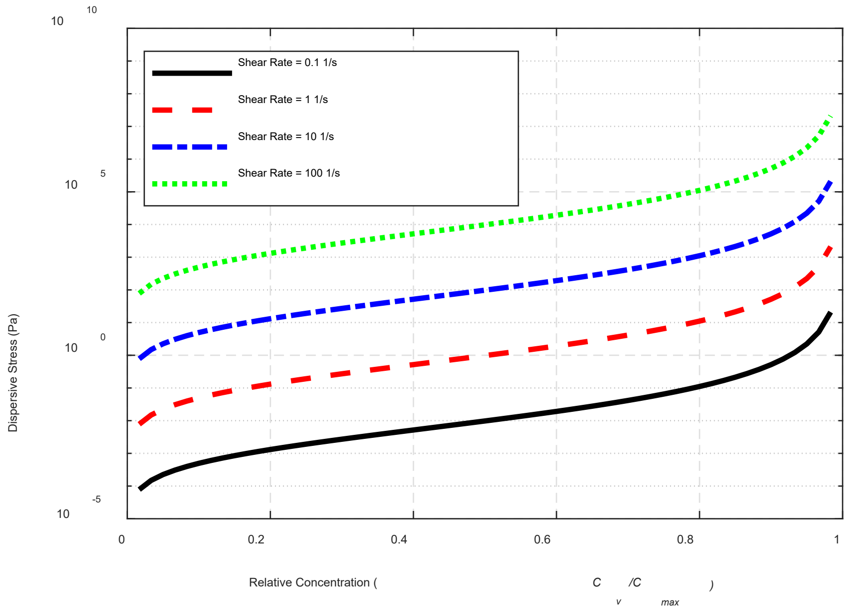


Figure 2-6: Dispersive stress based on Bagnold (1954) with d_s = 30 mm and c_{Bd} = 0.01.

Herschel-Bulkley

In the Bingham rheological resistance model, the relationship between shear rate and shear stress is linear. However experiment have shown that debris-flow mixtures can have non-linear relationships (Major and Pierson 1992; Jeffrey et al. 2001). A more general model which allows for this nonlinearity is the Herschel-Bulkley (HB) model:

$$\tau_{MD} = \tau_y + \tau_v$$

$$\tau_v = K\dot{\gamma}^n$$

where K is the consistency factor or index, and n is the power index or exponent. When $n < 1$ the fluid/mixture is shear-thinning and when $n > 1$ the fluid/mixture is shear thickening. The as with other rheological models, when the stress is less than the yield stress, the fluid/mixture behaves as a solid. One issue with the HB model is that the consistency factor as dimensional units which are a function of the power index. This makes estimating the parameter somewhat difficult. The HB model has been shown to work well for suspensions of fine sediments under high shear rates (Govier and Aziz, 1982).

Voellmy

The Voellmy resistance model combines a yield stress with a viscous/turbulent stress as (Voellmy 1955)

$$\tau_{MD} = \tau_y + \tau_{vd}$$

$$\tau_{vd} = \frac{\rho_m g |V|^2}{\xi}$$

where ξ is the Voellmy turbulence coefficient. Voellmy original proposed the formulation to simulate snow avalanches but it has since also been applied to simulate mud slides, debris flows, and rock avalanches (e.g. Hergarten and Robl 2015; Hungr and Mcdougall 2009; Körner 1976; Perla et al. 1980; Rickenmann and Koch 1997; Hussin et al. 2012). The Voellmy coefficient ξ is similar to a Chezy coefficient and has units of L/T^2. Common ranges for the coefficient are from 150 to 600 m/s^2. In the Voellmy model, the yield stress is typically computed with the Mohr-Coulomb yield stress with the cohesion set to zero.

Yield Stress

A Bingham plastic can absorb some stress without deforming the material. Deformation (strain) only occurs after stress exceeds a minimum threshold. That minimum threshold required before stress causes strain, is the yield stress (τ_y), which is the intercept of the stress-strain relationship. HEC-RAS provides three methods for yield stress:

1. User-specified constant
2. Exponential formulation
3. Mohr-Coulomb formula

Exponential

A widely used formula to estimate the yield stress is the exponential formulation (Chien and Ma 1958; Dai et al. 1980; O'Brien and Julien 1988)

$$\tau_y = a e^{b C_v}$$

where a and b are calibration coefficients, and C_v volumetric concentration between 0 and 1.

Table 2-2: Yield stress parameters for the Exponential equation from Julian (1995)

Material	a (Pa)	b
"Typical soil"	0.005	7.5
Kaolinite	0.05	9
Sensitive Clays	0.03	10
Bentonite	0.002	100

The exponential equation works relatively well for hyperconcentrated flows with concentrations between 5% and 30%. However, for high concentrations or very low concentrations the formulation does not work as well. For example a zero concentration produces a yield stress equal to the coefficient a and does not go to zero as it should theoretically.

Mohr-Coulomb

The Mohr-Coulomb yield stress model is given by

$$\tau_y = c + \mu\sigma$$

$$\sigma = (\rho_m - \rho_w) g h \cos^2\theta$$

$$\mu = \tan\phi$$

where c is the cohesion or cohesive strength, μ is the Coulomb friction coefficient, σ is the normal stress at the bottom of the mixture, θ is the bed slope angle, h is the vertical flow depth, and ϕ is the internal friction angle. The normal stress is computed assuming the flow is parallel to the bed as in Hergarten and Robl (2015). In addition the mixture is assumed to be fully saturated. The internal friction angle depends on mixture but its values are typically between 2.5º and 15º.

Mixture Density

The density of the water-sediment mixture is calculated with the following constitutive equation:

$$\rho_m = \rho_w + (\rho_s - \rho_w) C_v$$

where ρ_w is the water density, ρ_s is the particle density, and C_v volumetric concentration between 0 and 1. The above equation assumes that all of the voids between the particles are occupied water and that there is no air in the mixture.

Mixture Dynamic Viscosity

Sediment increases the viscosity of the flow mixture. There are many empirical and semi-empirical equations in literature to compute the viscosity of the mixture. HEC-RAS provides four ways of specifying the dynamic viscosity:

1. User-specified constant
2. User-specified ratio or relative viscosity
3. Exponential
4. Maron and Pierce (1956)

A comparison of the exponential (O'Brien et al., 1993) and Maron and Pierce (1956) formulas is shown in the figure below.

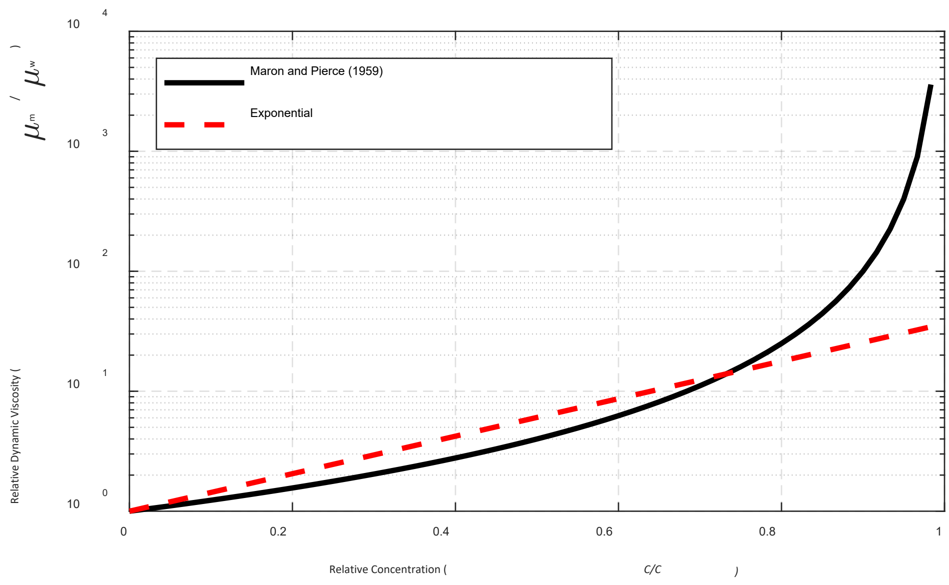


Figure 2-7: Dynamic viscosity based on the Maron and Pierce (1956) and O'Brien et al. (1993) formulas (β = 6).

Ratio

The ratio method in HEC-RAS basically specifies the relative dynamic viscosity. HEC-RAS computes the dynamic viscosity of the mixture as the water viscosity times the user-specified ratio. The water viscosity is computed internally based on the water temperature. The figure below shows the water dynamic viscosity as a function of temperature. Since the ratio is held constant and does not change with concentration or any other factors, the method is only recommended for simulations with constant concentration.

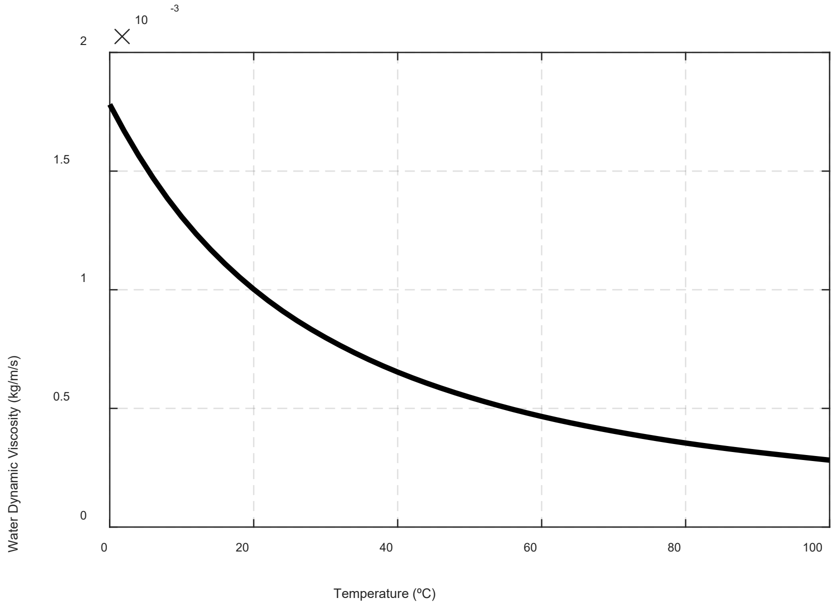


Figure 2-8: Water dynamic viscosity based on the table provided by Venard and Street (1975).

Exponential

A commonly used formulation for the mixture dynamic viscosity is the exponential expression (Chien and Ma 1958; Dai et al. 1980; O'Brien and Julien 1988). The formula is usually written as a two-parameter expression. However, here a simpler form is adopted to compute the relative dynamic viscosity as

$$\mu_r = \frac{\mu_m}{\mu_w} = e^{\beta C_v}$$

where μ_r is the relative mixture dynamic viscosity, μ_w is the water dynamic viscosity, μ_m is the mixture dynamic viscosity, C_v is again the volume concentration, and β is a coefficient fit to observed data and provided by the user. The advantage of the above formulation is that it only requires one empirical parameter and also satisfies the property $\mu_r = 1$ for C_v = 0. In addition, by using the relative dynamic viscosity, the formula automatically takes into account the variation in water viscosity due to temperature. The variability in the coefficient β accounts for the effects of particle size distribution and, in particular, the cohesion of the sediment. One limitation of the formula however is that the viscosity tends to be underestimated as the concentration reaches the maximum concentration.

Table 2-3: Sediment laden viscosity parameters for the Exponetial equation from Julien (1995)

Material	β
"Typical soil"	8
Kaolinite	8
Sensitive Clays	5
Bentonite	100

Maron and Pierce

Maron and Pierce (1956) proposed the following simple empirical expression for the relative dynamic viscosity

$$\mu_r = \frac{\mu_m}{\mu_w} = \left(1 - \frac{C_v}{C_{max}}\right)^{-2}$$

where C_{max} is the maximum concentration by volume (maximum packing volume fraction). The maximum concentration is the concentration where enough particles have been added for the mixture to behave as a solid. Li (2004) and Guazzelli and Pouliquen (2018) compared various experimental datasets of viscosities of suspensions and found that the above formulation fits a wide range of experiments relatively well for a wide range of concentrations. Another advantage of the formulation has the advantage is that it does not require additional calibration parameters as does the O'Brien equation. The formulation is a function of the maximum concentration; however this variable is a physical property of mixture which can be more readily measures or estimated and does not have such a large range of values as does the β coefficient in the O'Brien formulation. The maximum concentration is a function of the sediment size distribution, particle deformability, and the local flow conditions. In practice however, an approximate maximum concentration may be estimated as $C_{max} = 1 - p_m^{'}$ in which $p_m^{'}$ is the bed porosity. An example of the maximum concentration calculated from the proposed formula for bed porosity by Wooster et al. (2008) is shown in the Figure below. Natural sediments typically have porosities between 0.3 and 0.46.

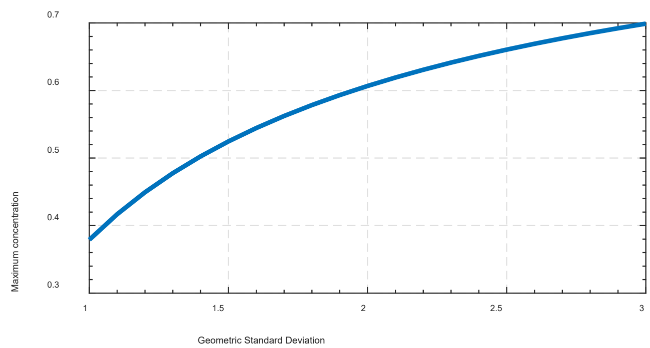


Figure 2-9: Maximum concentration as a function of the geometric standard deviation (based on the porosity equation by Wooster et al. (2008).

Shear Rate

The shear rate is an important variable which is utilized in the rheological models. Vertically averaged models - such as those in HEC-RAS – make vertical velocity profile assumptions in order to estimate a vertically averaged shear rate.

The two options in HEC-RAS include:

(1) Linear (e.g. Bird et al. 1960), and

(2) Parabolic (e.g. Julien 1995; Iverson and Denlinger 2001).

The general formula is:

$$\dot{\gamma} = \frac{B|V|}{h\cos\varphi\cos\psi}$$

Current versions of HEC-RAS simplify this to:

$$\dot{\gamma} = \frac{3|V|}{h}$$

where $|V|$ is the current velocity magnitude, h is the flow depth, φ is the water surface slope, and ψ is the inclination angle of the current velocity direction. The coefficient B is equal to 2 and 3 for the linear and parabolic profiles, respectively. The parabolic velocity profile was original used in the Quadratic model (O'Brien and Julien 1985). The Voellmy model does not require a shear rate and only utilizes the average velocity for the turbulent-dispersive stress.

References

Bagnold, R.A 1954. Experiments on a gravity-free dispersion of large solid spheres in a Newtonian fluid under shear. Proc. Royal Soc. Of London, Series A, 225, 49–63.

Bingham, E.C., 1922. Fluidity and plasticity, McGraw-Hill, New York, 439.

Bird, R.B., Stewart, W.E., and Lightfoot, E.N. 1960 Transport phenomena, John Wiley, New York, 780 pp.

Bozhinskiy, A.N., and Nazarov, A.N. 2000. Two-phase model of debris flow, in: Proc. 2nd Int. Conf. on Debris-Flow Hazards Mitigation: Mechanics, Prediction, and Assessment, Taipei, Taiwan, edited by: Wieczorek, G. F. and Naeser, N. D., Balkema, Rotterdam, 263–269.

Chien, N., and Mai, H. 1958. Properties of slurry flow. J. Sediment Res. 3(3), Beijing, China (in Chinese).

Dai, Z., Huang, Y., Cheng, H., and Xu, Q., 2014. 3D numerical modeling using smoothed particle hydrodynamics of flow-like landslide propagation triggered by the 2008 Wenchuan earthquake. Eng. Geol. 180, 21–33.

Govier, G.W. and Aziz, K. 1982. The flow of complex mixtures in pipes. Krieger Publishing Co. Melbourne, FL.

Guazzelli, E., and Pouliquen, O. 2018. Rheology of dense granular suspensions, J. Fluid Mech. 852.

Hergarten, S., and Robl, J. 2015. Modelling rapid mass movements using the shallow water equations in Cartesian coordinates. Nat. Hazards Earth Syst. Sci. 15 671–685.

Huang, Y., and Dai, Z. 2014. Large deformation and failure simulations for geo-disasters using smoothed particle hydrodynamics method. Eng. Geol. 168 (1), 86–97.

Hunger, O., and McDougall, S. 2009. Two numerical models for landslide dynamic analysis. Computers & Geosciences, 35, 978–992.

Hussin, H.Y., Luna, B.Q., van Westen, C.J., Christen, M., Malet, J.-P., and van Asch, Th.W.J, 2012. Nat. Hazards Earth Syst. Sci., 12, 3075–3090.

Iverson, R.M., and Denlinger, R.P. 2001. Flow of variably fluidized granular masses across three-dimensional terrain 1. Coulomb mixture theory. J. Geophys. Res., 106(B1), 537–552.

Jeffrey, D.P., Kelin, X.W., Alessandro, S., 2001. Experimental study of the grain-flow, fluid-mud transition in debris flows. J. Geol., 109, 427–447.

Julien, P.Y. 1995. Erosion and Sedimentation, Cambridge University Press, 280 p.

Julien, P.Y., and Lan, Y.Q. 1991. On the rheology of hyperconcentrations, J. Hydr. Eng. ASCE, 117(3), 346–353.

Julien, P.Y., and Leon, 2000. Mudfloods, mudflows, and debris flow: classification, rheology and structural design, International Seminar on the Debris Flow Disaster of December 1999. Caracas, Venezuela.

Körner, H.J., 1976. Reichweite und Geschwindigkeitvon Bergstürzen und Flieβschneelawinen. (Reach and velocity of rockslides and powder snow avalanches). Rock Mechanics, 8, 225–256.

Li, S. 2004. A generalized mixture rule for estimating the viscosity of solid-liquid suspensions and mechanical properties of polyphase rocks and composite materials. J. Geophys. Res. 109(B10207), 1–18.

Lin, P.-S., Lee, J.-H., and Chang, C.-W. 2011. An application of the Flo-2D model to debris-flow simulation – A case study of Song-Her District in Taiwan. Italian Journal of Engineering Geology and Environment, 947–956.

Major, J.J., Pierson, T.C., 1992. Debris flow rheology: experimental analysis of fine grained slurries. Water Resour. Res. 28(3), 841–857.

Maron, S.H., and Pierce, P.E. 1956. Application of Ree-Eyring generalized flow theory to suspensions of spherical particles, J. Colloid Sci., 11, 80–90.

O'Brien, J.S., and Julien, P.Y 1988. Physical properties and mechanics of Hyperconcentrated sediment flows, Proc. ASCE Hyd. Deiv. Spec. Conf. Delineation of Landslides, Flash Flood, and Debris Flow Hazards, Logan Utah, June 1984, 260–279.

Perla, R., Cheng, T.T., McClung, D.M. 1980. A two-parameter model of snow-avalanche motion. Journal of Glaciology, 26, 197–207.

Rickenmann, D., and Koch, T. 1997. Comparison of debris flow modelling approaches. In: Chen, C.L. (Ed.) ,Proceedings of the 1stInternational Conference on Debris-flow Hazards Mitigation: Mechanics, Prediction and Assessment. American Society of Civil Engineers, New York, NY, 576–585 pp.

Takahashi, T. 1980. Debris flow on prismatic open channel. J. Hydr. Div., ASCE, 106(3), 381–396.

Venard, J.K., and Street, R.L. 1975. Elementary Fluid Mechanics, 5th ed., Wiley, New York.

Voellmy, A. 1955. Uber die Zerstörungskraft von Lawinen. Schweiz. Bauztg, 73, 212–217.

Wooster, J.K., Dusterhoff, S.R., Cui, Y., Sklar, L.S., Dietrich, W.E., and Malko, M. 2008. Sediment supply and relative size distribution effects on fine sediment infiltration into immobile gravels, Water Resources Research, 44, W03424.